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NUMERICAL ANALYSIS OF MECHANICAL BEHAVIOUR OF FGM USING FSDPT

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ABSTRACT—

Functionally Graded Materials (FGMs) are advanced composites designed so that their material properties vary continuously between two surfaces. This removes any abrupt changes in the material causing FGMs to be ideal to use in high performance aerospace and structural applications. Due to the transverse pressure loading on FGM plates, transverse shear effects that occur are often ignored by using classical plate theories. This can cause problems when trying to determine how much the plate will deflect, the stresses in the plate and when/where failure will start to occur. This project looks at determining the static behaviour of rectangular FGM plates under transverse pressure. The First-Order Shear Deformation Plate Theory (FSDPT) has been used to incorporate the shear effect into the calculations. Equations have been derived and solved numerically using MATLAB. These have then been validated using Finite Element Models within ANSYS. Suitable failure criteria were then applied to find the failure points. The results show that varying the volume fraction of the material has a positive effect on the stiffness of the structure. The maximum deflection at the centre of the plate has decreased due to the increased stiffness. Stress contours show that the maximum stress is well managed throughout the plate. Finally the failure analysis shows exactly where failure occurs and at what pressure the plate will begin to fail. The FSDPT numerical model has been developed to accurately and efficiently analyze the pure mechanical response of an FGM plate and the pressure it can take before failure. This will provide valuable information when designing aerospace structures.

Index Terms— Functionally Graded Materials (FGM), FSDPT, Static Analysis, MATLAB, ANSYS, Finite Element Analysis (FEA), Failure Analysis.

I. INTRODUCTION

Modern aerospace and structural engineering applications operate under extreme conditions such as high temperatures, aerodynamic forces, pressure variations and mechanical loading. Traditional materials like aluminium, titanium and steel alloys may experience stress concentrations, thermal stresses and failure at joints when subjected to these conditions. To overcome these limitations, Functionally Graded Materials (FGMs) have been developed as advanced composites in which material properties vary continuously from one surface to another. Typically, ceramic provides high stiffness, strength and thermal resistance, while metal provides ductility, toughness and better energy absorption.

FGMs are increasingly considered for aerospace applications such as aircraft wings, fuselage skins, pressure bulkheads and structural panels because of their improved mechanical properties and reduced stress concentrations. However, predicting the structural behaviour of FGM plates under transverse pressure loading remains challenging due to the continuous variation of material properties and the influence of transverse shear deformation.

Conventional Classical Plate Theory (CPT) neglects transverse shear deformation by assuming that the normal to the mid-plane remains perpendicular after deformation. Although this assumption is suitable for thin plates, it may produce inaccurate deflection and stress predictions for moderately thick FGM plates. Therefore, First-Order Shear Deformation Plate Theory (FSDPT), which accounts for transverse shear effects, provides a more suitable approach for analysing the mechanical behaviour of such structures.

II. OBJECTIVES AND SCOPE

1. To study the static behaviour of rectangular Functionally Graded Material (FGM) plates with continuously varying ceramic-metal material properties.
2. To formulate and analyse FGM plates under transverse pressure loading using First-Order Shear Deformation Plate Theory (FSDPT), considering transverse shear effects.
3. To develop a MATLAB numerical model for calculating transverse deflection and stress distribution in FGM plates.
4. To perform Finite Element Analysis using ANSYS Workbench 2026 R1 and validate the numerical results obtained from MATLAB.
5. To investigate the influence of material volume fraction on stiffness, deflection and stress distribution, and determine critical failure locations and failure pressure using suitable failure criteria.

III. LITERATURE REVIEW

Mamandi (2023) – This research was used as the main basis throughout this study. It investigated the bending of FGM rectangular plates using the finite

element method with First Order Shear Deformation Theory (FSDPT). This showed the effectiveness of the finite element method when predicting plate deflection and stresses. It provided a good basis for implementing the FEM in MATLAB.

Kashtalyan (2004) – This paper provided a fundamental solution to the bending of functionally graded rectangular plates using 3d elasticity. This provided exact solution for reference, showing how the material properties affect stresses in the plane of the plate.

Daouadji et al. (2013) – This research provided extensive analytical solutions for the bending of functionally graded plates. It accounted for thickness stretching, midplane shift and transverse shear.

Kulkarni et al. (2010) – This paper investigated the bending and buckling of FGM plates using an advanced inverse trigonometric shear deformation theory. This provided insight into how the displacement field can be improved by using a refined displacement field, therefore negating the need for a shear correction factor.

Nguyen, Sab and Bonnet (2008) – This research provided insight into how shear deformation affects transverse shear in heterogeneous plates. It looked at different first order shear deformation theories that can be applied to functionally graded materials. This provided insight into how to choose which shear correction factor to use.

