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An Adaptive Cognitive Probability Distortion Model for Modelling Student Teachers' Probabilistic Reasoning: Mathematical Formulation and MATLAB Implementation

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ABSTRACT:

Mathematical modelling offers a specific Framework for representing probabilistic reasoning under uncertain situations by integrating computational and cognitive principles. Existing studies on probability education and randomness have largely depended on empirical observation to analyse misconceptions, providing specific investigation and theoretical mechanisms for analysing how cognitive biases support probabilistic judgement. The study provides an Adaptive Cognitive Probability Distortion Model (ACPD), which involves formal probabilistic reasoning, heuristic reasoning and dynamic learning practices within a mathematical framework. The proposed model involves Bayesian belief updating, Markov cognitive state transition and weighted probability estimation to cover changes regarding probability reasoning throughout the reasoning tasks. Additionally, the mathematical formulation is supported through computational application and analytical support, empowering systematic analysis of learning patterns and biased behaviour. The proposed framework supports a foundation for predictive modelling of probabilistic reasoning and acknowledges a computational approach for analysing cognitive processes along with mathematical education.

Keywords: Adaptive Cognitive Probability Distortion Model, Bayesian Inference, Cognitive Modelling, Markov Chains, Mathematical Modelling, Probability Theory, MATLAB Simulation, Probabilistic Reasoning

1. Introduction

Mathematical modelling has been considered a necessary analytical approach for identifying complex cognitive processes because it supports abstract reasoning to be presented through mathematical structures (Vogelsanger-Holenstein et al., 2025). Within probability education, modelling techniques suggest opportunities to analyse individual uncertainty interpretation, revise beliefs and make necessary observations when objective formation is supported with intuitive judgement. Though previous educational research has accurately examined common misconceptions regarding probability and randomness, several investigations remain limited and descriptive in suggesting reasoning behaviour under cognitive condition differences (Ingram, 2022). Therefore, there is a requirement for a computational framework to incorporate mathematical theory with cognitive mechanisms and analyse how probabilistic analysis supports reasoning over time.

The present research navigates the requirement by suggesting an integrated mathematical model, which involves probability theory, adaptive learning practices and heuristic reasoning. Apart from considering cognitive biases as distributed phenomena, the Framework supports the combined influence on probability while analysing belief revision and transitions between reasoning states (Mirbagheri & Ata, 2025). Therefore, this mathematical interpretation supports systematic analysis of probabilistic reasoning through computational execution and offers a theoretical base for analysing how cognitive bias and learning interact with decision-making approaches. As argued by Felix et al. (2025), the feasibility of this proposed framework contributes largely to the dynamic interpretation of predictive mathematical model capability development and supports future computational analysis in the probability education aspect.

2. Mathematical Foundation

The proposed framework is analysed by mathematical principles, which describe the stochastic behaviour, uncertainty and adaptability regarding the decision-making approach. For instance, classical probability theory offers the objective measurement of event occurrence and prioritises the reference points against subjective probability judgement (Bajpai, 2024). However, human reasoning frequently deviates from normative probability because cognitive processes are supported by previous experience, heuristic judgement and contextual interpretation. Therefore, mathematical presentation of probabilistic reasoning needs a capable framework for identifying both objective probability and cognitive variation (Jiwa & Gottlieb, 2026).

Bayesian inference supports the initial mathematical component of the framework by depicting updating after new evidence becomes available (Felobes, 2024). This approach supports successive probability estimation to justify evolution according to the observed identification, rather than remaining static throughout the reasoning processes. In the words of Caicedo-Solano et al. (2025), Markov processes offered the second component by modelling transitions between formal reasoning and heuristic reasoning states as stochastic events influenced by task and learning complexity. These probabilistic transformations support repeated reasoning activities to be presented. Moreover, weighted probability formulation justifies a mathematical relationship between heuristic perspective and normative probability with the degree of formal reasoning achieved by an individual (Borovcnik & Kapadia, 2018). Together, these mathematical foundations offered a theoretical basis for constructing the adaptive cognitive probability distortion model and its computational execution.

3. Research Aims and Objectives

3.1 Aim

To develop a mathematical model that describes how student teachers conceptualise randomness and probability, incorporating cognitive biases and reasoning patterns without relying on empirical survey data.

