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Stochastic Solid Transportation Problem Multi- Objective Multi-Item by using Gamma Distribution

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Abstract:

In this research paper, the complex problem of Multi-objective Multi-item Stochastic Solid Transportation Problem (MOMSSTP) with random parameters as stochastic nature. Traditional transportation models often fail to capture the intricacies of modern logistical operations, where multiple conflicting objectives, diverse item types and uncertain conditions are prevalent. Our study formulates an advanced MOMSSTP framework that incorporates these factors, aiming to optimize cost of transportation and time of transportation. Stochastic elements are introduced to account for uncertainties in origin's supply, conveyance's capacity and destination's demand which making the model robust and adaptable to variable operational conditions. On the basis of chance constraint programming suggest an approach to tackle the multi-objective multi-item stochastic solid transportation problem with random parameters characterized through gamma distribution and also used fuzzy programming approach for optimization. The effectiveness of our approach is demonstrating through a numerical example.

KEYWORDS: *Multi-Objective Multi-Item Stochastic Solid Transportation Problem (MOMSSTP), Global Criteria Method, Chance Constraint Programming, Fuzzy Programming, Gamma Distribution.*

1. INTRODUCTION

The efficient transportation of goods is a critical aspect of supply chain management, influencing both cost efficiency and service quality. Traditional TP's have been extensively studied, typically focusing on minimizing costs while transporting homogeneous products from multiple sources to various destinations. Firstly, Hitchcock [1] in 19141 introduced the classical transportation problem, a particular type of LPP (linear programming problem. In this classical transportation problem, homogeneous goods are moved from various sources to several destination with the aim of minimizing transportation costs. However, in real world scenarios, the complexity increases with the need to consider multiple objectives, multiple items and uncertainties in supply, demand and transportation capacities. Also, referred to as the 3-dimesional transportation problem, is ideal for these scenarios because it accounts for three key constraints. Haley [2] expanded the modified distribution approach by providing a solution procedure for solid TP. [3] . [4] Applies fuzzy linear programming to tackle the multi-objective solid TP, offering optimal solutions and comparing its effectiveness to other methods using a Fortran-based implementation. [5] presented a multi-objective approach to goal programming that simplifies the process by eliminating extra constraints and weight factors, while also effectively handing non convex trade off regions. [6] modified techniques to adjust cost's coefficients in objective functions with multi- choice goals for binary variables. [7] used expected value operator to obtain crisp values and obtained compromised solution by applying fuzzy programming technique & global criteria method. [8]

suggested an approach to obtain compromised solution by using global criteria, fuzzy interactive satisfied method & convex combination method. [9] suggested a methodology for converting unbalanced MOMIST issue to balanced problem. [10] investigate solid TP with fixed prices in uncertain nature. [11] proposed a methodology to solve multi-objective solid TP under uncertainty on the basis of chance constrained programming, also proposed initial feasible condition and extended fuzzy programming. [12] under fuzzy environment, five new methods to defuzzied the fuzzy models. [13] the single goal solid TP has been extensively studied by a number of researchers. [14] applied expected value operator to transformed intuitionistic fuzzy multi-stage multi-objective fixed price solid TP under green supply chain to deterministic form. [15] proposed an approach to optimize MOMST problem under uncertainty. [16] suggested a new approach for obtaining POS results of fixed charge STP under intuitionistic nature. [17] discussed three models, optimistic value model, dependent optimistic constrained model and expected-value model. [18] proposed a new method to optimize unbalanced fully rough multiobjective fixed charge transportation problem. [19] suggested an approach for finding the deterministic ideal allocation of trucks fleet in Egypt private sector company. [20] proposed a methodology to obtain optimal results of sustainable multi-objective 4-dimensional MIST problem under triangular intuitionistic fuzzy environment. [21], [22] proposed a LP (linear programming) method to optimized the storage's capacity. [23] suggested an algorithm to tackle the solid stochastic TP in fuzzy environment. [24] obtained POS by proposed a new technique for optimize fuzzy bi- objective fixed charge multi-index TP. In this research, we introduced the mathematical model for a MOMSSTP where the origin's supply, conveyance's capacity and destination's demand follow gamma's distribution. Additionally, developed an effective technique for optimizing the proposed problem. The research framework is structured as follows: section 2 presented models for MOMSSTP with gamma's distribution & created a comparable crisp model for the issue. Section 3 discussed the fuzzy's programming approach for MOMSSTP model. A numerical issue is solved and discussed results in section 4. Finally, provides a summary of the study's conclusions and identify the future scope in section 5.

2. Mathematical Model For MOMSSTP

This section includes formulation of MOMSSTP and formulation of multi-objective multi-item stochastic solid transportation problem.

2.1. Formulation For Multi-Objective Multiple Items STP

A STP extends the traditional 2-D transportation problem by incorporating multiple modes of transport for moving multiple items from various production sites to multiple destinations. STP includes three set constraints: supply limits from sources, demand requirements at destinations and varying capacities of transportation modes. The primary goal in STP is to create an effective transport plan that minimizes costs while meeting all constraints. In real life scenarios, it often involves optimizing multiple objectives simultaneously rather than focusing on single objective. Also, these parameters such as origin's supply, destination's demand & conveyance's capacity are often imprecise and uncertain due to factors such as fluctuation in raw material availability, change in customer preferences, availability and reliability of different transport modes and limited information. Set theory of random variables is used to handle such uncertainties. Therefore, a model which is in mathematical form is developed for a multi-objective multiple items STP that incorporates random origin's supply, conveyance's capacity & destination's demand.

