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Fuzzy gain-scheduling sliding mode control for quadcopter

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A B S T R A C T :

This paper presents a novel Fuzzy Gain-Scheduling Sliding Mode Control system designed to address the challenges of high-performance trajectory tracking for a quadcopter. The inherent nonlinearity, underactuated dynamics, and susceptibility to external disturbances make precise quadcopter control a significant challenge for conventional linear control methods. While classical Sliding Mode Control (SMC) offers robust performance against these uncertainties, its major drawback is the undesirable chattering phenomenon caused by high-frequency switching, which can degrade performance and damage actuators. To mitigate this issue, the proposed FGS-SMC leverages a fuzzy logic system to dynamically adjust the SMC gains. This intelligent gain-scheduling strategy allows the control system to tune its parameters online based on the state of the quadcopter, striking a crucial balance between achieving rapid convergence and minimizing chattering. Simulation results demonstrate that the FGS-SMC not only provides superior trajectory tracking accuracy and disturbance rejection but also significantly reduces control chattering compared to a conventional, fixed-gain SMC. The proposed method proves to be a robust and effective solution for precise and stable quadcopter control in uncertain environments.

Keywords: Fuzzy Logic, Sliding Mode Control (SMC), Gain-Scheduling, Quadcopter, Trajectory Tracking

Introduction

Unmanned aerial vehicles (UAVs), especially quadrotors, have gained significant interest due to their wide applications in both civil and military fields [1], [2]. However, their nonlinear, underactuated, and coupled dynamics make controller design a challenging task. Various methods such as PID [3], [4], feedback linearization [5], and sliding mode control (SMC) [6] have been investigated. SMC ensures robustness by forcing system trajectories onto a sliding surface within finite time, while fuzzy logic control (FLC), first introduced by Zadeh [7], provides a model-free intelligent control approach [8]. In this work, an Adaptive Fuzzy Gain-Scheduled SMC (AFGS-SMC) is designed as the inner loop to stabilize quadrotor attitude, with a fuzzy logic system adaptively tuning the switching gain based on trajectory deviations. A proportional-derivative (PD) controller is employed as the outer loop to regulate position. Simulation results validate the proposed controller, showing improved robustness and tracking performance compared to conventional methods [2], [9].

Material and methods

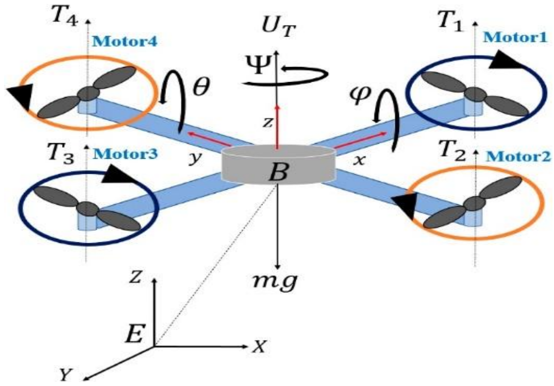

Figure 1. Euler angles in body frame

The quadcopter model is illustrated in Figure 1. The quadcopter has 6 degrees of freedom, three related to positional coordinates (*x, y, z*) and three related to angular positions (θ, ϕ, ψ). In this study, the degrees of freedom pertaining to positional coordinates relative to the ground are considered (x_E, y_E, z_E). The Newton-Euler equations of motion for the 6 degrees of freedom have been utilized to describe the system's dynamics [1].

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \\ \ddot{z} \\ \ddot{x} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + \dot{\theta}\frac{J_r}{I_{xx}}\Omega_r + \frac{l}{I_{xx}}U_2 \\ \frac{I_{zz}-I_{xx}}{I_{yy}}\dot{\phi}\dot{\psi} - \dot{\phi}\frac{J_r}{I_{yy}}\Omega_r + \frac{l}{I_{yy}}U_3 \\ \frac{I_{xx}-I_{yy}}{I_{zz}}\dot{\phi}\dot{\theta} + \frac{l}{I_{zz}}U_4 \\ g - \frac{\cos(\phi)\cos(\theta)}{m}U_1 \\ \frac{U_1}{m}\left[\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)\right] \\ \frac{U_1}{m}\left[\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi)\right] \end{bmatrix} + \begin{bmatrix} \xi_\phi(t) \\ \xi_\psi(t) \\ \xi_\theta(t) \\ \xi_H(t) \\ 0 \\ 0 \end{bmatrix} \qquad (1)$$

Where $\xi_\phi(t)$, $\xi_\psi(t)$, $\xi_\theta(t)$ and $\xi_H(t)$ correspond, respectively, to disturbances and uncertainties in the roll, pitch, yaw, and altitude angles. The value of Ω_r is obtained from the following equation where Ω_i is the rotational speed of each motor: $\Omega_r = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4$ and *g* represents the acceleration due to gravity and is considered to be 9.8 m/s².

