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Dynamical Complexity of the African Near-Equatorial and Low-Latitude Ionosphere Using GNSS-Derived TEC

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ABSTRACT

Unique ionospheric phenomena occur in the equatorial and low-latitude regions of Africa, such as equatorial plasma bubbles (EPB), ionospheric irregularities, and Equatorial Spread F (ESF). These phenomena, occurring around the equator's ±15° magnetic latitude, cause rapid fluctuations in the amplitude and phase of radio signals, leading to sudden and intense disturbances in the ionosphere, especially during geomagnetic storms. Collectively, they contribute to distinctive signatures of chaotic dynamics in the ionosphere. This study analyses the complex dynamics of the ionosphere over Africa's equatorial and low-latitude regions using Total Electron Content (TEC) data derived from GPS RINEX observation files spanning 2014, 2017, and 2019, which cover periods of high, medium, and low solar activity. The Largest Lyapunov Exponent (LLE) and Entropy (ENT) were used as indicators of chaotic behaviour, with entropy computed as Shannon information entropy. High entropy values suggest stability, while low values indicate instability in these regions. The results show that the equatorial region exhibits less chaotic behaviour than Africa's low-latitude region during periods of low solar activity. Conversely, higher chaotic behaviour was observed during periods of high solar activity compared to low solar activity. The outcomes of this research can serve as a benchmark for assessing the precision and reliability of ionospheric models tailored to these specific regions.

Keywords: LLE, ENT, TEC, solar activity index, chaotic dynamics

1.0 Introduction

The ionosphere acts as a dispersive medium, causing time delays in radio signals that vary with frequency (Olwendo et al., 2012). This effect diminishes and sometimes disrupts the transmission of radio signals. In practical terms, errors associated with the ionosphere are recognised as a primary contributor to inaccuracies in GNSS (Oladipo et al., 2012). This poses a significant challenge to the reliability and precision of Global Navigation Satellite Systems (GNSS) and their associated regional augmentation systems, such as the Satellite-Based Augmentation System (SBAS), particularly in safety-critical applications. This challenge is most pronounced in the Equatorial Ionisation Anomaly (EIA) region, where the ionisation distribution undergoes significant modifications during daytime due to the fountain effect. This effect results in a peak of electron density approximately ±15° to ±20° from the magnetic equator and a depletion at the magnetic equator during local noon hours. Furthermore, this phenomenon is linked to the emergence of ionospheric irregularities and plasma bubbles following local sunset, exacerbating the complexity of the situation (*Abe et al, 2019).* The total electron content (TEC) stands as a crucial parameter for elucidating the state and structure of the ionosphere. Examining the temporal variations of TEC allows for the theoretical exploration of different periods of ionospheric physical processes. TEC proves valuable in correcting radio wave propagation in space-based radio communication applications, such as satellite positioning, navigation, and orbit determination, as it is closely linked to the time delay experienced by radio waves penetrating the ionosphere. Currently, TEC assessments predominantly depend on GPS data. In contrast to earlier practices, where TEC measurements were performed utilising the Faraday rotation effect on a linear polarised plane wave during propagation (Klobuchar, 1985; Shim, 2009)), where dedicated transmitters on both geostationary and non-geostationary satellites were utilised for this purpose.

Estimating Total Electron Content (TEC) involves utilising dual-frequency GPS observations, a significant parameter for characterising the ionosphere and informing data assimilative models. Slant Total Electron Content (STEC) measures the total number of free electrons within a unit cross-sectional column along the electromagnetic wave's path between the satellite and the receiver. The total count of free electrons is proportional to the ionospheric differential delay between L1 (1575.42 MHz) and L2 (1227.60 MHz) signals, as described by Mala S. Bagiya et al., 2009.

In the realm of applied dynamics, the objective is to establish connections between mathematical systems and the physical or biological systems of interest. Typically, this involves model construction, where our comprehension of the physical system is utilized to formulate dynamic equations that depict the evolutionary behavior of the system. Alternatively, when only a series of measurements taken at specific intervals is available, a reverse approach is adopted. This method begins with the sequence of measurements, known as a time series, and employs numerical or analytical tools to process the data. This processing aims to unveil the dynamical behavior inherent in the physical system represented by the measured data. The time series essentially captures the measurements of the processes and dynamics within the natural system under investigation. In most instances, it reveals the behavioural characteristics of the system, such as whether it is linear, periodic, quasi-periodic, or chaotic. The nonlinear approach is often deemed more suitable for studying these processes, as Hegar et al. (1999) and Unnikrishnan (2010) highlighted.