Consider there are multiple items q transported from origin a_i^q to destination b_j^q through different conveyance e_k^q. In this scenario, the decision maker aims to finding the optimal transportation strategy by including number of objective functions which is denoted as Z_s. Also, assuming that all parameters are random variable except cost of transportation. To formulate the mathematical model for problem, the following parameters and variables are defined as below:

i: number of sources $(i = 1,2, \dots, m)$.
j: number of destinations $(j = 1,2, \dots, n)$.
k: number of conveyance modes $(k = 1,2, \dots, K)$.

q: number of items $(q = 1,2, \dots, l)$.
x_{ijk}^q: number of q^{th} items transported to destination j from sources i by using conveyance k.
c_{ijk}^{qs}: cost of s^{th}objective function for transportation of one unit of q^{th} items transported to destination j from sources i by using k^{th} conveyance.
a_i^q: amount of q^{th} items available at source i.
b_j^q: demand for q^{th} items at destination j.
e_k: capacity of k^{th}conveyance mode.
Z_s: s^{th} number of objective functions $(s = 1,2, \dots, S)$.

Problem for MOMSTP can be formulated by using above definitions as follows:

$$\text{Minimize } Z_s = \sum_{q=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} c_{ijk}^{qs} x_{ijk}^{q}, \quad (1)$$

Subject to constraints

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^{q} \le a_i^q, \; i = 1,2, \dots, m; q = 1,2, \dots, l, \quad (2)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^{q} \ge b_j^q, \; j = 1,2, \dots, n; q = 1,2, \dots, l, \quad (3)$$

$$\sum_{q=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^{q} \le e_k, \; k = 1,2, \dots, K, \quad (4)$$

$$x_{ijk}^{q} \ge 0 \; \forall \; i, j, k \text{ and } q, \quad (5)$$

Here, uncertain parameters follow Gamma's distribution which is widely used in continuous distributions for modelling the uncertainties in destination's demands, origin's supply and conveyance's capacities over time in STP. Specifically, it captures scenarios where the destination's demands for certain products is initially high and gradually decreases, reflecting real-world patterns such as monthly consumption trends. This is particularly useful when the demand for commodities like rice. The gamma distribution's flexibility in handling different shapes and scales allows for a more accurate representation of these dynamic changes. By incorporating gamma distributed variables, the STP can account for fluctuation and provide a robust framework for optimizing transportation strategies, ensuring that origin's supplies and conveyance's capacities are adjusted appropriately to meet varying demand efficiently. In problem (1-5), variables which assumed as random form are with independent gamma's distributions have distinct shape parameters and scale parameter remains same with known mean and variance. TP is unbalanced because of uncertainty. There are 2 feasible situations for an unbalanced STP. First is total supply of each item q from all sources should be maximum or equal to the total destination's demand for that items across all destinations and second is total conveyance's capacities should be maximum than or equal to the total destination's demand for each item. These situations are mathematically indicating as follows:

$$\sum_{i=1}^{m} a_i^q \geq \sum_{j=1}^{n} b_j^q \;\; \forall q = 1, 2, \dots, l \tag{6}$$

$$\sum_{k=1}^{K} e_k \geq \sum_{q=1} \sum_{j=1}^{n} b_j^q. \tag{7}$$

When the parameters are crisp then inequality relations are well defined. But all the parameters in our situation is random. Consequently, these standards ordered relation do not applied. In next section we address these challenges by proposing novel feasibility situations for the problem.

2.2. Formulation of Equivalent Crisp Model

We develop a deterministic model which is equivalent to the stochastic problem. For this we present the initial feasibility situations for the problem. Initial feasibility situations represented in eq. (6) and (7) for unbalanced STP with multiple items. Following postulates outlines, if one of these parameters are stochastic in nature then follows gamma's distribution.

Postulate 1: If the origin's supply parameter a_i^q of a STP follow gamma distribution (α_i^q, β) for i=1, 2, …, m, then the feasibility situations of the problem are defined as

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)} \left(\sum_{h=0}^{\sum_{q=1}^{l}\sum_{j=1}^{n}\alpha_i^q - 1} \left(\frac{-\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)^h \frac{1}{h!}\right) \geq 1 - \gamma', \tag{8}$$

and $\sum_{k=1}^{K} e_k \geq \sum_{q=1} \sum_{j=1}^{n} b_j^q$, (9)

here γ' representing the level of significance and remaining symbols are defined above.

For an unbalanced STP, the total supply and capacity of conveyance should be maximum than or equal to the total demand for each item. Hence, for deterministic STP feasibility conditions are

$$\sum_{i=1}^{m} a_i^q \geq \sum_{j=1}^{n} b_j^q \;\; \forall q, \;\; \sum_{k=1}^{K} e_k \geq \sum_{q=1}^{l} \sum_{j=1}^{n} b_j^q.$$

In this case only supply parameter a_i^q are not deterministic. With known mean, variances and same scale that is $a_i^q \sim$gamma (α_i^q, β), follow independent gamma's distribution. To determine the deterministic form of first feasible situation by apply chance constraint programming. Therefore, the first situation is:

$$Pr\left(\sum_{i=1}^{m} a_i^q \geq \sum_{j=1}^{n} b_j^q\right) \geq 1 - \gamma', \tag{10}$$

Here, γ' represented the significance level, for example $\gamma' = 0.01$, it implies 99% surety that the total supply of each item q from all sources will be maximum than total destination's demands for those items across all destinations. The sum of independent gamma's distribution along common scale parameter follows gamma's distribution it is already known. Therefore, supply $S = \sum_{q=1}^{l} \sum_{i=1}^{m} a_i^q$ follow gamma's distribution along shape parameter $\alpha = \sum_{q=1}^{l} \sum_{i=1}^{m} \alpha_i^q$ and scale parameter β. Consequently, the feasibility condition in equation (10) becomes

$$\left(\int_{\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}^{\infty} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} S^{\alpha-1} e^{\left(\frac{-S}{\beta}\right)} dS\right) \geq 1 - \gamma_i^q, \tag{11}$$

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)} \left(\sum_{h=0}^{\alpha-1} \left(\frac{\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)^h \frac{1}{h!}\right) \geq 1 - \gamma' \tag{12}$$