3.2 Objectives

1. To define key variables and constructs related to randomness perception, such as intuitive understanding, misconceptions, and decision-making under uncertainty
2. To formulate equations / models representing how these constructs interact, including how biases distort probabilistic reasoning
3. To use probabilistic and statistical principles (e.g., Bayesian inference, Markov chains, or cognitive weights) to simulate student-teacher responses in hypothetical scenarios
4. To validate the proposed model through the comparison with different conceptual models on decision-making in classroom and real-life contexts
5. To conduct sensitive analysis of model parameters to examine the impact of different biases on decision outcomes

4. Methodology

The research methodology will elaborate on how the research will be designed and incorporate mathematical models for teacher reasoning. It will include five individual steps such as theoretical modelling, variable definition, mathematical formulation, scenario simulation, and output analysis.

The mathematical model is implemented to address a central limitation in existing probability-education research: the predominance of descriptive and empirical approaches that identify misconceptions but do not formally explain how they interact, evolve, or influence decision-making across contexts. Mathematics provides a rigorous and generalisable framework to represent cognitive processes underlying randomness perception, allowing theoretical prediction without reliance on primary survey data. By formalising reasoning patterns, the model enables controlled experimentation with cognitive biases, task difficulty, and feedback, which is not possible through interviews or classroom observations alone.

Phase 1: Mathematics

In the first phase, mathematics is used to construct a formal representation of student teachers' reasoning. Key variables—true probability, perceived probability, cognitive state, bias parameters, and decision outcomes—are defined and linked through equations. A weighted probability formulation captures how heuristic judgments distort normative probability, while Bayesian components model belief updating in response to evidence. Markov chains are employed to represent transitions between heuristic and formal reasoning states across successive tasks. These mathematical structures allow the simulation of cognitive dynamics such as persistence of misconceptions, regression under task complexity, and gradual learning effects. MATLAB will be used to implement equations, estimate parameters, and run simulations, ensuring internal consistency and sensitivity testing.

Phase 2: Application

In the second phase, the mathematical model is applied to hypothetical classroom and real-life scenarios, such as dice experiments, coin tosses, or media-based probability judgments. Simulated trials generate response patterns that reflect common misconceptions and reasoning shifts. The outputs are analysed to identify stable bias configurations, critical transition points, and conditions under which formal reasoning dominates heuristic thinking. These findings have direct applications in teacher education, informing instructional design, diagnostic assessment, and targeted interventions. Thus, mathematics functions not merely as a computational tool but as an explanatory and predictive mechanism bridging cognitive theory and educational practice.

5. Theoretical model development

The initial step will be two developer theoretical frameworks, where it will connect cognitive science, educational psychology, and probability theory. Five different sub-models will empower this model development.

- **Cognitive state representation** will showcase how teachers transform between formal and heuristic reasoning.
- **Perceived probability models** will elaborate on how subjective beliefs change into probability dynamics.
- **The decision function** will include perceived probability with necessary judgmental responses.
- **Bias modelling** will integrate distortions like the Gambler's fallacy or base-rate neglect.
- **Simulation of responses** will identify how these models interact across the different tasks.

6. Adaptive Cognitive Probability Distortion Model (ACPDM)

6.1 Cognitive System

Cognitive System

Assuming,

At trial t ,

the student teacher exists in a cognitive state:

$$C_t \in \{F, H\} \quad (1)$$

Where,

- F = Formal probabilistic reasoning
- H = Heuristic/intuitive reasoning

True Probability

Supposing an event

E = "rolling a 6"

True probability:

$$P(E) = \frac{1}{6} \quad (2)$$

6.2 Heuristic Distortion Model

Heuristic Distortion

Human cognition distorts the true probability

$$H_t(E) \quad (3)$$

This is heuristic perception.

Original Heuristic Distortion Equation

$$H_t(E) = P(E) + \alpha R_t + \beta G_t + \gamma A_t + \delta B_t \quad (4)$$

Where:

R_t = Representativeness bias

G_t = Gambler's fallacy

A_t = Anchoring effect

B_t = Base-rate neglect

$\alpha, \beta, \gamma, \delta$ are Bias strengths

6.3 Dynamic Formal Reasoning Weight

Creating Dynamic Formal Reasoning Weight

$$w_t \in [0,1] \quad (5)$$

Where:

$w_t = 1 \rightarrow$ purely formal reasoning

$w_t = 0 \rightarrow$ purely heuristic reasoning

Original Learning-Weight Equation

$$w_{t+1} = w_t + \lambda(L_t - E_t) - \kappa D_t \quad (6)$$

Where:

L_t = learning gain

E_t = cognitive error

D_t = task difficulty

λ = learning rate

κ = cognitive overload parameter

6.4 Adaptive Cognitive Probability Distortion Model

Master Perceived Probability Equation

After combining formal and heuristic reasoning,

$$P_t'(E) = w_t P(E) + (1 - w_t) H_t(E) \quad (7)$$

Substituting the Heuristic Model

$$H_t(E) = P(E) + \alpha R_t + \beta G_t + \gamma A_t + \delta B_t \quad (8)$$

$$\Rightarrow P_t'(E) = w_t P(E) + (1 - w_t) [P(E) + \alpha R_t + \beta G_t + \gamma A_t + \delta B_t] \quad (9)$$

$$\Rightarrow P_t'(E) = P(E) + (1 - w_t)(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) \quad (10)$$

As formal reasoning increases

$$w_t \rightarrow 1$$

Bias effects disappear.

When heuristic reasoning dominates

$$w_t \rightarrow 0$$

Bias distortion becomes maximal.

Adaptive Cognitive Probability Distortion Model (ACPDMD)

$$P_t'(E) = P(E) + (1 - w_t)(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) \quad (11)$$

7. Model Implementation

7.1 Decision Model

Decision Function

Logistic decision theory

$$Pr(D_t = 1) = \frac{1}{1 + e^{-\theta(P_t'(E) - \tau)}} \quad (12)$$

Where:

θ = sensitivity

τ = decision threshold

7.2 Cognitive State Transition

Markov Cognitive Transition Model

State Transition Matrix

$$T = \begin{bmatrix} P_{FF} & P_{FH} \\ P_{HF} & P_{HH} \end{bmatrix} \quad (13)$$

Where:

P_{FH} = transition from Formal $\rightarrow$ Heuristic

P_{HF} = transition from Heuristic $\rightarrow$ Formal

Dynamic Transition Probability

$$PHF(t) = \frac{e^{\eta_1 + \eta_2 L_t - \eta_3 D_t}}{1 + e^{\eta_1 + \eta_2 L_t - \eta_3 D_t}} \quad (14)$$

Transition toward formal reasoning increases with learning and experience and decreases with task complexity

7.3 Bayesian Learning

Bayesian Update

$$P_{t+1}(H | E) = \frac{P(E|H)P_t(H)}{P(E)} \quad (15)$$

This model updates beliefs after evidence.

Student teachers revise beliefs after observing outcomes.

7.4 Integrated Mathematical Model

Full Integrated Thesis Model

ACPDm

$$P'_t(E) = P(E) + (1 - w_t)(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) \quad (16)$$

Learning Adaptation

$$w_{t+1} = w_t + \lambda(L_t - E_t) - \kappa D_t \quad (17)$$

Master Perceived Probability

$$P'_t(E) = w_t P(E) + (1 - w_t) H_t(E) \quad (18)$$

Decision Model

$$Pr(D_t = 1) = \frac{1}{1 + e^{-\theta(P'_t(E) - \tau)}} \quad (19)$$

Cognitive Transition

$$PHF(t) = \frac{e^{\eta_1 + \eta_2 L_t - \eta_3 D_t}}{1 + e^{\eta_1 + \eta_2 L_t - \eta_3 D_t}} \quad (20)$$

7.5 Numerical Illustration

Calculation

Supposing,

$$P(E) = 0.5$$

Bias values,

$$R_t = 0.2,$$

$$G_t = 0.1,$$

$$A_t = 0.15,$$

$$B_t = 0.05$$

Bias weights,

$$\alpha = 0.4,$$

$$\beta = 0.3,$$

$$\gamma = 0.2,$$

$$\delta = 0.1$$

Formal reasoning weight,

$$w_t = 0.6$$

Calculating Heuristic Estimate

$$H_t(E) = P(E) + \alpha R_t + \beta G_t + \gamma A_t + \delta B_t \quad (21)$$

$$\begin{aligned} H_t(E) &= 0.5 + 0.4(0.2) + 0.3(0.1) + 0.2(0.15) + 0.1(0.05) \\ &= 0.5 + 0.08 + 0.03 + 0.03 + 0.005 \\ &= 0.645 \end{aligned}$$

Calculating Perceived Probability

$$P'_t(E) = P(E) + (1 - w_t)(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) \quad (22)$$

$$\begin{aligned} P'_t(E) &= 0.5 + (1 - 0.6)[0.4(0.2) + 0.3(0.1) + 0.2(0.15) + 0.1(0.05)] \\ &= 0.558 \end{aligned}$$

Objective probability = 50%

Subjective perceived probability = 55.8%

The distortion arises from cognitive biases.