Singha, Prakash and Ganapathi (2011) – This research looked at the finite element method for functionally graded plates under transverse loading. This showed the importance of having an effective mesh.

Praween et al. / Natarajan et al. (2012-2015) – This research provided extensive contribution to the numerical solution of FGM plates using high and first order shear deformation theory.

They looked at the free vibration of functionally graded plates using the finite element method. They investigated the bending of functionally graded plates and looked at the damage to cracked or homogenous plates.

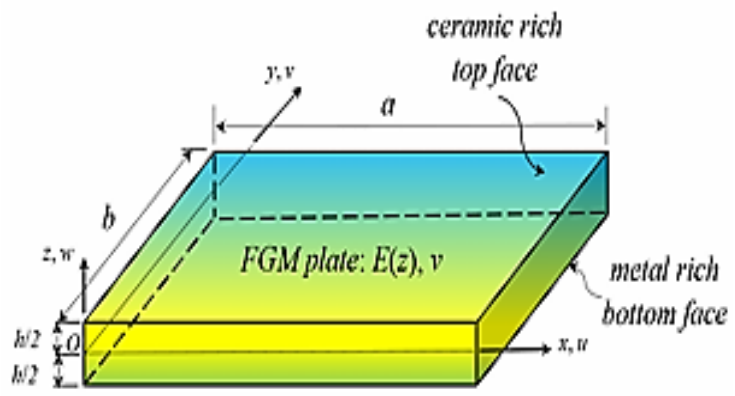
Reddy (2000) – This is a landmark paper in the field of functionally graded plates. J.N Reddy developed the theory of analysing functionally graded plates. He developed classical and shear deformation theories such as FSDPT and HSDPT using the Principle of Virtual Work. This provided the fundamental mathematics behind any finite element methods.

Talha and Singh (2010) – This paper looked at the static bending of FGM plates using a higher order shear deformation theory finite element model. They showed how varying the power law volume fractions (η) affects plate deflection and bending stresses.

Yang and Shen (2002/2003) – This paper looked at the nonlinear bending and buckling of functionally graded plates under transverse pressures and thermal. This provided a good basis for finding stresses and load deflection curves.

IV.MATERIALS USED FOR THE ANALYSIS

In this study, three different materials, namely Aluminium, Ceramic and Functionally Graded Material (FGM), were used for analysis and comparison. Aluminium was selected due to its lightweight, low density, ductility, corrosion resistance and good strength-to-weight ratio. Ceramic was chosen for its high stiffness, hardness, wear resistance and excellent thermal properties. Functionally Graded Material (FGM) was developed by combining aluminium and ceramic, with continuously varying material properties throughout the thickness of the plate, from a metal-rich region to a ceramic-rich region. This combination helps achieve the advantages of both materials, providing better stiffness, strength, toughness and thermal resistance while reducing stress concentration and deformation.



Geometry of a rectangular FGM plate and considered cartesian coordinate system

V.NUMERACIAL ANALYSIS

4.4 MATERIAL GRADATION MODELLING

In order to suppress abrupt interfacial bonding and stress discontinuities, the effective material properties of the FGM plate (Young's modulus E(z) and Poisson's ratio v(z)) are assumed to vary continuously through the thickness direction z (from -h/2 to +h/2) based on the standard distribution of the volume fractions.

$P(z)=P_m + (P_c - P_m)\, V_c(z)$

where P_c and P_m are the material properties of the ceramic and metal constituents.

The volume fraction of the ceramic constituent:

$V_c(z)=(2z+h/2h)^n$

To include accurate representation of the transverse shear deformations, the First-Order Shear Deformation Plate Theory (FSDPT) is used. The displacement field components (u, v, w) at any point (x, y, z) are expressed as:

$u(x,y,z) = u_0(x,y) + z\phi(x,y)$ eq 4.3

$v(x,y,z) = v_0(x,y) + z\phi(x,y)$ eq 4.4

$\omega(x,y,z) = \omega_0(x,y)$ eq 4.5

By using the eq (4.3), (4.4), (4.5) Derivation the Normal strains along the x-axis-axis, and z-axis. And Derivation of Shear Strains along xy-plane and Transverse shear strains along xz and yz planes.

$Q11 = \frac{E(z)}{1-v^2}$ eq 4.6

$Q_{12} = \frac{vE(z)}{1-v^2}$ eq 4.7

$Q_{44} = Q_{55} = Q_{66} = \frac{E(z)}{2(1+v)}$ eq 4.8

Where, eq (4.6) In-plane elastic stiffness components under a plane stress state. And eq (4.7) Transverse shear stiffness coefficients. Eq (4.8) In-plane shear stiffness coefficient, equivalent to the material's shear modulus.