Hence, this is the deterministic form of first feasibility condition in equation (9)

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)} \left(\sum_{h=0}^{\sum_{q=1}^{l}\sum_{i=1}^{m}\alpha_i^q - 1} \left(\frac{\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta}\right)^h \frac{1}{h!}\right) \geq 1 - \gamma'.$$

Postulate 2: If the demand parameter b_j^q of a STP follow gamma distribution $(\alpha_j^{q\prime}, \beta')$ for j=1, 2, …, n, then the feasibility situations of the problem are defined as

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{i=1}^{m} a_i^q}{\beta'}\right)} \left(\sum_{h=0}^{\sum_{q=1}^{l}\sum_{j=1}^{n}(\alpha_j^q)'-1} \left(\frac{\sum_{q=1}^{l}\sum_{i=1}^{m} a_i^q}{\beta'}\right)^h \frac{1}{h!}\right) \geq 1-\eta_1', \tag{13}$$

and
$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{k=1}^{K} e_k}{\beta'}\right)} \left(\sum_{h=0}^{\sum_{q=1}^{l}\sum_{j=1}^{n}(\alpha_j^q)'-1} \left(\frac{\sum_{q=1}^{l}\sum_{k=1}^{K} e_k}{\beta'}\right)^h \frac{1}{h!}\right) \geq 1-\eta_2', \tag{14}$$

here η_1' and η_2' representing the significance's level.

Postulate 3: If the parameter, conveyance's capacities e_k of a STP follow gamma's distribution (α_k'', β'') for k=1, 2, …, K, then the feasibility situations of the issue are defined as

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta''}\right)} \left(\sum_{h=0}^{\sum_{k=1}^{K}(\alpha_k)''-1} \left(\frac{\sum_{q=1}^{l}\sum_{j=1}^{n} b_j^q}{\beta''}\right)^h \frac{1}{h!}\right) \geq 1-\xi', \tag{15}$$

and
$$\sum_{i=1}^{n} a_i^q \geq \sum_{j=1}^{n} b_j^q \tag{16}$$

here ξ' representing the level of significance.

In equations 1 to 5, parameters in right hand side of the constraints are in the form of random variables which represented the model of multi-objective stochastic problem. This model is converted into deterministic form through eliminating the randomness from the parameters for optimization. For handling these stochastic parameters, apply CCP (chance constraint programming). This technique permitting the constraints to be violated up to specified probability level. By applying CCP to all uncertain constraints, we obtain the following model:

$$\text{Minimize } Z_s = \sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^{qs} x_{ijk}^q, \tag{17}$$

Subject to constraints

$$Pr\left(\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^q \leq a_i^q\right) \geq 1-\gamma_i^q,\ i=1,2,\dots,m;\ q=1,2,\dots,l, \tag{18}$$

$$Pr\left(\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^q \geq b_j^q\right) \geq 1-\eta_j^q,\ j=1,2,\dots,n;\ q=1,2,\dots,l, \tag{19}$$

$$Pr\left(\sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^q \leq e_k\right) \geq 1-\xi_k,\ k=1,2,\dots,K, \tag{20}$$

$$x_{ijk}^q \geq 0\ \forall\ i,j,k \text{ and } q \tag{21}$$

Here, $(1-\gamma_i^q)$, $(1-\eta_j^q)$, and $(1-\xi_k)$ represents the probability levels given by the DM as suitable safety margins. Any vector x belongs to $\mathbb{R}^n$ maximum than or equal to zero is considered a feasible solution if it satisfies all constraints (18) to (20). It is deemed a pareto optimal solution if there $\nexists$ a feasible x' such that $Z_s(x') \leq Z_s(x)$ and at least one objective $\exists$ for which the inequality is strictly applicable.

Uncertainty in the models arises from various factors and doesn't always include all parameters as random variables. Therefore, the problem is categorized into distinct models based on specific scenarios:

(i) Only supply parameter with multiple items $a_i^q (i=1,2,\dots,m), (q=1,2,\dots,l)$ follows gamma distribution.

(ii) Only demand parameter with multiple items $b_j^q (j=1,2,\dots,n), (q=1,2,\dots,l)$ follows gamma distribution.

(iii) Only capacity of conveyance $e_k (k=1,2,\dots,K)$ follows gamma distribution.

(iv) Supply $a_i^q (i=1,2,\dots,m), (q=1,2,\dots,l)$, demand $b_j^q (j=1,2,\dots,n), (q=1,2,\dots,l)$ and conveyance capacity $e_k (k=1,2,\dots,K)$ follows gamma distribution.

The equivalent crisp model varies based on the nature of the parameters, which can be either deterministic or random. If we assuming the parameters follows gamma's distribution with shape parameters greater than one, the crisp form becomes nonlinear. The four identified models identified are structured so that the first 3 models are subsets of fourth model. This means that by treating some parameters as crisp, the first 3 models can obtain from the fourth. When all parameters of a chance- constraint are crisp, the probability of satisfying the constraint is one. Therefore, the crisp model for the fourth model is discussed. To tackle this problem using traditional methods, deterministic equivalents to the chance-constraints are required, but this approach is complex and often only feasible in special cases. The following postulates present the crisp equivalent for chance constraints with gamma- distributed random variables.

Postulate 4: Assuming that the supply parameters $a_i^q (i = 1,2, \dots, m), (q = 1,2, \dots, l)$ are independent gamma's random variables, then $Pr\left(\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^q \le a_i^q\right) \ge 1 - \gamma_i^q$ iff

$$e^{\left(\frac{-\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^q}{\beta}\right)} \left(\sum_{h=0}^{\alpha_i^q - 1} \left(\frac{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^q}{\beta}\right)^h \frac{1}{h!}\right) \ge (1 - \gamma_i^q), \tag{22}$$

Here gamma distribution's the shape & scale parameters denoted as α_i^q & β corresponding to $a_i^q (i = 1,2, \dots, m), (q = 1,2, \dots, l)$.