7.6 Model Constraints

Domain Conditions

Probability bounds

$$0 \leq P(E) \leq 1 \quad (23)$$

$$0 \leq P_{t'}(E) \leq 1 \quad (24)$$

Weight constraint

$$0 \leq w_t \leq 1 \quad (25)$$

Bias parameters

$$\alpha, \beta, \gamma, \delta \geq 0 \quad (26)$$

7.7 Mathematical Interpretation

Model Interpretation

If $w_t > 1$

Then

$$P_{t'}(E) \rightarrow P(E) \quad (27)$$

formal reasoning dominates

7.8 Sensitivity and Stability

Sensitivity to representativeness bias

$$\frac{\partial P'_t(E)}{\partial \alpha} = (1 - w_t)R_t \quad (28)$$

Distortion increases with representativeness

Distortion decreases with formal reasoning

Sensitivity to gambler's fallacy

$$\frac{\partial P'_t(E)}{\partial \beta} = (1 - w_t)G_t \quad (29)$$

Sensitivity to anchoring

$$\frac{\partial P'_t(E)}{\partial \gamma} = (1 - w_t)A_t \quad (30)$$

Sensitivity to base-rate neglect

$$\frac{\partial P'_t(E)}{\partial \delta} = (1 - w_t)B_t \quad (31)$$

Differentiate with respect to formal reasoning

$$\frac{\partial P'_t(E)}{\partial w_t} = -(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) \quad (31)$$

This shows,

$$\frac{\partial P'_t(E)}{\partial w_t} < 0 \quad (32)$$

This proves increasing formal reasoning ALWAYS reduces probabilistic distortion.

Second-Order Analysis

$$\frac{\partial^2 P'_t(E)}{\partial w_t^2} = 0 \quad (33)$$

linear suppression effect.

Equilibrium Analysis

$$P'_t(E) = P(E) \quad (34)$$

Then

$$(1 - w_t)(\alpha R_t + \beta G_t + \gamma A_t + \delta B_t) = 0 \quad (35)$$

Case 1

$$w_t = 1$$

Perfect formal reasoning.

Case 2

All biases vanish

$$\alpha = \beta = \gamma = \delta = 0 \quad (36)$$

This creates a theoretical equilibrium condition.

Stability Analysis

$$w_{t+1} = w_t + \lambda(L_t - E_t) - \kappa D_t \quad (37)$$

When

$$w_t \rightarrow 1$$

This shows stable formal cognition develops.

Sensitivity Calculation

$$w_t = 0.7$$

$$R_t = 0.4$$

$$\frac{\partial P'_t(E)}{\partial \alpha} = (1 - w_t)R_t \quad (38)$$

$$\begin{aligned}\frac{\partial P'_t(E)}{\partial \alpha} &= (1 - 0.7)0.4 \quad (39) \\ &= 0.3(0.4) \\ &= 0.12\end{aligned}$$

8. Computational Analysis and Discussion

The computational execution shows that the proposed Adaptive Cognitive Probability Distortion Model (ACPDM) offers a mathematical representation of probabilistic reasoning by involving objective probability as an adaptive learning technique within a unified framework. Rather than traditional probability models, which estimate rational decision-making, the ACPDM accounts for cognitive biases for systematic distortion of subjective probability estimation (Yang et al., 2023).

