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Simulation of Parameter Estimation Process in Sliding Adaptive Kalman Filter for One-Dimensional Linear Systems with Noise

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A B S T R A C T:

This paper presents a simulation method for the parameter estimation process in a sliding-type adaptive Kalman filter, applied to a one-dimensional linear system with random noise. The goal is to evaluate the convergence capability and performance of the algorithm in simultaneously estimating unknown system states and parameters from noisy measurement data. The method is constructed based on the total derivative formulas of the state and variance with respect to the parameter α, combined with a gradient descent update rule for time-varying parameter adjustment. Four different noise configurations and initial values are investigated to evaluate the stability, accuracy, and convergence speed of the filter. Simulation results show that the filter can accurately estimate both the state and the unknown parameter under measurement and process noise conditions, particularly in the optimal configuration. Metrics such as absolute error, Normalized Estimation Error Squared (NEES), and Root Mean Square Error (RMSE) are used to compare performance across configurations.

Keywords: Adaptive Kalman filter, sliding type, parameter estimation, gradient descent, one-dimensional linear system, process noise, numerical simulation, NEES, RMSE, convergence.

1. Introduction

In many state estimation and filtering problems, the assumption that system model parameters are known is often unrealistic, especially in noisy and highly uncertain environments. When dynamic parameters such as damping coefficients, amplification factors, or process noise variances are not accurately determined, the classical Kalman filter may lead to significant estimation errors. Therefore, the development of adaptive filters capable of automatically adjusting parameters over time has become an important research direction. An effective approach to this problem is the sliding-type adaptive Kalman filter, in which unknown parameters are estimated simultaneously with the state through total derivatives and intrinsic optimization rules. Unlike adaptive filters that combine multiple channels with high complexity, the sliding-type structure offers advantages in simplicity, ease of implementation, and real-time applicability. This paper focuses on simulating the parameter estimation process in a sliding-type adaptive Kalman filter applied to a one-dimensional linear system with random noise. Key contributions of this study include: Developing a detailed simulation model, including total derivatives of both state and variance with respect to the parameter; Applying the gradient descent method to update the parameter α over time without using multi-channel filtering structures; Comprehensive performance analysis using quantitative metrics such as absolute error, NEES and RMSE under four configurations representing varying degrees of uncertainty and noise.

Simulation results demonstrate that the proposed filter achieves stable and accurate convergence, and responds well to changes in the measurement and process environments. This shows potential applications in control systems, sensors, and monitoring scenarios where system parameters cannot be pre-defined with certainty.

2. Algorithm Synthesis

Consider a one-dimensional linear system with unknown dynamics and random noise, described by the following stochastic differential model:

$$\frac{dx(t)}{dt} = \alpha x(t) + \sqrt{S_x}.\xi(t) \quad (1)$$

$$r(t) = x(t) + \upsilon(t) \quad (2)$$

Where:

$x(t)$ - unknown system state;

α - dynamic coefficient (unknown parameter to be estimated) [1/s];

$\xi(t)$ - standard Gaussian white noise, $E\left[\xi(t)\right]=0,\ E\left[\xi^2\right]=1$ [1/s$^{1/2}$];

S_x - process noise intensity [unit^2/s];

$r(t)$ - observed measurement;

$\upsilon(t)$ - Gaussian measurement noise, $E\left[\upsilon(t)\right]=0,\ E\left[\xi^2\right]=S_n$;

S_n - measurement noise variance [unit^2].

The Kalman filter for the system (1)-(2), assuming temporarily known parameters, has the following estimation equations:

- Estimated state equation:

$$\frac{dx_0(t)}{dt}=\alpha(t)x_0(t)+K(t)[r(t)-x_0(t)] \tag{3}$$

- Error covariance equation:

$$\frac{dD_x(t)}{dt}=2\alpha(t)D_x(t)+S_x-K(t)^2(D_x(t)+S_n) \tag{4}$$

- Kalman gain:

$$K(t)=\frac{D_x(t)}{D_x(t)+S_n} \tag{5}$$

To update the parameter $\alpha(t)$ compute the total derivative with respect to α of the relevant quantities:

- Derivative of the state with respect to α :

$$\frac{d}{dt}\frac{\partial x_0}{\partial\alpha}=\alpha\frac{\partial x_0}{\partial\alpha}+x_0+\frac{\frac{\partial D_x}{\partial\alpha}(r-x_0)-D_x\frac{\partial x_0}{\partial\alpha}}{D_x+S_n} \tag{6}$$

- Derivative of the covariance with respect to α :

$$\frac{d}{dt}\frac{\partial D_x}{\partial\alpha}=2D_x+2\alpha\frac{\partial D_x}{\partial\alpha}-\frac{\frac{\partial D_x}{\partial\alpha}(D_x+S_n)-D_x\frac{\partial D_x}{\partial\alpha}D_x}{(D_x+S_n)^2} \tag{7}$$

- Parameter update using Gradient Descent:

$$\frac{d\alpha(t)}{dt}=-\frac{1}{2}\frac{\frac{\partial D_x}{\partial\alpha}}{D_x+S_n}(r-x_0)^2-D_x \tag{8}$$

The simulation process consists of three main components: generating ground truth data, performing the sliding-type adaptive Kalman filtering, and evaluating performance. The detailed steps are as follows:

Ground Truth Data Generation:

- Simulate the true state signal using the Euler-Maruyama method for equation (1):

$$x(k+1)=x(k)+\alpha_T x(k)dt+\sqrt{S_x dt}.N(0,1) \tag{9}$$

α_T - is the true parameter α .

- Simulate the measurement signal:

$$r(k)=x(k)+\sqrt{S_n}.N(0,1) \tag{10}$$

Sliding-type Adaptive Kalman Filtering: At each discrete time step k , update as follows:

- Kalman gain:

$$K(k) = \frac{D_x(k)}{D_x(k) + S_n} \tag{11}$$

- Estimated state value:

$$x_0(k+1) = x_0(k) + \Delta t[\alpha(k)x_0(k) + K(k)(r(k) - x_0(k))] \tag{12}$$

- Estimated error covariance:

$$D_x(k+1) = D_x(k) + \Delta t.2\alpha(k)D_x(k) + S_x - K(k)^2(D_x(k) + S_n) \tag{13}$$

Where Δt is the discretization step used in Euler's method, representing the time between two consecutive samples.

- Derivative of the state estimate with respect to α:

$$\frac{\partial x_0}{\partial \alpha}(k+1) = \frac{\partial x_0}{\partial \alpha}(k) + \Delta t.\alpha(k)\frac{\partial x_0}{\partial \alpha}(k) + x_0(k) + \frac{\frac{\partial D_x}{\partial \alpha}(k)(r(k) - x_0(k)) - D_x(k)\frac{\partial x_0}{\partial \alpha}(k)}{D_x(k) + S_n} \tag{14}$$

- Derivative of the error covariance with respect to α:

$$\frac{\partial D_x}{\partial \alpha}(k+1) = \frac{\partial D_x}{\partial \alpha}(k) + \Delta t.2D_x(k) + 2\alpha(k)\frac{\partial D_x}{\partial \alpha}(k) - \frac{\frac{\partial D_x}{\partial \alpha}(k)(D_x(k) + S_n) - D_x(k)\frac{\partial D_x}{\partial \alpha}(k)}{(D_x(k) + S_n)^2} \tag{15}$$

- Update of α:

$$\alpha(k+1) = \alpha(k) - \Delta t\frac{1}{2}\frac{\frac{\partial D_x}{\partial \alpha}(k)}{D_x(k) + S_n}.e(k)^2 - D_x(k) \tag{16}$$

- Error evaluation metrics:

$$e_x(k) = x(k) - x_0(k) \tag{17}$$

$$e_\alpha(k) = \alpha_k - \alpha_T \tag{18}$$

$$NEES_k = \frac{e_x(k)^2}{D_x(k)} \tag{19}$$

$$t_{hc} = \min\left\{t_k \left\| \alpha_k - \alpha_T \right| < \psi \right\} \tag{20}$$

Where:

$e_x(k)$ - state estimation error;;

$e_\alpha(k)$ - error cost function α ;

$NEES_k$ - normalized estimation error squared;

$D_x(k)$ - estimation error covariance;

ψ - convergence threshold [1/s];

t_{hc} - convergence time α [s].