Proof: Gamma distribution's Pdf (probability density function) with shape & scale parameter as α_i^q & β respectively corresponding to a_i^q is

$$f(a_i^q) = \frac{1}{\Gamma(\alpha_i^q)\beta^{\alpha_i^q}} (a_i^q)^{(\alpha_i^q - 1)} e^{\left(\frac{-a_i^q}{\beta}\right)}, 0 < a_i^q < \infty, \ a_i^q, \beta \in \mathbb{R}^+, i = 1,2, \dots m.$$

Let $g_i = \sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk}^q$, then in equation (18) becomes

$$Pr\ (g_i \le a_i^q) \ge 1 - \gamma_i^q, \tag{23}$$

$$\left(\int_{g_i}^{\infty} \frac{1}{\Gamma(\alpha_i^q)\beta^{\alpha_i^q}} (a_i^q)^{(\alpha_i^q - 1)} e^{\left(\frac{-a_i^q}{\beta}\right)} d\alpha_i^q\right) \ge 1 - \gamma_i^q, \tag{24}$$

$$e^{\left(\frac{-\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^q}{\beta}\right)} \left(\sum_{h=0}^{\alpha_i^q - 1} \left(\frac{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^q}{\beta}\right)^h \frac{1}{h!}\right) \ge (1 - \gamma_i^q) \tag{25}$$

So, equivalently deterministic of the chance-constraint in equation (22). Hence, postulate is proved.

Postulate 5: Assuming that the demand parameters $b_j^q (j = 1,2, \dots, n), (q = 1,2, \dots, l)$ are independent gamma's random variables, then $Pr\left(\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^q \ge b_j^q\right) \ge 1 - \eta_j^q$ iff

$$e^{\left(\frac{-\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^q}{\beta'}\right)} \left(\sum_{h=0}^{(\alpha_j^q)' - 1} \left(\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^q}{\beta'}\right)^h \frac{1}{h!}\right) \ge \eta_j^q, \tag{26}$$

Here gamma distribution's the shape & scale parameters denoted as $(\alpha_j^q)'$ & β' corresponding to $b_j^q (j = 1,2, \dots, n), (q = 1,2, \dots, l)$.

Proof: Gamma distribution's Pdf corresponding to b_j^q is

$$f(b_j^q) = \frac{1}{\Gamma(\alpha_j^q)'(\beta')^{(\alpha_j^q)'}} (b_j^q)^{[(\alpha_j^q)' - 1]} e^{\left(\frac{-b_j^q}{\beta'}\right)}, 0 < b_j^q < \infty, j = 1,2, \dots n.$$

Let $w_j = \sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk}^q$, then in equation (19) becomes

$$Pr\ (w_j \le b_j^q) \ge \eta_j^q,$$

Through integration we get, the equivalent deterministic form

$$e^{\left(\frac{-\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^q}{\beta'}\right)} \left(\sum_{h=0}^{(\alpha_j^q)' - 1} \left(\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^q}{\beta'}\right)^h \frac{1}{h!}\right) \ge \eta_j^q. \tag{27}$$

So, equivalently crisp of the chance-constraint in equation (27). Hence, postulate is proved.

Postulate 6: Assuming that the capacity of conveyance $e_k (k = 1,2, \dots, K)$ are independent gamma's random variables, then $Pr\left(\sum_{q=1}^{l} \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}^q \le e_k\right) \ge 1 - \xi_k$, iff

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^q}{\beta''}\right)} \left(\sum_{h=0}^{(\alpha_k)'' - 1} \left(\frac{\sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^q}{\beta''}\right)^h \frac{1}{h!}\right) \ge 1 - \xi_k, \tag{28}$$

Here gamma distribution's the shape & scale parameters denoted as $(\alpha_k)''$ & β'' corresponding to $e_k (k = 1,2, \dots, K)$.

Proof: Through the above proved postulates, we can also prove this postulate with the same logic. Utilizing postulates 4 to 6, an equivalent crisp model is developed for the issue in which all parameters adhere to gamma distributions with specified mean & variance. So, the model which is deterministic is given below:

$$\text{Minimize } Z_p = \sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^{qp}\, x_{ijk}^{q}, \tag{29}$$

Subject to constraints

$$e^{\left(\frac{-\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{q}}{\beta_i^q}\right)}\left(\sum_{h=0}^{\alpha_i^q-1}\left(\frac{\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk}^{q}}{\beta_i^q}\right)^h \frac{1}{h!}\right) \geq (1-\gamma_i^q),\; i = 1\; to\; m;\; q = 1\; to\; l, \tag{30}$$

$$e^{\left(\frac{-\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{q}}{(\beta_j^q)'}\right)}\left(\sum_{h=0}^{(\alpha_j^q)'-1}\left(\frac{\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk}^{q}}{(\beta_j^q)'}\right)^h \frac{1}{h!}\right) \geq \eta_j^q,\; j = 1\; to\; n;\; q = 1\; to\; l, \tag{31}$$

$$e^{\left(\frac{-\sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{q}}{(\beta_k)''}\right)}\left(\sum_{h=0}^{(\alpha_k)''-1}\left(\frac{\sum_{q=1}^{l}\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk}^{q}}{\beta''}\right)^h \frac{1}{h!}\right) \geq 1-\xi_k,\; k = 1\; to\; K, \tag{32}$$

$$x_{ijk}^{q} \geq 0\ \forall\ i,j,k\ and\ q \tag{33}$$

A non-linear equivalent deterministic` model of multi-objective programming problem is obtained. Assumed that this model has feasible optimal compromise result. For determining compromise results of this MOMSSTP, we apply the fuzzy's programming method. Fuzzy programming for optimizing MOMSSTP is shown in next section. Hence the model become special case when the parameter's follow Erlang distribution.