Using Hamilton's Principle

$\int \left(\delta u_{\frac{h}{2}} + \delta w\right) d\mathrm{t} = 0$ eq 4.9

$\delta U = \left[\int_\Omega \int_{-h/2}^{h/2} \delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\gamma_{xy} + \tau_{yz}\delta\gamma_{yz} + \tau_{xz}\delta\gamma_{xz}\right]$

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ r_{xy}\\ r_{yz}\\ r_{xz}\end{bmatrix} = \begin{bmatrix}Q_{11} & Q_{12} & 0 & 0 & 0\\ Q_{21} & Q_{22} & 0 & 0 & 0\\ 0 & 0 & Q_{166} & 0 & 0\\ 0 & 0 & 0 & Q_{141} & 0\\ 0 & 0 & 0 & 0 & Q_{16}\end{bmatrix}\begin{bmatrix}r_{111}\\ r_{111}\\ r_{11}\\ r_{11}\\ r_{11}\end{bmatrix}$$

$$N_{xx} = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sigma_{xx} dz$$

$$Nyy = \int_{-1/2}^{1/2} \sigma_{yy} dz$$

$$N_{xy} = \int_{-h/2}^{h/2} \tau_{xy} dz$$

$$\mathrm{Mxx} = \int_{-\frac{h}{2}}^{\frac{h}{2}} z\sigma_{\mathrm{xx}} dz$$

$$Myy = \int_{-h/2}^{h/2} z\sigma_{yy} dz$$

$$Mxy = \int_{-\pi/2}^{\pi/2} z\tau_{xy} dz$$

Consider: $\begin{bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{bmatrix}$ = In -plane force resultants.

$[Q]$ = Transvesse force result

$\begin{bmatrix}Mxx\\ Myy\\ Myy\end{bmatrix}$ = Moment resultants

Euler-lagrange equations

(1) $\delta u_0 \Rightarrow \frac{\partial Nxx}{\partial x} + \frac{\partial Nxy}{\partial y} = 0$

(2) $\delta v_0 \Rightarrow \frac{\partial N_{yy}}{\partial y} + \frac{\partial N_{xy}}{\partial x} = 0$

(3) $\delta\phi_x \Rightarrow \frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$

(4) $\delta\phi y \Rightarrow \frac{\partial Myy}{\partial y} + \frac{\partial Mxy}{\partial x} - Qy = 0$

(5) $\delta w_0 \Rightarrow \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q_b + q_t = 0$

$$\begin{bmatrix} Nxx \\ Nyy \\ Nxy \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} dz$$

The Constitutive Relation Where eq (5.0) ABD Matrix.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \quad \text{eq 5}$$

Formulation of the 9-node isoparametric quadrilateral element(q9)

$$N_1 = \frac{1}{4}[(1-\xi_1)(1-\eta_1)(-\xi_1-\eta_1-1)+(1-\xi_1^2)(1-\eta_1^2)]$$

$$N_2 = \frac{1}{4}[(1+\xi_2)(1-\eta_2)(\xi_2-\eta_2-1)+(1-\xi_2^2)(1-\eta_2^2)]$$

$$N_3 = \frac{1}{4}[(1+\xi_3)(1+\eta_3)(\xi_3+\eta_3-1)+(1-\xi_3^2)(1-\eta_3^2)]$$

$$N_4 = \frac{1}{4}[(1-\xi_4)(1+\eta_4)(-\xi_4+\eta_4-1)+(1-\xi_4^2)(1-\eta_4^2)$$