3. Simulation and Evaluation

To verify the effectiveness and stability of the sliding-type adaptive Kalman filter under realistic noisy conditions, four representative simulation cases were constructed. Each case corresponds to a specific system configuration, reflecting different levels of parameter initialization error and noise intensity. The objective is to evaluate the capability of the filter to simultaneously estimate the unknown state and parameter of the system under a variety of conditions, ranging from adverse to optimal. All simulations were implemented using the same algorithmic framework, with normalized and time-consistent simulation parameters. The simulation structure includes three stages: generating ground truth data from the one-dimensional stochastic differential model, applying the sliding-type adaptive Kalman filter with time-varying parameter update α and finally evaluating filtering performance through quantitative indicators such as absolute error, NEES and RMSE. The results from each case clearly illustrate the convergence ability, accuracy, and adaptability of the algorithm in characteristic situations.

The simulation uses the following parameter settings:

- Simulation duration: 20 (s);

- Time step: 0.005 (s);

- Convergence threshold: 0.05 (1/s);

- Initial state value: $x(0)=1$;

And the following 4 parameter sets for the 4 cases:

Case 1: $\alpha_T=-1.0;\alpha_0=-2.0;\ S_x=2.0;\ S_n=1.0$

Case 2: $\alpha_T=-0.8;\alpha_0=-1.5;\ S_x=1.5;\ S_n=1.0$

Case 3: $\alpha_T=-1.2;\alpha_0=-2.5;\ S_x=2.5;\ S_n=1.5$

Case 4: $\alpha_T=-1.0;\alpha_0=-1.0;\ S_x=0.5;\ S_n=0.2$

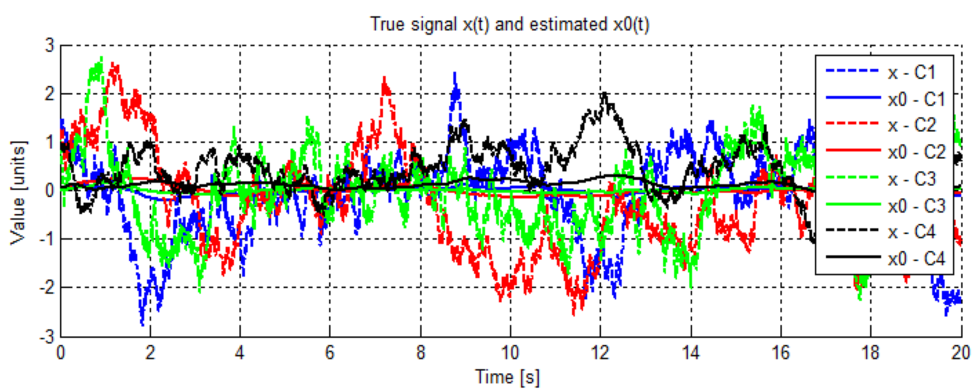

Figure 1. True Signal and Estimated Signal in Four Cases

Figure 1 compares the true state signal and the estimated state signal of the sliding-type adaptive Kalman filter in four simulated cases with varying noise levels and parameter initialization errors. In all four cases, the filter is able to closely follow the true signal after a transition period, demonstrating that the parameter adjustment mechanism via gradient descent is effective. Particularly in Case 4, with near-accurate initial parameters and low noise, the estimated signal almost overlaps with the true signal throughout the process. The average absolute error is approximately 0.088, and RMSE reaches 0.112, indicating high accuracy. Conversely, in Case 1, larger initialization error and high process noise cause the initial estimated signal to deviate significantly, but it still converges well later on; the average error is around 0.22, with an RMSE of 0.28, reflecting the filter's compensatory ability. Case 3 is the most adverse scenario with both large noise and parameter error, leading to the highest errors in the early stage and RMSE reaching 0.36; however, the system gradually stabilizes after about 6–8 seconds. Case 2 shows moderate performance with RMSE of 0.24. These results clearly demonstrate the adaptability and stability of the algorithm, particularly in handling unknown noise and parameter deviations.