2.0 Materials and Methods

The data in this paper comprises Vertical Total Electron Content (VTEC) derived from actual observations made by Global Navigation Satellite Systems (GNSS), specifically the Global Positioning System (GPS). The GNSS observational data were employed to extract VTEC values at the respective measurement sites, serving as a reference for calculating LLE and ENT. The observation data of the stations used are in receiver independent exchange format (RINEX) with a sampling interval of 30 s and are obtained from the CDDIS website (https://cddis.nasa.gov/); UNAVCO websites (https://unavco.org/), and AFREF Reference Station(http://afrefdata.org/)

VTEC was calibrated using the Gg software and method described by Ciraolo *et al*, 2007

Figure 1 shows the map depicting the locations of the stations used for this research

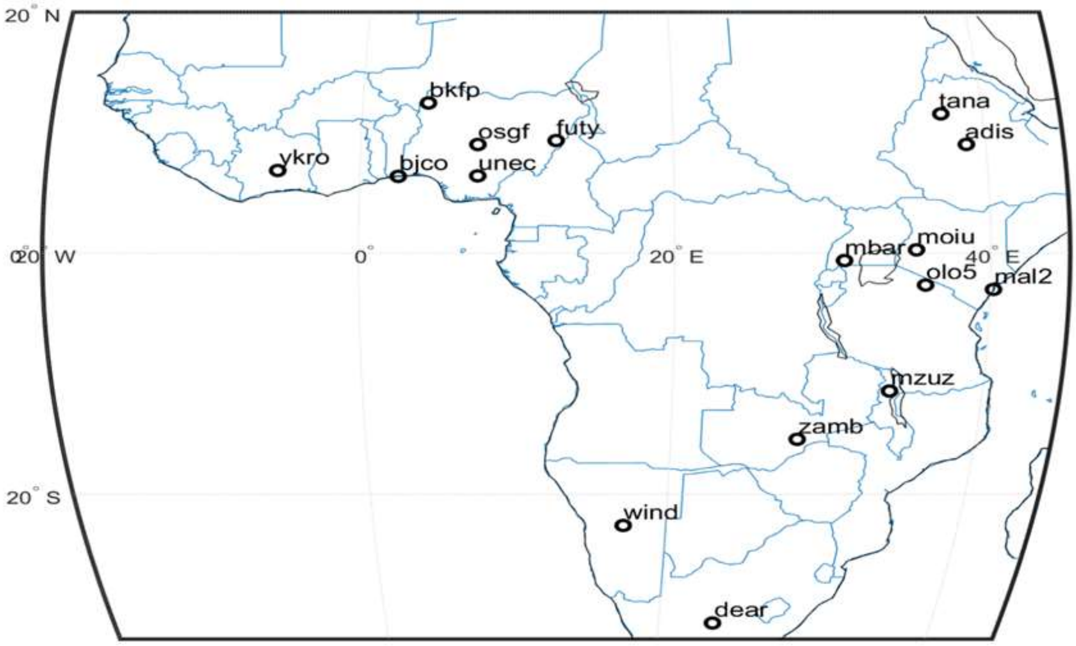

Figure 1: Distribution of observation stations over Africa map

2.1 Computation of VTEC

The Total Electron Content (TEC), which is the quantity of electrons present along the path of an electromagnetic wave traversing from a satellite, through the ionosphere, to a ground GNSS receiver, is determined by

$$\textbf{TEC} = \int_{reciever}^{satellite} N.ds \qquad 2.1a$$

where N is the electron density, TEC is measured in TECU where 1TECU is 10^{16} e/m^2

The relationship between VTEC and STEC is given as

VTEC=MF X STEC, 2.1b

where MF is a mapping function, given as

$$\text{MF} = \left[1 - \left(\frac{\cos(e)}{1+{}^{h}/_{R_E}}\right)^2\right]^{1/2} \qquad 2.1d$$

where e is the elevation angle of a satellite, h is the ionospheric shell height, and R_E is the Earth's mean radius. Theoretical detail of the calibration of STEC, and hence VTEC, from GPS observable data in RINEX format can be found in (Ciraolo *et al*,2007), and a practical guideline can be found at https://indico.ictp.it/event/a13251/session/6/contribution/46/material/0/0.pdf.