$$N_5 = \frac{1}{4}[2(1-\xi_5^2)(1-\eta_5)-2(1-\xi_5^2)(1-\eta_5^2)]$$

$$N_6 = \frac{1}{4}[2(1+\xi_6)(1-\eta_6^2)-2(1-\xi_6^2)(1-\eta_6^2)]$$

$$N_7 = \frac{1}{4}[2(1-\xi_7^2)(1+\eta_7)-2(1-\xi_7^2)(1-\eta_7^2)]$$

$$N_8 = \frac{1}{4}[2(1-\xi_8)(1-\eta_8^2)-2(1-\xi_8^2)(1-\eta_8^2)]$$

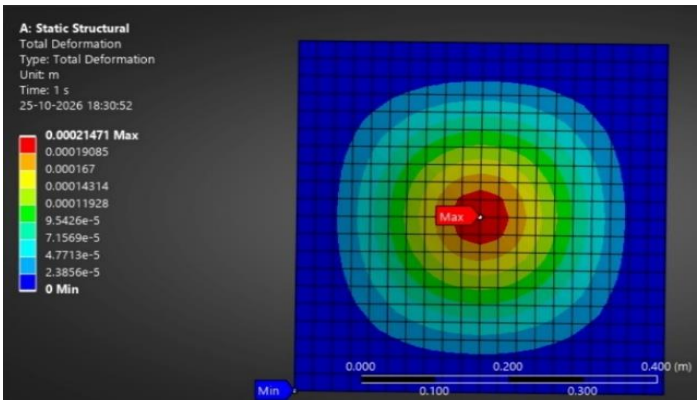
$$N_9 = (1-\xi_9^2)(1-\eta_9^2)$$

VI.MATERIAL PROPERTIES AND MODEL DETAILS

AL-GLASS CERAMIC FGM PLATE DEFLECTIONS

The combination of aluminium and glass ceramic FGM plate with a young's modulus of 71GPa and 73GPa respectively and passions ratio of 0.3 with a plate geometry of length 50cm and breadth 50cm and thickness of 1cm with 10 layers of square plate and an increment load of 1000N.

Fig Ansys Deflection value-0.0002147m



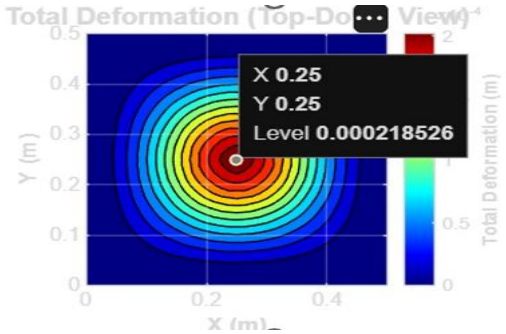


Fig 2 MATLAB Deflection value- 0.00021852m

The deflection values gained for the structure under 1000N loading are shown in figures Under the 1000N loading, figures 1 and 2 show that the deflections given by ANSYS software and those calculated using MATLAB closely match, giving approximate values of 0.000215m. This shows that the structure behaves elastically and that the MATLAB numerical implementation is well validated against the ANSYS FEA model.

The combination of aluminium and glass ceramic FGM plate with a young's modulus of 71GPa and 73GPa respectively and passions ratio of 0.3 with a plate geometry of length 50cm and breadth 100cm and thickness of 1cm with 10 layers of square plate of load 1000N.

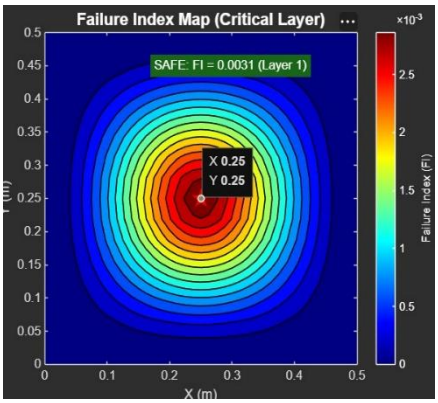


Fig 3 Failure safe index of square geometry

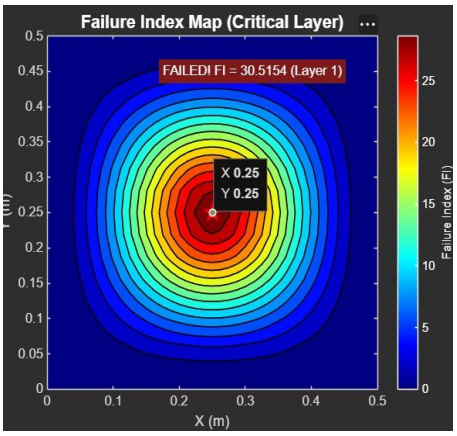


Fig 4 Failure index of square geometry

The failure index and safety factor for the structure has been shown in figure.3 Figure 4 shows that failure has occurred since the failure index is above 1.0. Note that the failure index is up to 30.5154 under high loading on Layer 1. Figures 1 shows that the structure is safe and within the limits with a failure index of 0.0031.

AL-ZIRCONIUM FGM PLATE DEFLECTIONS

The combination of aluminium and Zirconium FGM plate with a young's modulus of 71GPa and 200GPa respectively and passions ratio of 0.3 with a plate geometry of length 50cm and breadth 50cm and thickness of 1cm with 10 layers of square plate and load of 1000N.