When all four motors spin at the same speed, a force is exerted on the quadcopter in the positive direction of the *z* axis (U_1). If the rotational speed of motors 1 and 3 is the same while the speed of motor 4 exceeds that of motor 2, the quadcopter will rotate about the angle ϕ (U_2). Similarly, if the rotational speed of motors 2 and 4 is the same while the speed of motor 1 exceeds that of motor 3, the quadcopter will rotate about the angle θ(U_3). When the sum of the squares of the speeds of motors 2 and 4 surpasses that of motors 1 and 3, the quadcopter will rotate about the angle ψ (U_4).

The mathematical expression for the relationships between the speeds and the mentioned forces is given by:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} b\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ b\left(\Omega_4^2 - \Omega_2^2\right) \\ b\left(\Omega_1^2 - \Omega_3^2\right) \\ d\left(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2\right) \end{bmatrix} \qquad (2)$$

3. Controller design

The control system diagram is shown in Figure 2. The system states (12 states) include 6 degrees of freedom in terms of position and angles, as well as their derivatives (linear and angular velocities). The rotational speeds of the rotors are also sensed and recorded by sensors on the quadcopter system. The controller consists of two parts: the altitude controller and the attitude controller. The input to the altitude controller is the quadcopter's position and the reference position, and its output is three control commands: u_x, u_y, and u_z. These values are transformed into three outputs by the conversion block, to form the reference angles θ and ϕ and first control command (U_1). The reference angles θ_d and ϕ_d are then sent to the attitude controller. The attitude controller takes the reference values for angles θ and φ from the conversion block and for ψ from the user as inputs, and generates the control commands U_2, U_3, and U_4.

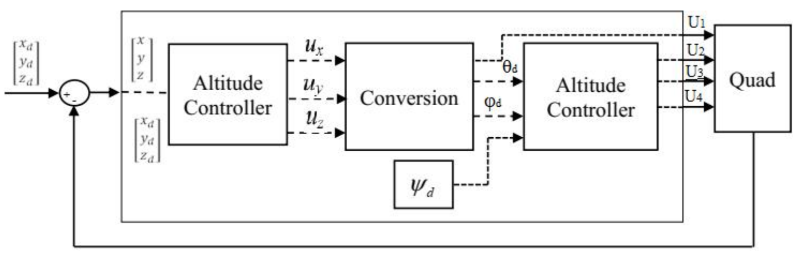

Figure 2. Control system diagram

a. Altitude Controller

A linear controller has been used to determine the control commands ux, uy, and uz. It is assumed that the positional error dynamics are represented by the following linear relationship.

$$\begin{bmatrix} (\ddot{x}_d - \ddot{x}) + k_{d\_x}(\dot{x}_d - \dot{x}) + k_{p\_x}(x_d - x) \\ (\ddot{y}_d - \ddot{y}) + k_{d\_y}(\dot{y}_d - \dot{y}) + k_{p\_y}(y_d - y) \\ (\ddot{z}_d - \ddot{z}) + k_{d\_z}(\dot{z}_d - \dot{z}) + k_{p\_z}(z_d - z) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \left| \ddot{e}_v + k_{d\_v}\dot{e}_v + k_{p\_v}e_v \right| = \left| 0 \right| \qquad (3)$$

b. Conversion Block

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} \ddot{x}_d + k_{d\_x}\dot{e}_x + k_{p\_x}e_x \\ \ddot{y}_d + k_{d\_y}\dot{e}_y + k_{p\_y}e_y \\ \ddot{z}_d + k_{d\_y}\dot{e}_y + k_{p\_y}e_y \end{bmatrix} = \begin{bmatrix} \frac{U_1}{m}\left[\cos(\phi)\sin(\theta)\cos(\psi) + \sin(\phi)\sin(\psi)\right] \\ \frac{U_1}{m}\left[\cos(\phi)\sin(\theta)\sin(\psi) - \sin(\phi)\cos(\psi)\right] \\ g - \frac{\cos(\phi)\cos(\theta)}{m}U_1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \theta_d \\ U_1 \\ \phi_d \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{\ddot{x}}{\ddot{z}+g}\cos(\psi_d) + \frac{\ddot{y}}{\ddot{z}+g}\sin(\psi_d)\right) \\ \sqrt{\ddot{x}^2 + \ddot{y}^2 + (\ddot{z}+g)^2} \\ \tan^{-1}\left(\frac{\ddot{x}\sin(\psi_d) + \ddot{y}\cos(\psi_d)}{U_1}\right) \end{bmatrix} \qquad (4)$$

c. Attitude Controller

The sliding surface is described by a nonlinear relationship as expressed as:

$$s = e + \beta|\dot{e}|^{\lambda}\,\mathrm{sign}(\dot{e}), \quad \beta > 0; \; 1 < \lambda < 2 \qquad (5)$$