2.2 Computation of LLE

Lyapunov exponent or Lyapunov characteristic exponent of a dynamical system is a quantity that characterises the rate of separation of infinitesimally close trajectories. It is common to refer to the maximum one as the Largest Lyapunov Exponent (LLE) because it determines a notion of predictability for a dynamical system. It is often used to ascertain the state of chaoticity in the system represented by the set of measured data. (Boeing, 2016). The average rate of divergence of trajectories representing a time series is given by ((Wolf *et al*., 1985; Unnikrishnan 2010; Ogunsua *et al*., 2014) as:

$$\lambda_1 = \lim_{r\to\infty}\frac{1}{t}\ln\frac{\Delta x(t)}{x(0)} = \lim_{r\to\infty}\frac{1}{t}\sum_{i=1}^{t}\ln\left(\frac{\Delta x(t_i)}{\Delta x(t_{i-1})}\right) \tag{2.2}$$

where r represents a small space of evolution in phase space, Δx represents the expansion of trajectories in r , and t represents time of evolution in phase space.

2.3 Computation of ENT

Unlike Lyapunov exponents, which measure local instability in terms of geometric distances between orbits, entropy is a purely probabilistic way to quantify dynamical complexity (Unnikrishnan 2010). Entropy (ENT) is the Shannon information entropy (Shannon 1948) of all diagonal line lengths distributed over integer bins in a histogram. It is a measure of signal complexity which shows the richness of deterministic structuring. The higher the value of Entropy, the more complex of certainty structure of the system.

$$ENT = -\sum_{l=l_{\min}}^{N} P(l) InP(l) \tag{2.3}$$

Where $P(l)$ is the frequency distribution of the diagonal line lengths (for a diagonal parallel

to the main diagonal); l is the length of the line structure.

3.0 Results and Discussion

3.1 Analysis of ionospheric characteristics in the African region

Some unique ionospheric variability is exhibited in the equatorial and low-latitude regions of Africa as depicted in Figures 1(a-c). The analysis shown in the figures was done by using latitude, time, and VTEC of GNS-derived TEC for at least 15 stations across the regions. The data were interpolated into a grid and plotted as contour maps, with time on the abscissa and the latitude on the ordinate. From our analysis, we deduced that:

i. The maximum VTEC occurs during the daytime, which could be a result of solar irradiance and solar zenith angle usually observed during the daytime at low and mid-latitudes according to (Bolaji et al., 12).

ii. Regions near the equator show higher electron content, with VTEC showing the highest values at the spring and autumn equinoxes. This could be a result of a higher degree of ionization in the ionosphere as a result of the Earth's axial tilt, giving more exposure to solar irradiance. In contrast, VTEC is lower for stations away from the equator.

iii. Higher TECU in the high solar activity years shows that TEC is strongly influenced by solar activity