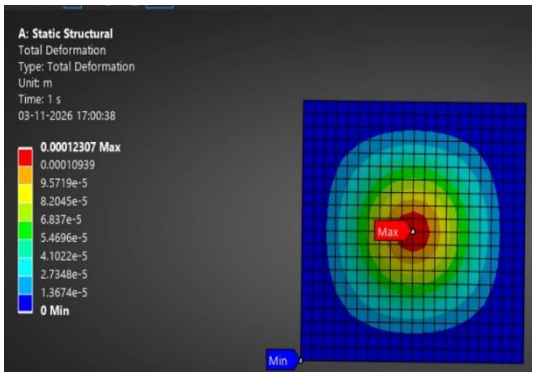


Fig 1 Ansys Deflection Value-0.00012307m

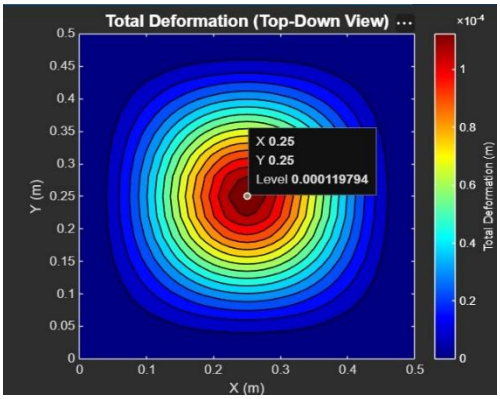


Fig 1MATLAB Deflection Value-0.0011979m

The deflection values for 1000N loading are shown in figures 1 and 2. Figure 1 and 2 show how closely matched the deflection values are when comparing ANSYS and MATLAB values for a 1000N loading. The deflection values were very similar, around 0.00012m. This shows that the structure behaves in a linear elastic manner. It can also be seen that the numerical MATLAB solution is very well validated with the ANSYS FEA model.

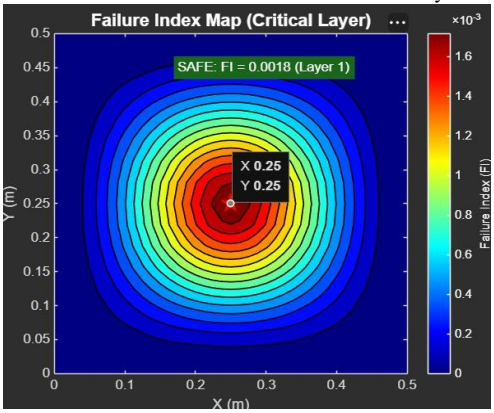


Fig 3Failure safe index of square geometry

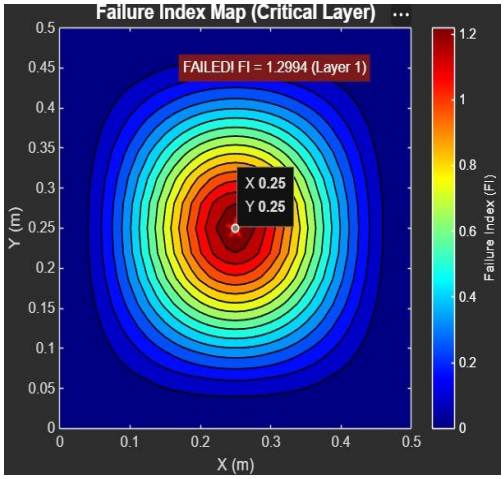


Fig 3 Failure index of square geometry

The failure safe index and failure index values for the rectangle and square geometries are shown in figures 3 we can see failure has occurred due to the failure index being greater than 1.0, under critical loading conditions. Figures 4 show a lower failure safe index meaning the structure is safe and within the limits.

AL-SILICON CARBIDE FGM PLATE DEFLECTIONS

The combination of aluminium and silicon carbide FGM plate with a young's modulus of 71GPa and 410GPa respectively and passions ratio of 0.3 with a plate geometry of length 50cm and breadth 50cm and thickness of 1cm with 10 layers of square plate and an increment load of 1000N.

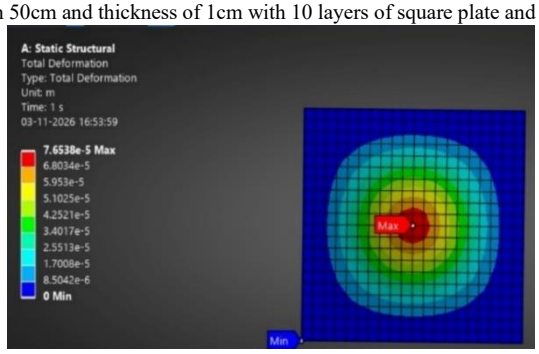


Fig 1 Ansys Deflection Value-0.000076538m

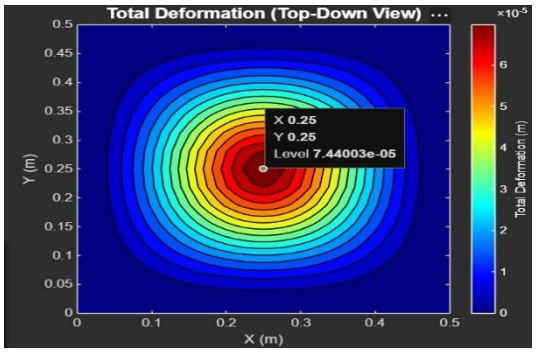


Fig 2 MATLAB Deflection Value-0.0000744m

The deflection values for the structure are displayed in figures 1show the deflections calculated using both the ANSYS and MATLAB methods under a 1000N loading. As can be seen the results are very similar with a value of approximately 0.000075m This proves the structure behaves in a linear elastic manner and that the results are well validated with the MATLAB numerical method producing similar results to the ANSYS model.