3. Fuzzy Programming Technique For MOMSSTP Model

The MOMSSTP involves optimizing multiple conflicting objectives under uncertainty, considering various items and transportation routes. Fuzzy programming is a useful method for tackling such complex problems by incorporating the vagueness and imprecision inherent in real world scenarios. For optimization of MOMISSTP we apply fuzzy's programming method. The steps of the fuzzy's programming method for optimization of issue are outlined here:

Step 1- Address each objective individually to optimize the model, with the resulting optimal results being the ideal results for that specific objective. The collection of optimal values for all various objectives will form the pareto ideal solution point in the multi-objective crisp model.

Step 2- The objectives' pay-off matrix is constructed as follows:

$$\begin{array}{c c} & \begin{array}{ccccc} Z_1 & Z_2 & Z_3 & \dots & Z_s \end{array} \\ \begin{array}{c} X_1 \\ X_2 \\ X_3 \\ \dots \\ X_s \end{array} & \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & \dots & Z_{1s} \\ Z_{21} & Z_{22} & Z_{23} & \dots & Z_{2s} \\ Z_{31} & Z_{32} & Z_{33} & \dots & Z_{3s} \\ \dots & \dots & \dots & \dots & \dots \\ Z_{s1} & Z_{s2} & Z_{s3} & \dots & Z_{ss} \end{bmatrix} \end{array}$$

X_s represent the optimal results point for the single goal crisp issue with the s[th] objective function. In the pay off matrix, $Z_{ij} = Z_j(X_i)$ denotes the element located at the i[th] row and j[th] column, where i ranges from 1 to S and j ranges from 1 to S.

Step 3- On the basis of pay off matrix, determine the L_s (lower bound) and U_s (upper bound) for all objective functions.

Step 4- Create the linear membership function $\mu_{Z_s}(X)$ that corresponds to each goal as

$$\mu_{Z_s}(X) = \begin{cases} 1, & if\ Z_s \leq L_s, \\ \frac{U_s - Z_s(X)}{U_s - L_s}, & if\ L_s < Z_s < L_s, \\ 0, & if\ Z_s \geq U_s, \end{cases} \tag{34}$$

Step 5- An equivalent crisp model is constructed as below:

Max λ (35)

Subject to constraint

$\lambda \leq \mu_{Z_s}(X), s = 1,2, \dots, S,$ (36)

Also, original set of constraints (37)

We derive compromise result for the equivalent multi-objective model, with the help of following outlined steps. Hence, original issue has compromise results.

4. Numerical Example

To demonstrate the proposed research, we take an example of a sugar & jaggery transportation problem where origin's supplies, destination's demand and capacities of conveyance are modeled as gamma's random variables. A factory produces jaggery (1st item) and sugar (2nd item). Factory has two production plants, which are denoted as F_1, F_2 supplying jaggery and sugar to three warehouses W_1, W_2, W_3 through two different conveyances E_1, E_2. The cost of transportation and time of transportation for 1st item are provided in table 1 and 2 respectively, similarly price of transportation and time of transportation for 2nd item are given table 3 and 4 respectively.

Here,

(i) c_{ijk}^{q} represents the cost of transportation (10,000 per day) for both items to supplies 1000 tons of jaggery and sugar from ith sources to jth destinations through kth type of transportation's modes. For instance, transporting 1000 tons of jaggery and sugar from 1st factory to first warehouse by using 1st transportation's mode will cost 70,000 per day.

(ii) Total amount of jaggery a_i^1 and sugar a_i^2 is available at the ith factory (in 1000 tons).

(iii) Total amount of jaggery b_j^1 and sugar b_j^2 is demanded at the jth warehouse (in 1000 tons).

(iv) The capacity of conveyance e_k of kth transportation's mode (in 1000 tons).

Table 1. Cost of transportation for 1st item in rupees (10,000 per day).

c_{ijk}^{1}	E_1		E_1		E_1	
		E_2		E_2		E_2
	W_1		W_2		W_3	
F_1	7	4	11	5	6	4
F_2	5	15	8	21	10	2

Table 2. time of transportation for 1st item.

t_{ijk}^{1}	E_1		E_1		E_1	
		E_2		E_2		E_2
	W_1		W_2		W_3	
F_1	4	3	8	9	6	4
F_2	9	4	7	6	2	3

Table 3. Cost of transportation for 2nd item in rupees (10,000 per day).

c_{ijk}^{2}	E_1		E_1		E_1	
		E_2		E_2		E_2
	W_1		W_2		W_3	
F_1	8	16	3	14	7	8
F_2	6	5	6	17	10	4

Table 4. Time of transportation for 2nd item.

t_{ijk}^{2}	E_1		E_1		E_1	
		E_2		E_2		E_2
	W_1		W_2		W_3	
F_1	4	5	9	4	3	6
F_2	7	3	6	5	4	8

(v) t_{ijk}^{q} denoted the day's numbers required to supplies 1000 tons of jaggery and sugar from ith sources to jth destinations through kth type of transportation's modes. For example, it will take 7 days to supplies 1000 tons of jaggery and sugar form 1st factory to 1st warehouse through 1st transportation's mode.

Here, defined the decision variables as:

The quantity of jaggery (x^1_{ijk}) and sugar (x^2_{ijk}) that is supplies by k^th^ transportation's mode from i^th^ source to j^th^ destination (in1000 tons).

For example, on the basis of various parameters of the problem, developed 4 four models. Here is the description of the models.