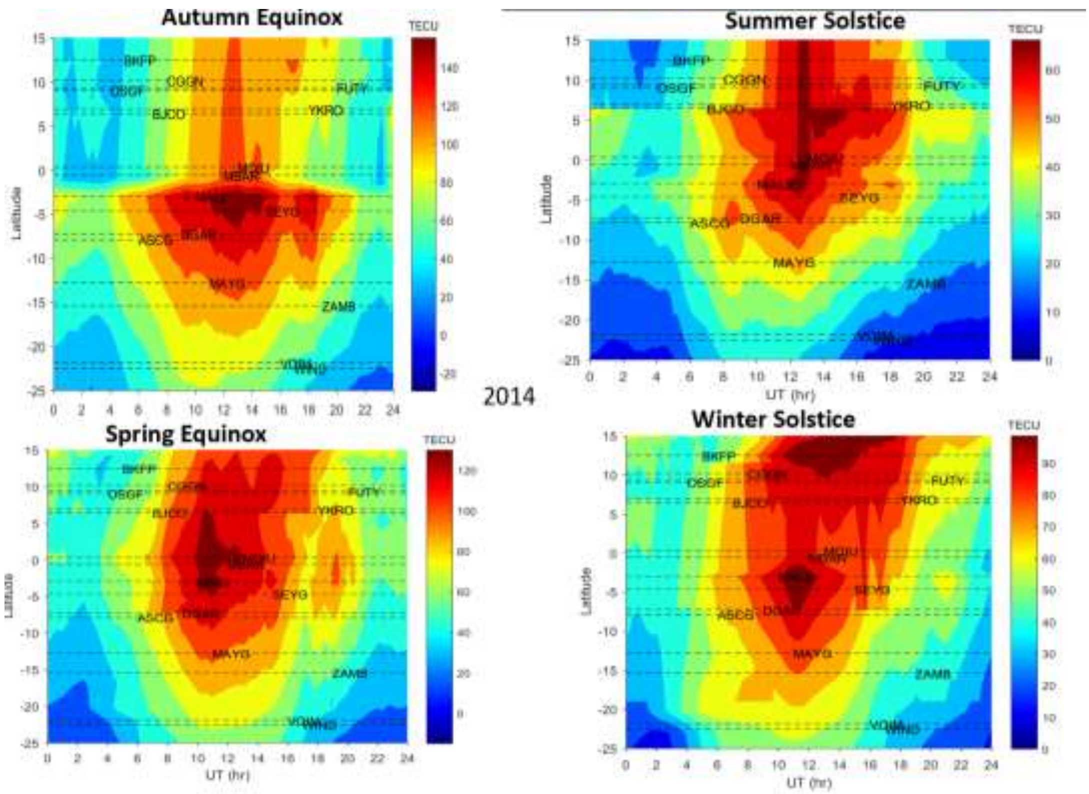

Figure 1a: Diurnal variation of VTEC for high solar activity year 2014

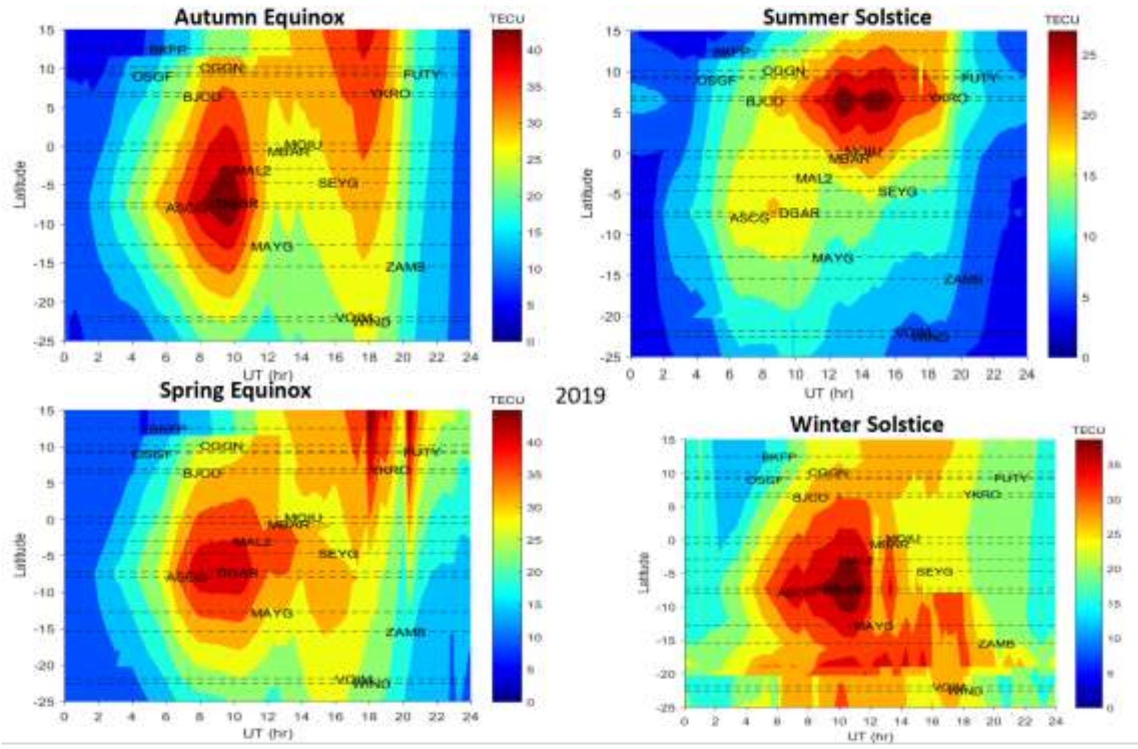

Figure 1b: Diurnal variation of VTEC for low solar activity year 2019

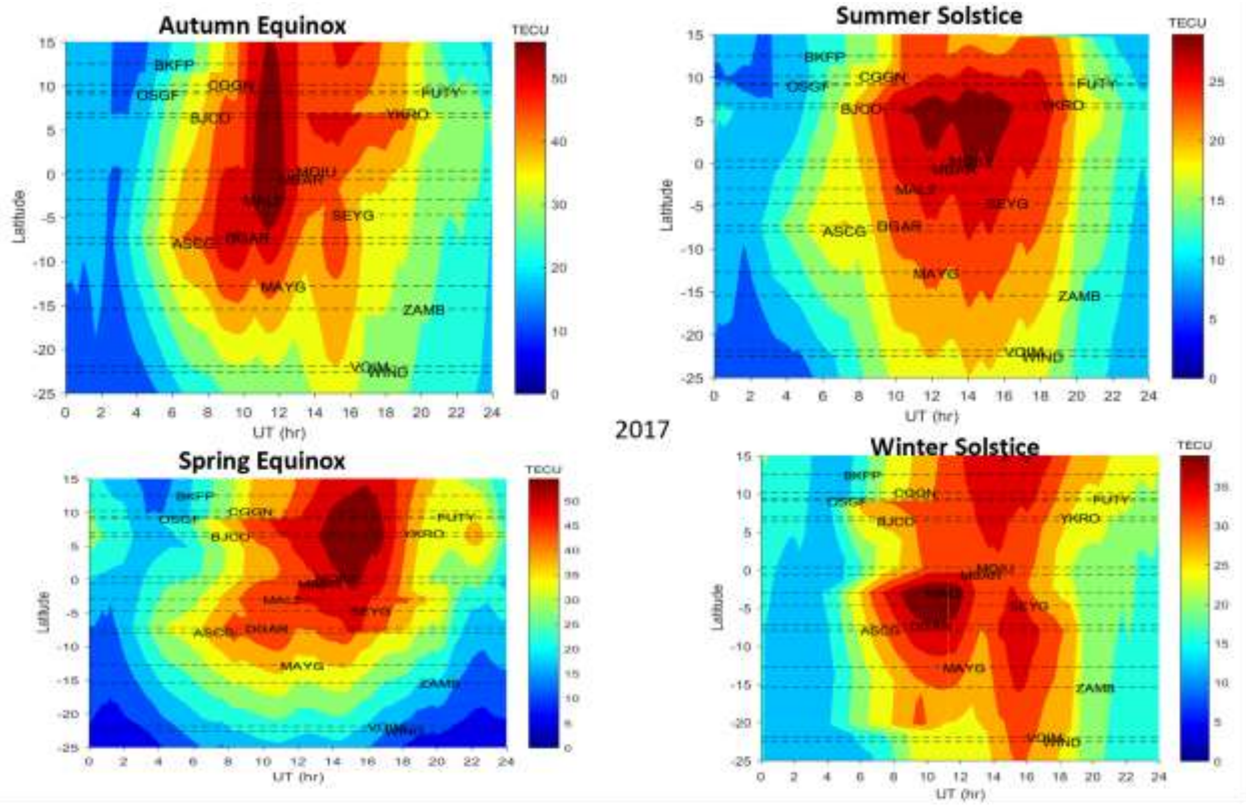

Figure 1c: Diurnal variation of VTEC for medium solar activity year 2017

3.2 Assessment of chaotic dynamics

To investigate the chaotic dynamics from the time series of the electron content, we computed two chaotic quantifiers (LLE and ENT). LLE was computed as described by (Wolf *et al*., 1985; Unnikrishnan 2010; Ogunsua *et al*., 2014) while the ENT was computed using the algorithm made available in the Cross Recurrence Plot toolbox as described by Marwan (2010).