To achieve the sliding surface, the following control law must be satisfied using the control command:

$$\dot{s} = -k_1 s - k_2 \times |s|^{\rho}\,\mathrm{sign}(s) \qquad (6)$$

Lyapunov theory is utilized to demonstrate stability. The Lyapunov function is defined as $V = s^2/2$. Equations for the control command U_2 are written, and U_3 and U_4 are obtained in a similar manner.

$$\dot{V} = s\left\{\dot{e} + \beta\lambda|\dot{e}|^{\lambda-1}\left[\ddot{\phi}_d - \left(\frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + \dot{\theta}\frac{J_r}{I_{xx}}\Omega_d + \frac{l}{I_{xx}}U_2\right)\right]\right\} \qquad (7)$$

and

$$U_2 = \frac{I_{xx}}{l}\left(-\dot{\theta}\frac{J_r}{I_{xx}}\Omega_d - \frac{I_{yy}-I_{zz}}{I_{xx}}\dot{\theta}\dot{\psi} + \ddot{\phi}_d\right) + \frac{-\dot{e}}{\beta\lambda|\dot{e}|^{\lambda-1}} - \frac{I_{xx}}{l}\left[k_1 s + k_2 \times |s|^{\rho}\,sign(s)\right] \qquad (8)$$

Hence, the control laws are obtained as following equation:

$$\begin{aligned} U_2 &= \frac{I_x}{l}\left(\ddot{\phi}_d - a_2\dot{\theta}\Omega_d - a_1\dot{\theta}\dot{\psi} - \frac{-\dot{e}}{\beta\lambda|\dot{e}|^{\lambda-1}} - k_{1\phi}s_{\phi} - k_{2\phi} \times |s_{\phi}|^{\rho}\,sign(s_{\phi})\right) \\ U_3 &= \frac{I_y}{l}\left(\ddot{\theta}_d - a_4\dot{\phi}\Omega_d - a_3\dot{\phi}\dot{\psi} - \frac{-\dot{e}}{\beta\lambda|\dot{e}|^{\lambda-1}} - k_{1\theta}s_{\theta} - k_{2\theta} \times |s_{\theta}|^{\rho}\,sign(s_{\theta})\right) \\ U_4 &= \frac{I_z}{l}\left(\ddot{\psi}_d - a_5\dot{\phi}\dot{\theta} - \frac{-\dot{e}}{\beta\lambda|\dot{e}|^{\lambda-1}} - k_{1\psi}s_{\psi} - k_{2\psi} \times |s_{\psi}|^{\rho}\,sign(s_{\psi})\right) \end{aligned} \qquad (9)$$

d. Fuzzy inference system

A Sugeno fuzzy system is employed to adjust the parameters k_1 and k_2, which determine the controller gains. In cases where the level of disturbance and uncertainty in the system is high, larger values for k_1 and k_2 are chosen. The fuzzy system as presented in Figure 3 has four inputs: θ, ϕ , and two additional parameters representing the distance from the sliding surface and the rate of change of this distance.

Inputs:

θ, ϕ: absolute values of pitch and roll angles.

s: sliding surface error.

$\dot{s}$: derivative of the sliding surface error.

Membership functions (MFs):
s: 5 MFs (NB, NS, ZE, PS, PB).

$\dot{s}$: 3 MFs (N, Z, P).

|θ|, |ϕ|: 3 MFs (Small, Medium, Large).

k: constant singleton outputs.

The rules designed according to deviation of state trajectories from sliding surface, where the controller gain is high when state trajectories are far from the sliding surface and low once state trajectories reached sliding surface. Table 1 illustrates fuzzy rules.

Table 1: Fuzzy rules

k		$\dot{s}$		
		N	Z	P
s	NB	M	L	L
	NS	M	M	M
	ZE	S	S	S
	PS	M	M	M
	PB	L	L	M

where each rule has the following form: If s is x and $\dot{s}$ is y, then output level k=c. (c *is a constant*). The FLS final output is given by the weighted average of the whole rules output.

$$output = \frac{\sum_{i=1}^{N} w_i k_i}{\sum_{i=1}^{N} w_i} \qquad (10)$$

where w_i represents rule firing strength and N is the rules number.