Case I: Only Supply Parameter Follows Gamma Distribution

Firstly, we assumed that the demand and capacities of conveyance are fixed but the supply parameter follows gamma's distribution. The "probability density function" of the supply parameter a_i^q follows $G(\alpha_i^q, 16)$, also $\alpha_1^1 = 2, \alpha_2^1 = 3, \alpha_1^2 = 3, \alpha_2^2 = 4$. Assume that probability levels addressing to the supply parameter as $\gamma_1^1 = 95\%, \gamma_2^1 = 96\%, \gamma_1^2 = 90\%, \gamma_2^2 = 93\%$. Demands (in 1000 tons) are given as $b_1^1 = 8, b_2^1 = 3, b_3^1 = 6, b_1^2 = 4, b_2^2 = 5, b_3^2 = 4$ and capacities of conveyance as $e_1 = 15, e_1 = 16$. By using the provided data and the model (29) to (33), we obtained the crisp model for the issue. Additionally, we observe that

$$e^{\left(\frac{-\sum_{q=1}^{2}\sum_{j=1}^{3} b_j^q}{\beta}\right)}\left(\sum_{h=0}^{\sum_{q=1}^{2}\sum_{j=1}^{3}\alpha_i^q-1}\left(\frac{-\sum_{q=1}^{2}\sum_{j=1}^{3} b_j^q}{\beta}\right)^h\frac{1}{h!}\right)$$

$= 0.999999 \geq 0.99$.

As a result, the initial condition of feasibility is fulfilled.

Optimal results of these objective functions are:

$Z_1 = 119, x^1_{112} = 2.061164, x^1_{122} = 3, x^1_{211} = 5.938836, x^1_{232} = 6, x^2_{121} = 5, x^2_{211} = 3.061164, x^2_{212} = 0.938836, x^2_{232} = 4.$

$Z_2 = 104.3142, x^1_{111} = 1.301884, x^1_{112} = 4.383929, x^1_{212} = 2.314187, x^1_{221} = 1.487849, x^1_{222} = 1.512151, x^1_{231} = 6, x^2_{111} = 1.210267, x^2_{122} = 5, x^2_{131} = 4, x^2_{212} = 2.789733.$

Hence, for this problem the pay-off matrix is

$$\begin{array}{cc} & \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \end{array} & \begin{bmatrix} 119 & 205.8777 \\ 286.6505 & 104.3142 \end{bmatrix} \end{array}$$

We obtain L_s (lower bound) and U_s (upper bound) for objective functions. Z_s $(s = 1,2)$ are configured as $L_1 = 119 \leq Z_1 \leq 286.6505 = U_1$ & $L_2 = 104.3142 \leq Z_2 \leq 205.8777 = U_2$. We generate membership function related to each objective by utilizing these objective function's bound. Thus, the model of fuzzy programming is

max λ

subject to constraints are

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} c_{ijk}^{qp} x_{ijk}^q + (U_1 - L_1)\lambda \leq U_1,$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} t_{ijk}^{qp} x_{ijk}^q + (U_2 - L_2)\lambda \leq U_2,$

$$e^{\left(\frac{-\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^q}{\beta_i^q}\right)}\left(\sum_{h=0}^{\alpha_i^q-1}\left(\frac{\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^q}{\beta_i^q}\right)^h\frac{1}{h!}\right) \geq (1-\gamma_i^q),\ i = 1,2\ ;\ q = 1,2,$$

$\sum_{i=1}^{2}\sum_{k=1}^{2} x_{ijk}^q \geq b_j^q,\ j = 1,2,3; q = 1,2,$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^q \leq e_k,\ k = 1,2,$

$x_{ijk}^q \geq 0\ \forall\ i, j, k\ and\ q.$

We get compromise solution by applying lingo software $x^1_{112} = 5.685784, x^1_{211} = 1.684239, x^1_{212} = 0.6299772, x^1_{221} = 3, x^1_{231} = 0.3157614, x^1_{232} = 5.684239, x^2_{131} = 4, x^2_{212} = 4, x^2_{221} = 5, Z_1 = 157.1401, Z_2 = 127.4196.$

Case II: Only Demand Parameter Follows Gamma Distribution

We assumed that the supplies and capacities of conveyance are fixed but the demands parameter follows gamma's distribution. Supplies for both items are $a_1^1 = 14, a_2^1 = 11, a_1^2 = 10, a_2^2 = 13$ & capacities of conveyance $e_1 = 15, e_1 = 14$. The demand parameters follow gamma distribution as $b^1_1 \sim G(2,0.7), b^1_2 \sim G(3,0.7), b^1_3 \sim G(3,0.7), b^2_1 \sim G(4,0.7), b^2_2 \sim G(2,0.7), b^2_3 \sim G(2,0.7)$. Assuming that probability levels addressing to the destination's demand parameter as $\eta^1_1 = 97\%, \eta^1_2 = 94\%, \eta^1_3 = 95\%, \eta^2_1 = 95\%, \eta^2_2 = 96\%, \eta^2_3 = 93\%$. We created the crisp model for the issue by utilizing of model (29)-(33) and the data provided in the tables. Optimal results of these objective functions are:

$Z_1 = 100.6135, x^1_{112} = 1.834166, x^1_{122} = 4.231352, x^1_{211} = 1.914998, x^1_{232} = 4.407056, x^2_{121} = 3.508932, x^2_{211} = 3.933383, x^2_{212} = 1.494176, x^2_{232} = 3.033250.$

$Z_2 = 86.78488, x^1_{112} = 3.749164, x^1_{221} = 0.03445836, x^1_{222} = 4.196894, x^1_{231} = 4.407056, x^2_{111} = 1.882550, x^2_{122} = 3.508932, x^2_{131} = 3.03325, x^2_{212} = 3.54501.$

Hence, for this problem the pay-off matrix is

$$\begin{array}{cc} & \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \end{array} & \begin{bmatrix} 100.6135 & 161.9034 \\ 250.6209 & 86.78488 \end{bmatrix} \end{array}$$