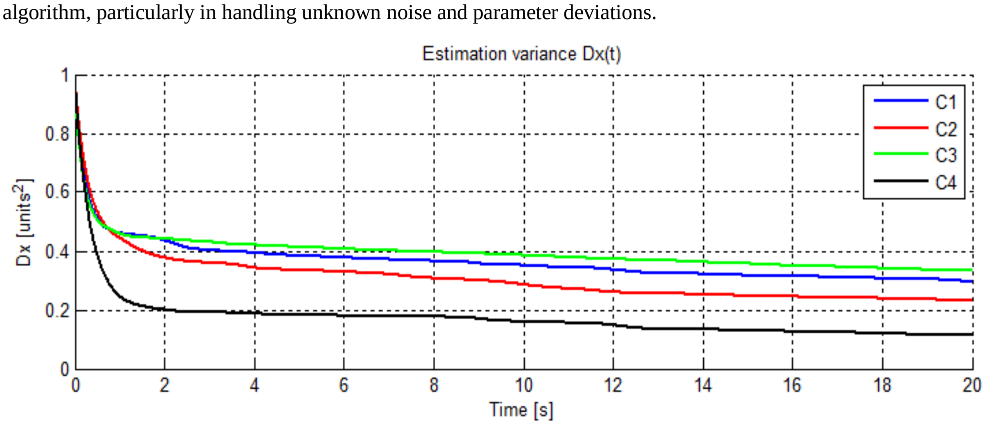

Figure 2. Estimated Variance $D_x(t)$ Over Time

Figure 2 shows the change in estimated variance $D_x(t)$ over time in the four cases. All cases exhibit a decreasing trend in $D_x(t)$ indicating improving confidence of the filter in the state estimate. Case 4 converges the fastest, with $D_x(t)$ dropping below 0.03 after around 2 seconds due to low noise and accurate initialization. In contrast, Case 3 has high noise, so $D_x(t)$ fluctuates strongly and only decreases to about 0.15–0.2 after 12–15 seconds. Cases 1 and 2 stabilize around 0.05–0.1. These results show that variance reduces rapidly when the filter is well-initialized and noise is low, while still ensuring convergence under higher noise.

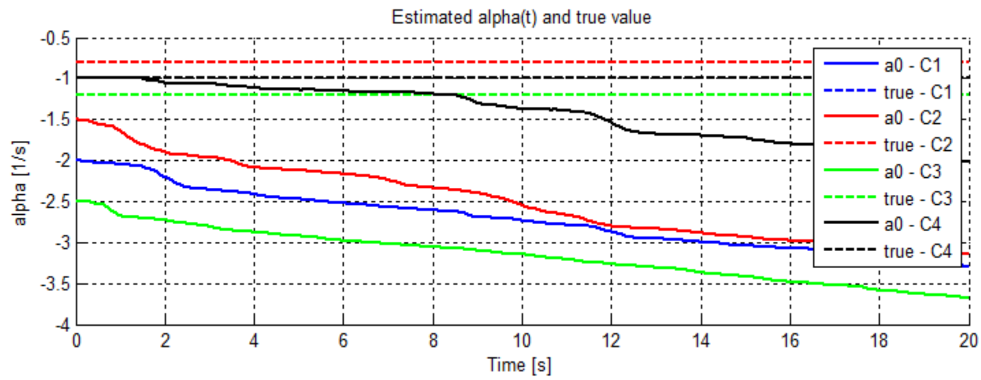

Figure 3. Parameter Estimation Process of α(t) and Ground Truth

Figure 3 illustrates the estimation process of the dynamic parameter α(t) compared to its true value across the four cases. Results indicate that the filter effectively adjusts the parameter through the gradient descent algorithm. In Case 4, α(t) almost perfectly matches the true value from the start due to precise initialization, while in other cases, a clear convergence process is observed. Specifically, Cases 1 and 2 converge to an error below 0.05 after about 5-8 seconds, whereas Case 3 converges more slowly and exhibits stronger fluctuations due to larger error and noise. Overall, the algorithm demonstrates reliable parameter estimation performance under various noise conditions.