As any complex dynamics investigation could not be achieved without a proper choice of delay and embedding dimension, we used a delay of 5 after a thorough application of the algorithm and procedure given by Takens (1981), and the embedding dimension using the method given by Abarbanel *et al.* (1993). Having employed the algorithms above, our suitable choice of delay and embedding dimension is, respectively, 5 and 3.

Figures 2(a-c) show the daily LLE and ENT for some stations across the equatorial and low-latitude regions of Africa. Since chaos investigation is sensitive to outliers, we only used stations with complete data for the sampled days. Below are our findings:

i. During the high solar activity year 2014, LLE values were observed to be higher near the equatorial region than in the low-latitude region. This could be a result of EIA which is more pronounced in the region. The higher values of LLE show that the region is more perturbed than the lower latitude. This behaviour was confirmed with lower values of ENT, as low ENT indicates less instability.

ii. For the low solar activity year 2019, the low-latitude region exhibits higher chaotic behaviour than the equatorial region (Fig. 2b(i)). This could be a result of a weakened Earth's magnetic field strength, thereby exposing the lower/polar latitude regions to more space weather events that could make the ionosphere more unstable at these regions. This was also confirmed with a lower ENT (Fig. 2b(ii)). Therefore, Low-latitude regions exhibit more chaotic behaviour than the equatorial region during the low solar activity year. This could be because Low-latitude regions are more susceptible to the effects of geomagnetic storms compared to areas closer to the equator. These storms can cause significant fluctuations in TEC and introduce non-linear behaviour that amplifies the sensitivity to initial conditions, potentially leading to a higher LLE. This shows that the prediction of ionospheric behaviour at low-latitude regions could be a challenging task in the region.

iii. LLE values were observed to be low during equinoxes (spring and autumn) for all the years. This could be a result of the ionosphere experiencing enhanced ionisation due to the more direct exposure to solar radiation. This can lead to improved radio wave propagation conditions as a result of stable or less variable ionospheric conditions. LLE values were, on the other hand, high during the solstices. However, the equatorial regions experience more significant ionospheric variability during equinoxes compared to latitudes far from the equator. This could be a result of a phenomenon known as the Equatorial Ionisation Anomaly (EIA), which is more pronounced in the equatorial region during the equinoxes. All these were confirmed with high/low ENT