We obtain L_s (lower bound) and U_s (upper bound) for objective functions. $Z_s\ (s = 1,2)$ are configured as $L_1 = 100.6135 \le Z_1 \le 250.6209 = U_1$ & $L_2 = 86.78488 \le Z_2 \le 161.9034 = U_2$. We generate membership function related to each objective by utilizing these objective function's bound. Thus, the model of fuzzy programming is

max λ

subject to constraints are

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} c^{qp}_{ijk} x^q_{ijk} + (U_1 - L_1)\lambda \le U_1,$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} t^{qp}_{ijk} x^q_{ijk} + (U_2 - L_2)\lambda \le U_2,$

$\sum_{j=1}^{3}\sum_{k=1}^{2} x^q_{ijk} \le a^q_i,\ i = 1,2; q = 1,2,$

$$e^{\left(\frac{-\sum_{i=1}^{2}\sum_{k=1}^{2} x^q_{ijk}}{(\beta^q_j)'}\right)} \left(\sum_{h=0}^{(\alpha^q_j)'-1} \left(\frac{\sum_{i=1}^{2}\sum_{k=1}^{2} x^q_{ijk}}{(\beta^q_j)'}\right)^h \frac{1}{h!}\right) \ge \eta^q_j,\ j = 1,2,3;\ q = 1,2,$$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x^q_{ijk} \le e_k,\ k = 1,2,$

$x^q_{ijk} \ge 0\ \forall\ i, j, k\ and\ q.$

We get compromise solution by applying lingo software $x^1_{112} = 3.749164, x^1_{221} = 4.231352, x^1_{232} = 4.407056, x^2_{122} = 0.08039964, x^2_{131} = 3.03325, x^2_{212} = 5.42756,, x^2_{221} = 3.428532,\ Z_1 = 127.7289, Z_2 = 100.3633.$

Case III: Only Conveyance Capacity Follows Gamma Distribution

We assumed that the supply & demand are fixed but the conveyance capacity follows gamma's distribution. Supplies for both items are $a^1_1 = 10, a^1_2 = 15, a^2_1 = 11, a^2_2 = 12$ & demand for both items are $b^1_1 = 11, b^1_2 = 8, b^1_3 = 5, b^2_1 = 4, b^2_2 = 6, b^2_3 = 9$ .The conveyance' capacity follow gamma distribution as $e_1 \sim G(3,15)$, $e_2 \sim G(4,15)$. Assume that probability levels addressing to the conveyance's capacity as $\xi_k = 89\%$, $\xi_k = 90\%$. We created the crisp model for the issue by utilizing of model (29)-(33) and the data provided in the tables. Optimal results of these objective functions are:

$Z_1 = 180.8285, x^1_{112} = 2, x^1_{122} = 8, x^1_{211} = 9, x^1_{232} = 5, x^2_{121} = 6, x^2_{131} = 1,\ x^2_{211} = 0.8284668, x^2_{212} = 3.171533, x^2_{232} = 8.$

$Z_2 = 161.8285, x^1_{112} = 10, x^1_{212} = 1, x^1_{221} = 2.828461, x^1_{222} = 5.171539, x^1_{231} = 5,\ x^2_{122} = 6, x^2_{131} = 5, x^2_{212} = 4, x^2_{231} = 4.$

Hence, for this problem the pay-off matrix is

$$\begin{array}{cc} & \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \end{array} & \begin{bmatrix} 180.8285 & 310.3139 \\ 415.23 & 161.8285 \end{bmatrix} \end{array}$$

We obtain L_s (lower bound) and U_s (upper bound) for objective functions. $Z_s\ (s = 1,2)$ are configured as $L_1 = 180.8285 \le Z_1 \le 415.23 = U_1$ & $L_2 = 161.8285 \le Z_2 \le 310.3139 = U_2$. We generate membership function related to each objective by utilizing these objective function's bound. Thus, the model of fuzzy programming is

max λ

subject to constraints are

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} c_{ijk}^{qp} x_{ijk}^{q} + (U_1 - L_1)\lambda \leq U_1,$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} t_{ijk}^{qp} x_{ijk}^{q} + (U_2 - L_2)\lambda \leq U_2,$

$\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^{q} \leq a_i^q,\ i = 1,2; q = 1,2,$

$\sum_{i=1}^{2}\sum_{k=1}^{2} x_{ijk}^{q} \geq b_j^q,\ j = 1,2,3; q = 1,2,$

$e^{\left(\frac{-\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{q}}{(\beta_k)''}\right)} \left(\sum_{h=0}^{(\alpha_k)''-1} \left(\frac{\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{q}}{\beta''}\right)^{h} \frac{1}{h!}\right) \geq 1 - \xi_k,\ k = 1,2,$

$x_{ijk}^{q} \geq 0\ \forall\ i, j, k\ and\ q.$

We get compromise solution by applying lingo software $x_{112}^1 = 10, x_{212}^1 = 1, x_{221}^1 = 8, x_{232}^1 = 5, x_{121}^2 = 0.9164443, x_{122}^2 = 1.411589, x_{131}^2 = 4.671966,\ x_{212}^2 = 4, x_{221}^2 = 3.671966, x_{232}^2 = 4.328034, Z_1 = 243.5593, Z_2 = 201.5663.$

Case IV: Supply, Demand and Conveyance's Capacity Follow Gamma Distribution

In this case, we assumed that the origin's supply, destination's demand and conveyance's capacity follow gamma's distribution. On the basis of data used in previous cases, we created the crisp model for the issue by utilizing of model (29)-(33) and the data provided in the tables. Optimal results of these objective functions are:

$Z_1 = 101.0601, x_{112}^1 = 1.066549, x_{122}^1 = 4.619235, x_{211}^1 = 3.017196, x_{232}^1 = 4.407056, x_{121}^2 = 3.508932, x_{212}^2 = 5.42756,\ x_{232}^2 = 3.03325.$

$Z_2 = 88.19891, x_{112}^1 = 4.083745, x_{222}^1 = 4.619235, x_{231}^1 = 4.407056, x_{122}^2 = 3.508932, x_{131}^2 = 3.03325, x_{212}^2 = 5.42756.$