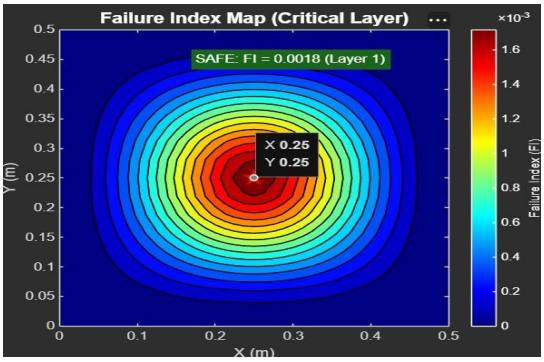


Fig 3 Failure safe index of square geometry

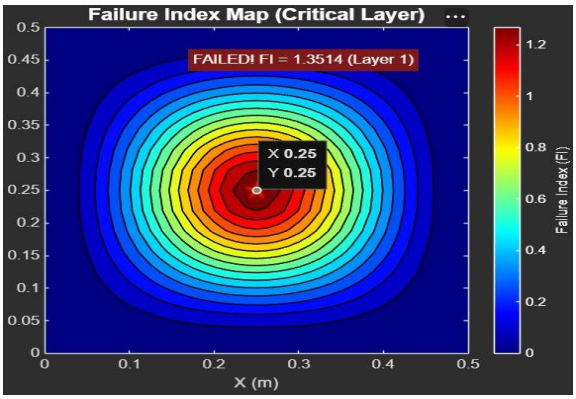


Fig 3 Failure index of square geometry

The failure safe index and failure index for the rectangle and square geometries have been shown in figures 3 Figures 4 show that failure has occurred since the failure index is above 1.0 under high loading. Figures 6.3 show that the structure is safe and within the limits.

VI. RESULTS AND DISCUSSION

The mechanical behaviour of structural component depends on its material, geometrical, loading and boundary conditions. Aluminium, Ceramics and Functionally Graded Material (FGM) plates are selected for numerical analysis and finite element analysis. The selected materials are subjected to numerical programming in MATLAB and finite element analysis in ANSYS using First Order Shear Deformation Plate Theory (FSDPT). The FGM plate consists of a mixture of metal and ceramic phases with different properties along its thickness.

The static structural response of a functionally graded material (FGM) rectangular plate was analysed using a First-Order Shear Deformation Plate Theory (FSDPT) bespoke MATLAB solver and validated using the simulation software ANSYS Workbench 2026 R1. The Aluminium-Glass-Ceramic, Aluminium-Zirconium and Aluminium-Silicon Carbide combinations were used for testing with the transverse forces ranging from 1,000N to 1,000,000 N and the dimensions of the rectangular plate being 50*50*1cm3 .

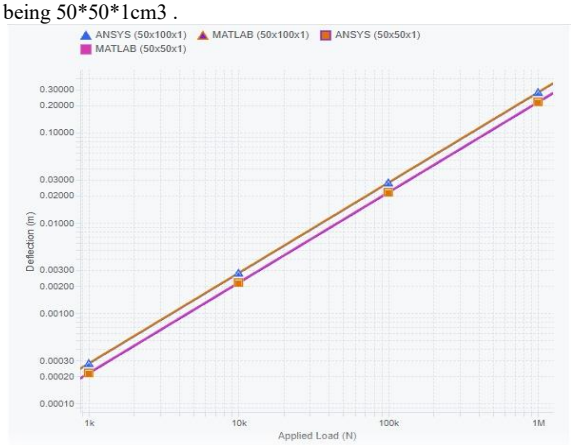
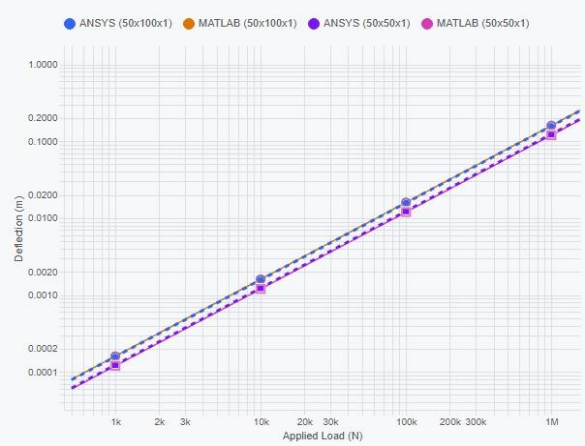