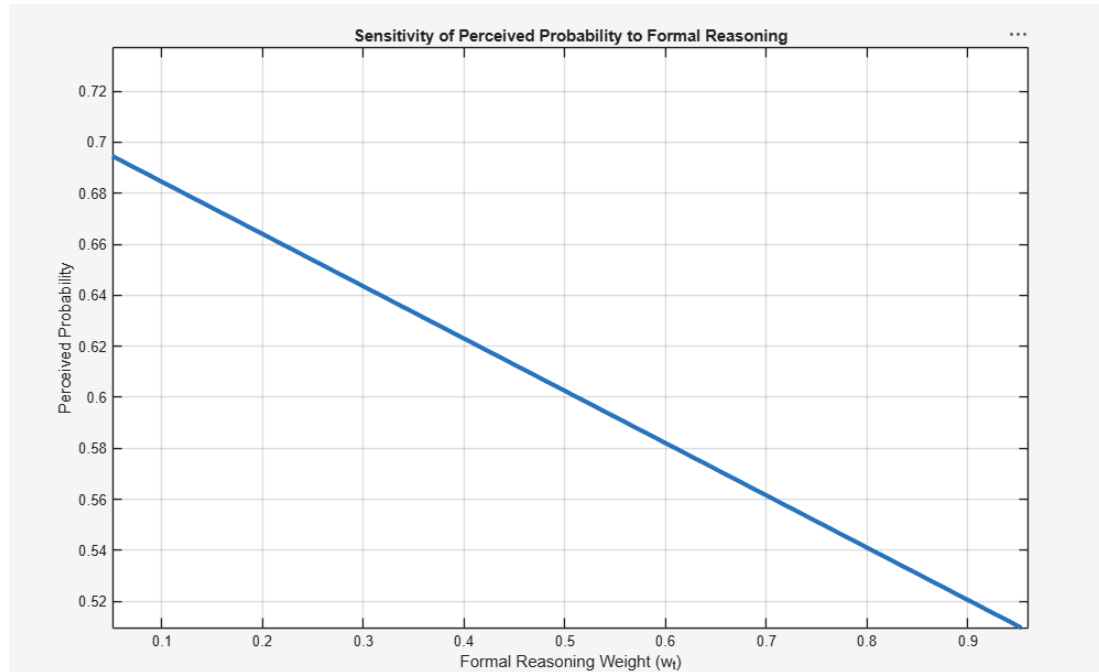


Figure 1: Sensitivity Analysis

The numerical representation and shares show that obtained probability changes from objective probability when heuristic reasoning dominates. In this regard, reflecting the significance of base-rate neglect, gamblers' fallacy and the influence of representativeness are crucial (AlKhars et al., 2019). From figure 1, the finding is consistent with the practical assumption and probabilistic judgement if not analysed by statistical information by cognitive processes.

The sensitivity analysis supports the validation of the same work by focusing on individual bias parameter contribution and the obtained probability. As mentioned by Ben-Baruch & Cohen (2025), the partial deviation refers to the fact that individual bias provides a measurement of proportional distortion and probabilistic effect, while an increment in formal reasoning constantly suppresses these differences.

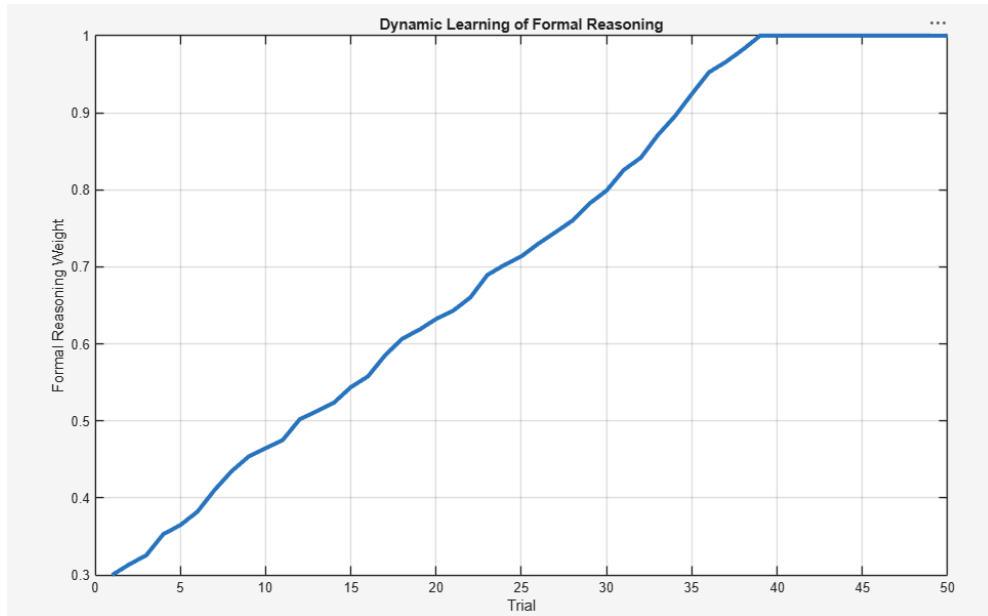


Figure 2: Dynamic Learning Model

Therefore, the model supports that improvements in terms of probabilistic understanding and learning practices decrease dependency on intuitive analysis. Moreover, the equilibrium interpretation supports two different political conditions under subjective probability that refer to objective probability (Başar, 1983; Battigalli et al., 2022). It is namely the complete cognitive bias neglect or the achievement of formal reasoning. The stability analysis also recommends that learning repetition experiences convergence contribution towards cognitive state stability, acknowledging the nature of the proposed model.