Hence, for this problem the pay-off matrix is

$$\begin{array}{cc} & \begin{array}{cc} Z_1 & Z_2 \end{array} \\ \begin{array}{c} X_1 \\ X_2 \end{array} & \begin{bmatrix} 101.0601 & 157.2778 \\ 254.9051 & 88.19891 \end{bmatrix} \end{array}$$

We obtain L_s (lower bound) and U_s (upper bound) for objective functions. Z_s $(s = 1,2)$ are configured as $L_1 = 101.0601 \leq Z_1 \leq 254.9051 = U_1$ & $L_2 = 88.19891 \leq Z_2 \leq 157.2778 = U_2$. We generate membership function related to each objective by utilizing these objective function's bound. Thus, the model of fuzzy programming is

max λ

subject to constraints are

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} c_{ijk}^{qp} x_{ijk}^{q} + (U_1 - L_1)\lambda \leq U_1,$

$\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3}\sum_{k=1}^{2} t_{ijk}^{qp} x_{ijk}^{q} + (U_2 - L_2)\lambda \leq U_2,$

$e^{\left(\frac{-\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^{q}}{\beta_i^q}\right)} \left(\sum_{h=0}^{\alpha_i^q-1} \left(\frac{\sum_{j=1}^{3}\sum_{k=1}^{2} x_{ijk}^{q}}{\beta_i^q}\right)^{h} \frac{1}{h!}\right) \geq (1 - \gamma_i^q),\ i = 1,2\ ;\ q = 1,2,$

$e^{\left(\frac{-\sum_{i=1}^{2}\sum_{k=1}^{2} x_{ijk}^{q}}{(\beta_j^q)'}\right)} \left(\sum_{h=0}^{(\alpha_j^q)'-1} \left(\frac{\sum_{i=1}^{2}\sum_{k=1}^{2} x_{ijk}^{q}}{(\beta_j^q)'}\right)^{h} \frac{1}{h!}\right) \geq \eta_j^q,\ j = 1,2,3;\ q = 1,2,$

$e^{\left(\frac{-\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{q}}{(\beta_k)''}\right)} \left(\sum_{h=0}^{(\alpha_k)''-1} \left(\frac{\sum_{q=1}^{2}\sum_{i=1}^{2}\sum_{j=1}^{3} x_{ijk}^{q}}{\beta''}\right)^{h} \frac{1}{h!}\right) \geq 1 - \xi_k,\ k = 1,2,$

$x_{ijk}^{q} \geq 0\ \forall\ i, j, k\ and\ q.$

We get compromise solution by applying lingo software $x^1_{112} = 4.083745, x^1_{221} = 4.619235, x^1_{232} = 4.407056, x^2_{122} = 0.4227329, x^2_{131} = 3.03325, x^2_{212} = 5.42756, x^2_{221} = 3.086199, Z_1 = 134.909, Z_2 = 103.3976.$

Result and Discussion

Developed deterministic models for the Multi-Objective Multi-item Stochastic Solid Transportation Problem (MOMSSTP) also, addressing the complexities associated with random parameters in a stochastic environment. Non-linear constraints are the corresponding deterministic form's chance constraints. The model was tested using a numerical example, demonstrating its ability to optimize transportation cost and time effectively while accounting for uncertainties in supply, demand and conveyance's capacity. The incorporation of stochastic elements through gamma distribution and extend fuzzy programming technique provided a comprehensive solution that is both adaptable and efficient. All models are solved through Lingo 20.0 software.

Compromise solutions for first case shows that, it takes about 129 days to deliver 17 thousand tones sugar and 13 thousand tons of jaggery from two factories to three warehouses, addressing to the optimal values of the first model's objectives are $Z_1 = 157.1401$ and $Z_2 = 127.4196$. The lowest possible cost of transportation being Rs. 1,571,401. The achievement rate is 0.7725024 for objectives in fuzzy's programming approach.

In second case of compromise solution shows that, it takes about 101 days to deliver 12.387572 thousand tones sugar and 11.9697416 thousand tons of jaggery from 2 factories to 3 warehouses, addressing to the optimal values of the second model's objectives are $Z_1 = 127.7289$ and $Z_2 = 100.3633$. The lowest possible cost of transportation being Rs. 1,277,289. The achievement rate is 0.8192394 for objectives in fuzzy's programming approach.

From the compromise solutions for third case shows that, it takes about 202 days to deliver 24 thousand tones sugar and 19 thousand tons of jaggery from 2 factories to 3 warehouses, addressing to the optimal values of the third model's objectives are $Z_1 = 243.5593$ and $Z_2 = 201.5663$. The lowest possible cost of transportation being Rs. 2,435,593. The achievement rate is 0.7323789 for objectives in fuzzy's programming approach.

Finally, compromise solutions for last case shows that, it takes about 104 days to deliver 13.110036 thousand tones sugar and 11.9697419 thousand tons of jaggery from 2 factories to 3 warehouses, addressing to the optimal values of the fourth model's objectives are $Z_1 = 134.9090$ and $Z_2 = 103.3976$. The lowest possible cost of transportation being Rs. 1,349,090. The achievement rate is 0.7799807 for objectives in fuzzy's programming approach.

Conclusions

This research presents a solution method for MOMSSTP where origin's supplies, destination's demands and conveyance's capacities follow gamma's distribution. A chance-constraint programming method is used to convert the issue into crisp model, which becomes nonlinear under this distribution. Due to presence of random variables, standard feasibility conditions can't be applied, so the chance-constraint method is used to ensure feasibility. Fuzzy programming is then employed to obtain a compromise results, balancing conflicting objectives. In this research all parameters are uncertain except coefficient of objectives. So, the issue can be further explored by considering coefficient of goals in stochastic or fuzzy environment.

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