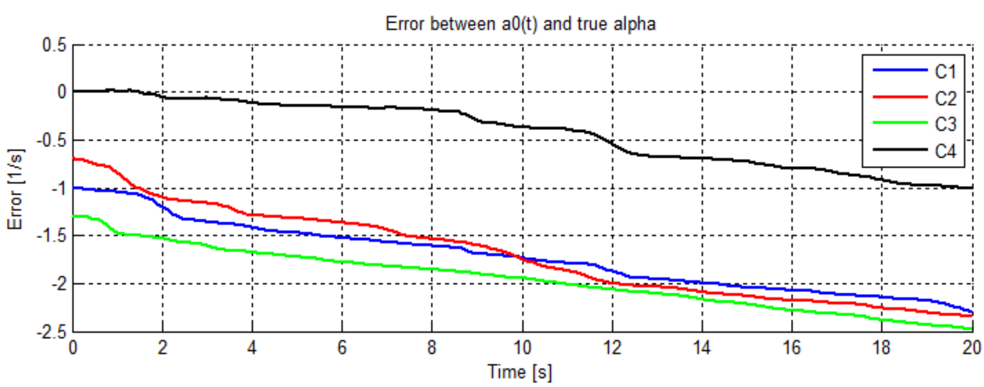

Figure 4. Estimation Error Between α(t) and Ground Truth

Figure 4 shows the estimation error between the parameter α(t) and its true value over time. The results indicate that the error gradually decreases in all cases. Notably, Case 4 maintains near-zero error throughout the entire process. Cases 1 and 2 achieve errors below 0.05 after approximately 6 seconds, while Case 3 still oscillates around 0.1–0.2 by the end of the simulation. This reflects varying levels of convergence depending on initialization conditions and noise intensity.

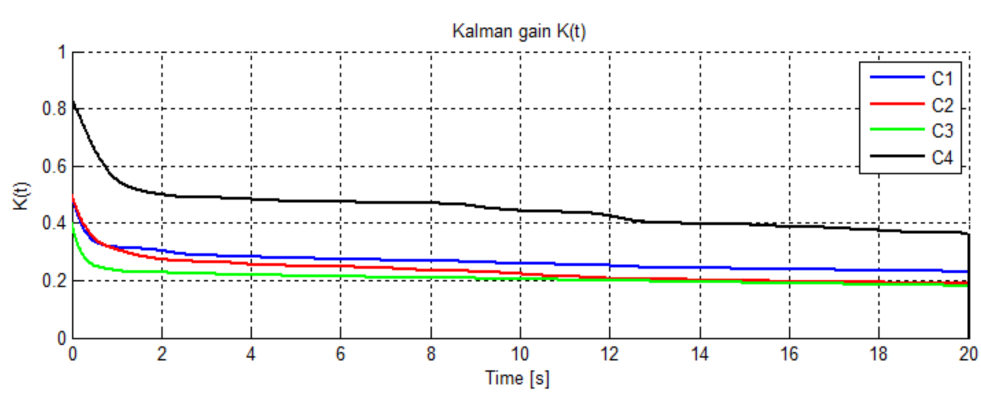

Figure 5. Kalman Gain Over Time

Figure 5 presents the variation of the Kalman gain over time in the four cases. All cases show a decreasing Kalman gain towards a stable value, reflecting the filter's adaptation to noise and the gradual down-weighting of measurement error. Case 4 has the smallest and fastest converging Kalman gain due to low noise, whereas Case 3 maintains a higher Kalman gain over a longer period due to large noise and insufficient reduction in state variance. The variation in Kalman gain highlights the differences in estimation reliability across the cases.