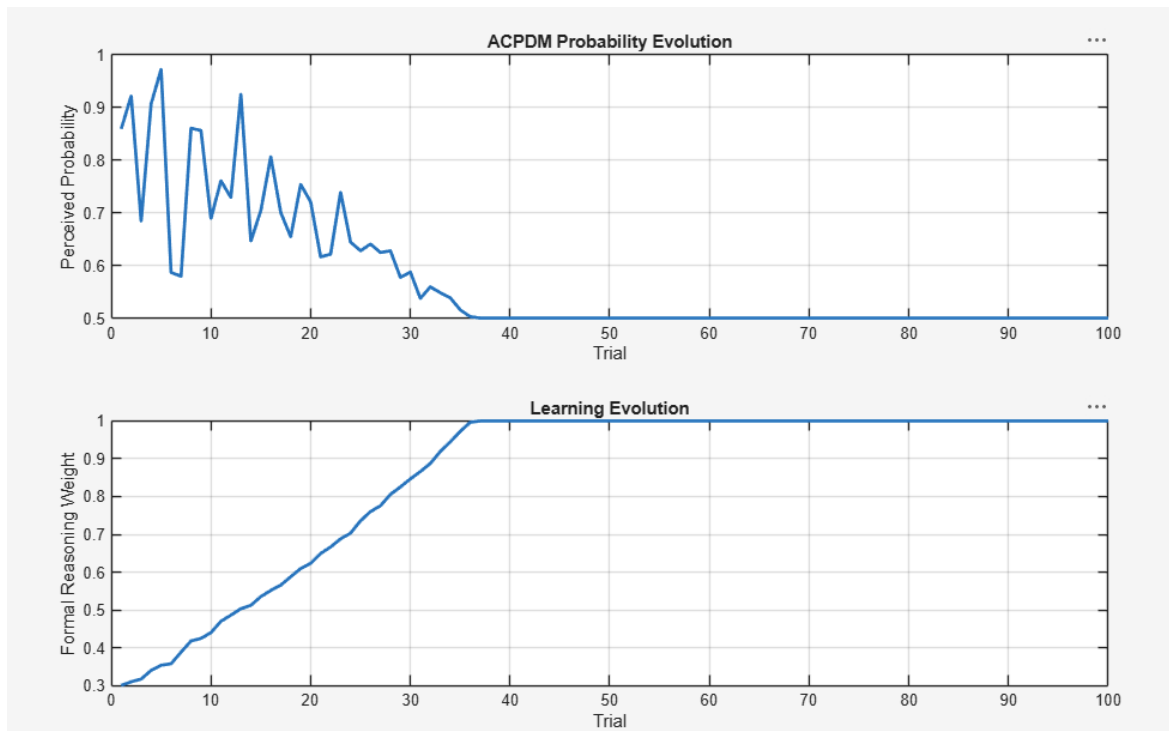


Figure 3: Full Integrated Model

The Bayesian updating mechanism supports the computational capacity of the framework by supporting revised and systematic follow-up for new evidence (McCann, 2020). Therefore, it suggests the iterative nature of probabilistic learning. Moreover, the Markov heuristic and formal reasoning define cognitive modification to be presented at transitions rather than distributed events (Zylberberg & Yang, 2021). Consequently, these mathematical components ensure the framework to analyse both reasoning behaviour and cognitive development in both the short and long term. According to figure 3, the computational analysis supports that the ACPDM is mathematically constant, flexible and logically stable for executing the simulation techniques.

Therefore, the framework offers a rigid foundation of probabilistic reasoning and educational intervention to support computational research approaches to mathematical education (Osei-Wusu et al., 2026).

9. Conclusion

The study represented the mathematical Framework for modelling student teachers' probabilistic reasoning via the Adaptive Cognitive Probability Distortion Model. By integrating cognitive bias parameters, Bayesian belief updating, adaptive learning approaches and Markov cognitive transition, the proposed model moves beyond descriptive research towards a predictive computational approach. This computational execution reflected formal reasoning to decrease heuristic distortion, whereas equilibrium, sensitivity and stability support internal consistency transformation and mathematical formulation. Such findings suggest that probabilistic reasoning can be presented as a dynamic cognitive process supported by previous beliefs, learning approaches and contextual factors apart from behavioural outcomes. Additionally, the proposed framework supports both computationally and theoretically the study of education by suggesting a structured and empirical validation. As a result, it has been targeted to improve cognitive reasoning and understanding under uncertain situations.

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