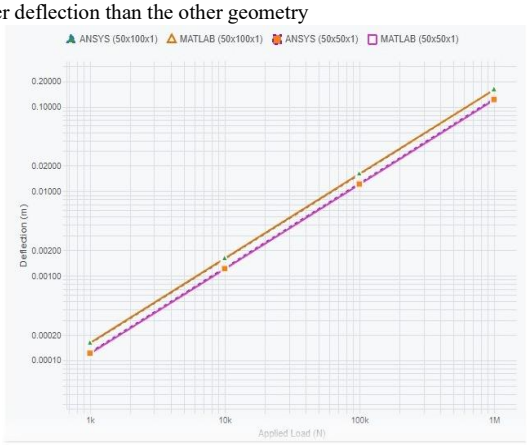
Fig 1 Comparison of ansys and matlab defection values for Al-glass ceramic material

From Fig:7.1 The graph shows that as more force is applied, deformation of the structure occurs in a perfectly linear fashion, demonstrated by plotting the Applied load (X-axis) against Deflection (Y-axis). The vertical spacing between the two different geometries also clearly shows that the less stiff 50×100×1 geometry will always have a larger deflection than the other geometry.



2 Comparison of ANSYS and MATLAB defection values for Al-zirconium material

From Fig:.2 The graph shows that as more force is applied, deformation of the structure occurs in a perfectly linear fashion, demonstrated by plotting the Applied load (X-axis) against Deflection (Y-axis). The vertical spacing between the two different geometries also clearly shows that the less stiff 50×100×1 geometry will always have a larger deflection than the other geometry



3 Comparison of ANSYS and MATLAB defection values for Al-silicon carbide material

From Fig:3 The graph shows that as more force is applied, deformation of the structure occurs in a perfectly linear fashion, demonstrated by plotting the Applied load (X-axis) against Deflection (Y-axis). The vertical spacing between the two different geometries also clearly shows that the less stiff 50×100×1 geometry will always have a larger deflection than the other geometry

COMPARISON OF MATERIAL AND GEOMETRY

The Aluminium & Glass-Ceramic FGM plate had a small 1.3% – 1.8% error margin between MATLAB and ANSYS (Table 6.1). Both had perfect linear elastic properties. This can be seen by increasing the load ten times and seeing the deflection also increase perfectly in proportion by ten times. Increasing the geometry of the Aluminium & Glass-Ceramic FGM plate from square (50501cm3) to rectangular (501001cm3) increased the centre deflection.

The Aluminium & Zirconium FGM plate had a small error margin of between 1.3% – 2.7% (Table 6.2). The results had an interesting trend. For the Aluminium & Zirconium FGM plate using a square geometry (50501cm3) the results from MATLAB gave a smaller deflection value than ANSYS however for the rectangular geometry (501001cm3) it gave a larger deflection value.

The Aluminium & Silicon Carbide FGM plate had a small error margin of between 1.2% – 2.8%. The centre deflection was significantly less due to the high Young's Modulus of the Silicon Carbide (410GPa). This shows that having a ceramic component gives more stiffness to the structure.

FAILURE ANALYSIS

Analysis of the progressive failure using failure index maps showed that the structure remains intact at operating loads of (FI< 1.0). However, when applying the maximum transverse force it caused critical stresses to occur in the structure (FI > 1.0). The failure occurred due to the bottom layer (Layer 10) being under tension and having a high metal content. This caused micro cracking within the matrix and initial yielding. The results from ANSYS and MATLAB were very similar which proves that the FSDPT numerical model developed is accurate.

VII. FUTURE WORK

The present study can be further extended in several ways to improve the analysis and understanding of the mechanical behaviour of Functionally Graded Material (FGM) plates. Future work can focus on thermal-structural analysis by investigating the behaviour of FGM plates under combined high-temperature and pressure conditions, which are commonly experienced in aerospace applications. The existing MATLAB numerical solver can be further developed to perform dynamic analysis, including the determination of natural frequencies, vibration modes, modal responses and other dynamic effects. In addition, Higher Order Shear Deformation Plate Theory (HSDPT) can be implemented to improve the accuracy of the analysis, particularly for thicker plates, by providing a more detailed representation of shear deformation. Further investigations can also be carried out using different metal-ceramic material combinations, volume fraction distributions and material gradation patterns to study their effects on stiffness, strength and deformation. Finally, experimental testing can be performed to validate the numerical results obtained from MATLAB and ANSYS, thereby improving the reliability and practical applicability of the proposed analysis for aerospace structural applications.