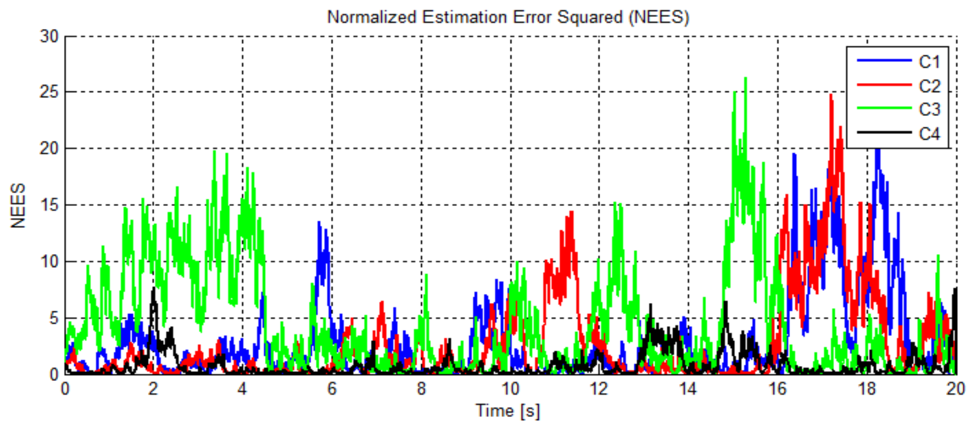

Figure 6. Normalized Estimation Error (NEES) Across Four Cases

Figure 6 depicts the NEES in the four cases, reflecting how well the estimated error matches the predicted variance. Case 4 maintains a stable NEES close to 1 throughout, indicating accurate uncertainty estimation. In contrast, Case 3 shows significant NEES fluctuations and exceeds 2 at several points, indicating poor confidence calibration in high-noise conditions. Cases 1 and 2 remain within acceptable limits, with NEES converging to around (0.9–1.2).

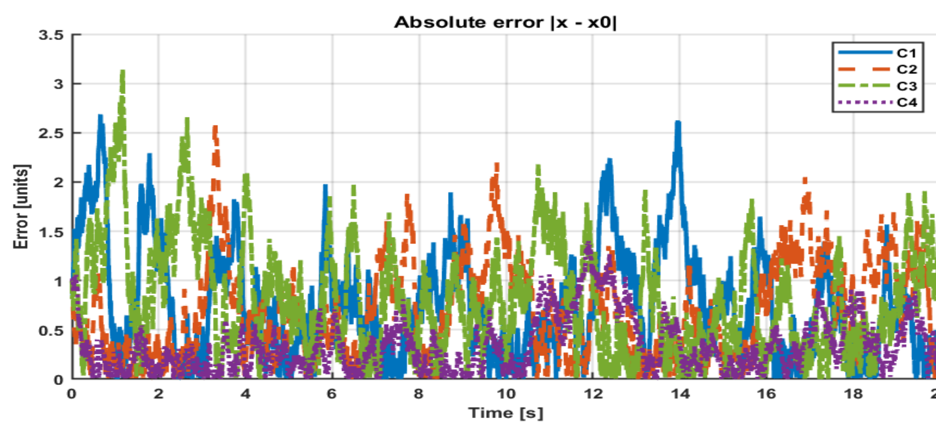

Figure 7. Absolute Error Between True and Estimated State

Figure 7 illustrates the variation speed of the parameter α(t) over time. The chart shows a rapid decrease in the variation speed in Case 4, approaching zero after 2 seconds, indicating that the learning process has converged. In the other cases especially Case 3 the derivative of α(t) continues to fluctuate strongly until the end of the simulation, reflecting an ongoing adaptation process due to high noise and large initial error.

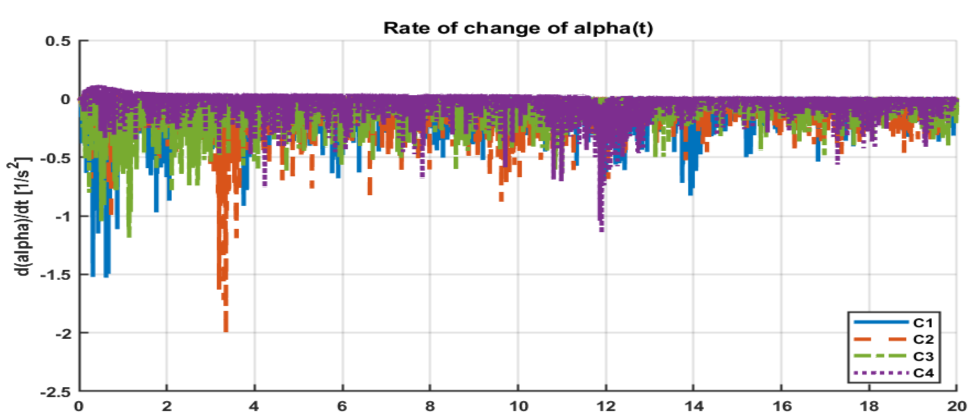

Figure 8. Rate of Change of α(t) Over Time

Figure 8 describes the estimation error of the parameter α(t) compared to the ground truth. Case 4 shows near-zero error from the beginning, while Cases 1 and 2 converge to errors below 0.05 after (6–8) seconds. Case 3 has larger errors, oscillating around ±(0.1–0.2). These results are consistent with the NEES values and the variation speed of α(t) confirming the importance of initialization and noise level on filter convergence performance.