Conclusion

The quadcopter mathematical model was simulated using Matlab/Simulink environment, based on (1) and the model parameter used in the simulation are taken from, as listed in Table 2.

Table 2: Parameters of the quadcopter

Name	Parameter	Value	Unit
Mass	m	0.65	kg
Inertia on x axis	I_x	7.5e-3	kgm^2
Inertia on y axis	I_y	7.5e-3	kgm^2
Inertia on z axis	I_z	1.3e-2	kgm^2
Thrust coefficient	b	3.13e-5	Ns^2
Drag coefficient	d	7.5e-7	Nms^2
Rotor inertia	J_r	6e-5	kgm^2
Arm length	l	0.23	m

The control results for the quadcopter are presented in MATLAB, demonstrating its performance in the absence of external disturbances.

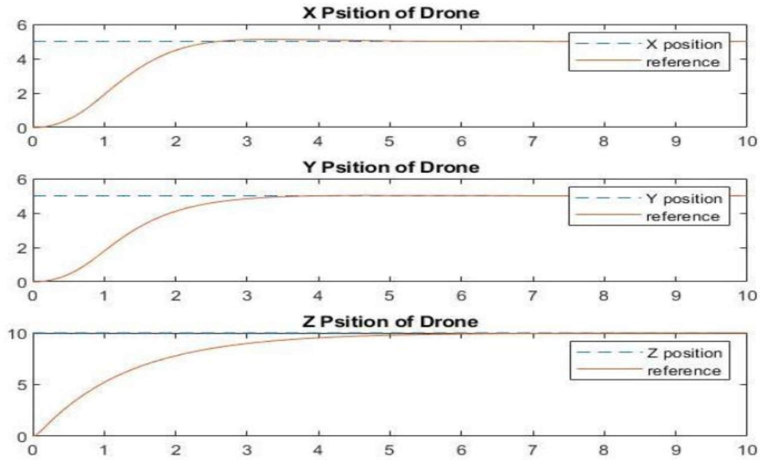

Figure 3. Quadcopter instantaneous coordinates and set values

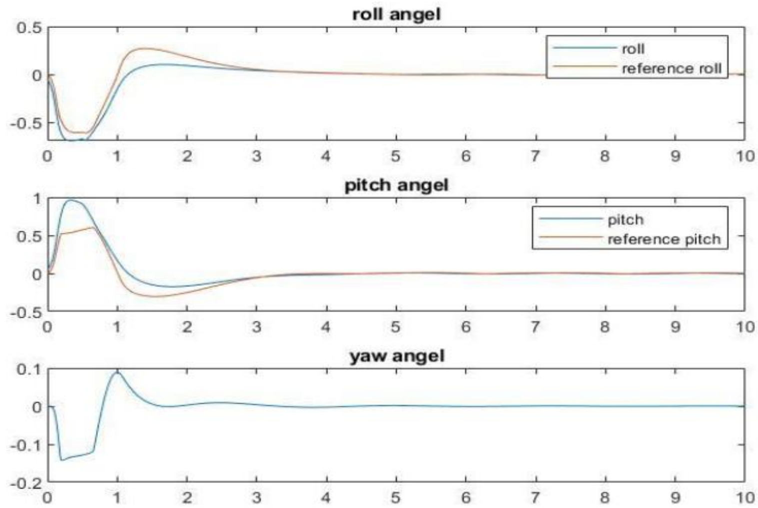

Figure 4. Angular of Quadcopter and set values

The dynamic model of the quadrotor unmanned aerial vehicle (UAV) has been presented. Subsequently, a conventional sliding mode controller (SMC) is designed as the inner-loop controller for the quadrotor. While this controller provides inherent robustness, its major drawback is the undesirable chattering problem, which can negatively impact performance and cause actuator wear. To mitigate this issue, an adaptive fuzzy gain-scheduling SMC technique was proposed. This novel approach utilizes a fuzzy inference system to intelligently tune the controller's gains online, achieving a crucial balance between convergence speed and chattering reduction. The results of the controller's performance were evaluated through comprehensive simulations conducted in MATLAB/Simulink. These initial simulations demonstrate the controller's effectiveness in achieving robust trajectory tracking while significantly reducing chattering under ideal, disturbance-free conditions. The promising results from this study lay the groundwork for a more extensive analysis. The performance of this control system under the influence of various external disturbances, such as wind gusts and payload variations, will be thoroughly addressed and presented in a subsequent paper.

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