VIII. CONCLUSION

The static bending and progressive failure behaviour of Functionally Graded Material (FGM) rectangular plates was successfully analysed using First-Order Shear Deformation Plate Theory (FSDPT) and a custom MATLAB finite element solver. The results were validated using ANSYS Workbench 2026 R1 for different material combinations, showing a close agreement with a small percentage difference of 1.2%–2.8%. The parametric study showed that transverse deflection increases with applied load and plate aspect ratio, while material properties significantly influence structural stiffness. Among the materials studied, Silicon Carbide showed higher stiffness and lower deflection due to its high elastic modulus. Progressive failure analysis indicated that the plates remain safe under normal operating loads, while failure initiates at the bottom metal-rich layer due to matrix cracking and yielding under higher loads. Future improvements can include thermal-structural and dynamic analysis, along with the implementation of Higher-Order Shear Deformation Plate Theory (HSDPT) for better accuracy in thicker plates.

REFERENCES

1. *Mamandi, A. (2023). Finite element based bending analysis of rectangular FGM plates using first-order shear deformation theory. Journal of Mechanical Science and Technology, 37(5), 2491–2506.*
2. *Kashtalyan, M. (2004). Three-dimensional elasticity solution for bending of functionally graded rectangular plates. Composites Part B: Engineering, 35(7–8), 653–664.*
3. *Daouadji, T. H., Tounsi, A., & Adda Bedia, E. A. (2013). Analytical solution for bending analysis of functionally graded plates. Composite Structures, 97, 133–143.*
4. *Kulkarni, K., Singh, B. N., & Maiti, D. K. (2010). Analytical solution for bending and buckling analysis of functionally graded plates using inverse trigonometric shear deformation theory. Applied Mathematical Modelling, 34(3), 582–599.*
5. *Nguyen, T. K., Sab, K., & Bonnet, G. (2008). First-order shear deformation plate models for functionally graded materials. Composite Structures, 83(1), 25–36.*
6. *Singha, M. K., Prakash, T., & Ganapathi, M. (2011). Finite element analysis of functionally graded plates under transverse load. Finite Elements in Analysis and Design, 47(5), 482–489.*
7. *Reddy, J. N. (2000). Analysis of functionally graded plates. International Journal of Numerical Methods in Engineering, 47(1–3), 663–673.*
8. *Talha, M., & Singh, B. N. (2010). Static response and stress analysis of functionally graded plates using higher order shear deformation theory. Applied Mathematical Modelling, 34(11), 3991–4011.*
9. *Yang, J., & Shen, H. S. (2002). Non-linear bending of functionally graded plates under transverse load in thermal environments. International Journal of Solids and Structures, 39(3), 585–602.*
10. *Natarajan, S., Manickam, G., & Barkanov, E. (2012). Bending and free vibration of functionally graded plates using an es-pFEM. Finite Elements in Analysis and Design, 59, 1–11.*
11. *Praveen, G. N., & Reddy, J. N. (1998). Nonlinear transient thermoelastic analysis of functionally graded ceramic-metal plates. International Journal of Solids and Structures, 35(30), 4457–4476.*
12. *Zenkour, A. M. (2005). A comprehensive analysis of functionally graded sandwich plates: Part 1—Deflection and stresses. International Journal of Solids and Structures, 42(18–19), 5224–5242.*
13. *Matsunaga, H. (2008). Vibration and stability of functionally graded plates subjected to in-plane stresses based on a global higher-order deformation theory. Composite Structures, 82(4), 540–550.*
14. *Qian, L. F., Batra, R. C., & Chen, L. M. (2004). Free and forced vibrations of functionally graded plates using higher-order shear and normal deformable plate theory. Composites Part B: Engineering, 35(7–8), 645–658.*
15. *Chakraborty, A., Gopalakrishnan, S., & Reddy, J. N. (2003). A new element for analysis of functionally graded plates based on higher-order shear deformation theory. International Journal for Numerical Methods in Engineering, 57(5), 571–589.*
16. *Ferreira, A. J. M., Roque, C. M. C., & Martins, P. A. L. (2003). Analysis of functionally graded plates using radial basis functions. Composite Structures, 57(1–4), 407–414.*
17. *Shariat, B. S., & Eslami, M. R. (2006). Thermal buckling of functionally graded rectangular plates based on first-order shear deformation theory. Composite Structures, 72(4), 470–480.*

18. *Mahi, A., Bedia, E. A. A., & Tounsi, A. (2015). A new refined plate theory for bending and free vibration analysis of functionally graded plates. Mechanics of Advanced Materials and Structures, 22(4), 275–285.*

19. *Kim, J. S., & Lee, J. Y. (2009). Bending and free vibration analysis of functionally graded plates using a nine-node element. Journal of Sound and Vibration, 321(3), 579–595.*

20. *Voigt, W. (1928). Lehrbuch der Kristallphysik. BG Teubner, Leipzig.*