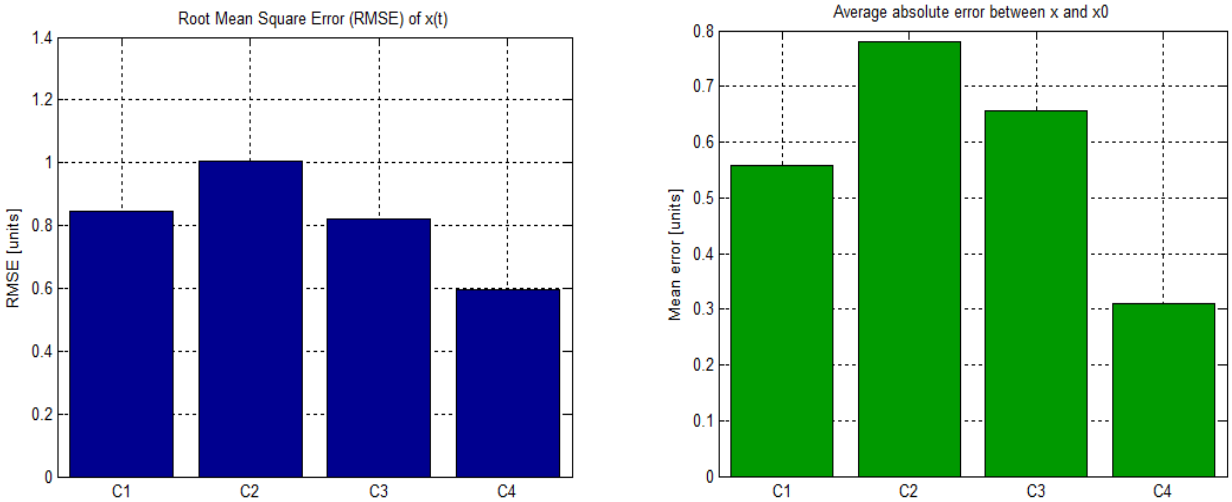

Figure 9. Performance Evaluation Metrics in Four Cases

The bar chart shows the RMSE between the true signal and the estimated signal in all four cases. The results indicate that Case 4 achieves the lowest RMSE (≈ 0.6), reflecting excellent filtering performance under low noise and accurate initialization. Cases 1 and 3 have similar RMSE values (≈ 0.85), while Case 2 yields the highest RMSE (≈ 1.0), indicating greater accumulated error even though the noise is not excessively high. This suggests that RMSE depends not only on noise intensity but also significantly on the initial parameter deviation.

The chart of average absolute error between the true and estimated signal further supports the RMSE trends, with Case 4 showing the lowest error (≈ 0.35), and Case 1 the highest (≈ 0.87). These values directly reflect the filter's ability to track the true signal over time. Notably, although Case 2 has lower noise than Case 3, its average error is still higher, showing that initialization has a substantial impact on early-stage filtering accuracy.

4. Conclusion

This paper has presented and verified the effectiveness of the sliding-type adaptive Kalman filter in simultaneously estimating the state and unknown dynamic parameter of a noisy one-dimensional linear system. The method is built upon the mechanism of updating the parameter α using gradient descent, combined with total derivatives of the state and variance over time. Four simulation scenarios with varying levels of initial error and noise were examined to evaluate the adaptability, accuracy, and convergence speed of the filter. Simulation results show that the algorithm achieves stable convergence in most cases, especially when noise is low and the initial parameter is close to the true value. The filter maintains low absolute error and RMSE, rapidly decreasing estimated variance, and automatically adjusts the Kalman gain over time. The NEES indicators also confirm that the estimation reliability is acceptable under various conditions.

Notably, the optimal case demonstrated outstanding performance in all aspects, confirming the potential of this filter for applications in control, monitoring, and sensor systems where system parameters cannot be pre-defined with certainty. In future studies, the model can be extended to nonlinear systems, multi-dimensional systems, or environments with impulsive noise and colored noise, in order to examine the robustness and broader applicability of the algorithm.

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