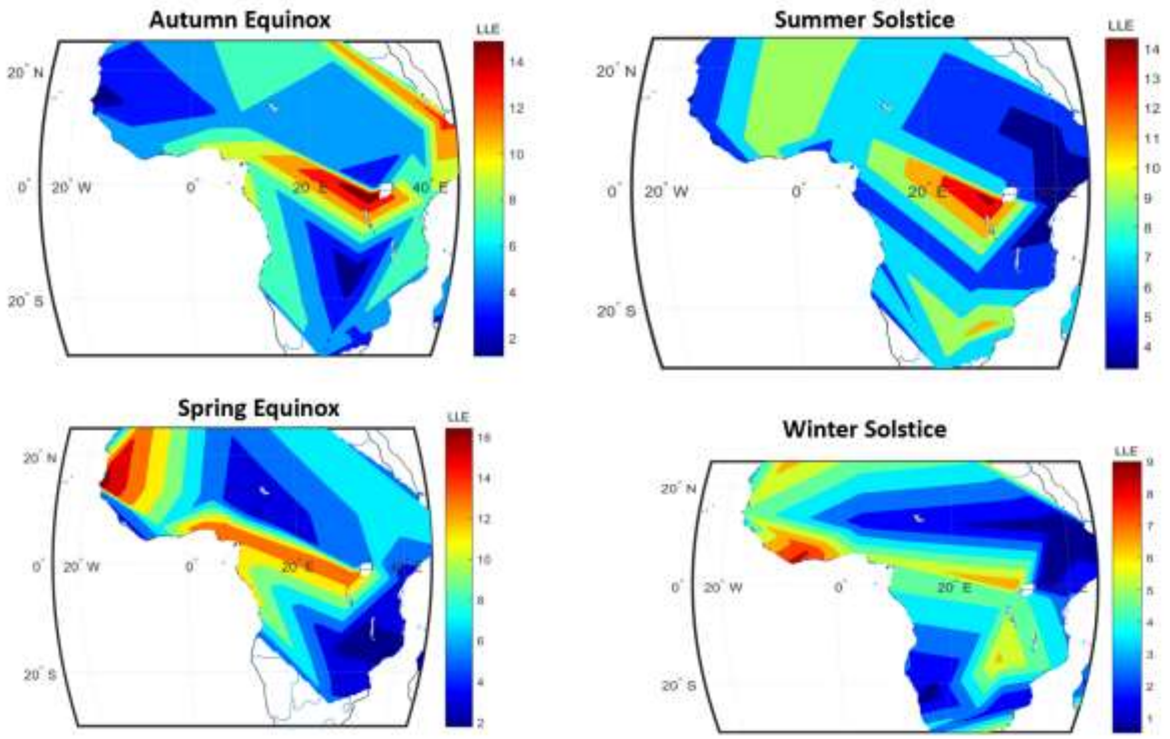

Figure 2a(i): Geographical Variation of LLE for High Solar Activity Year 2014

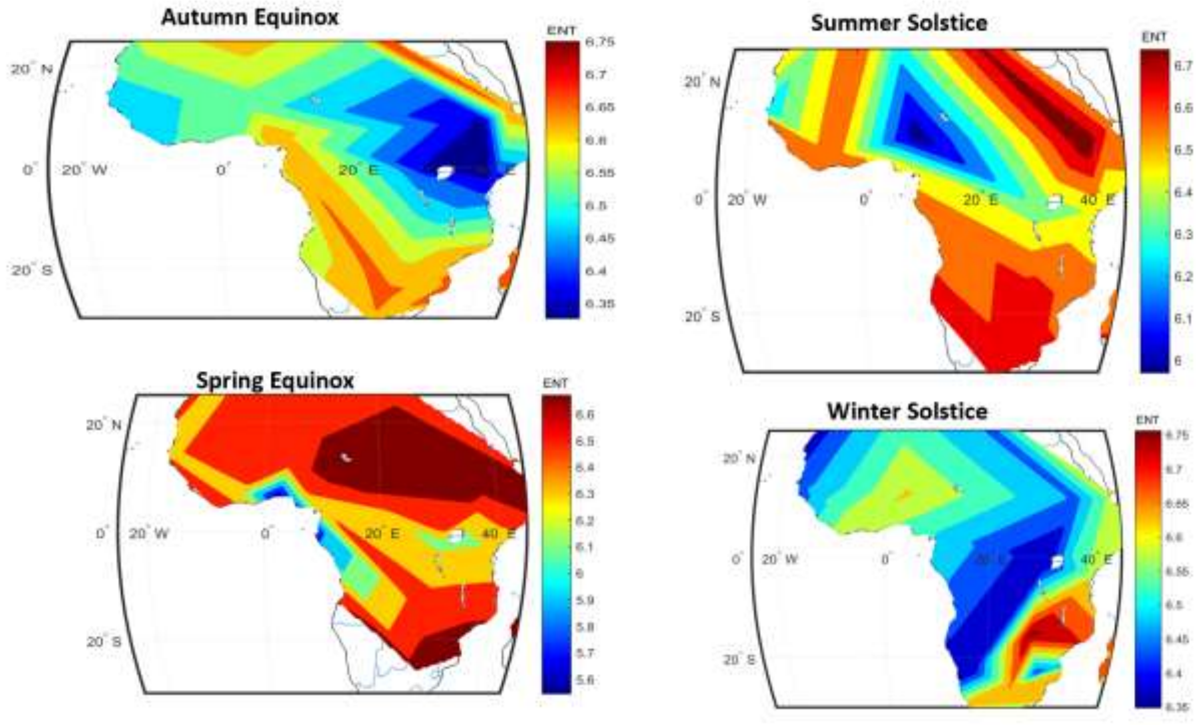

Figure 2a(ii): Geographical Variation of ENT for High Solar Activity Year 2014

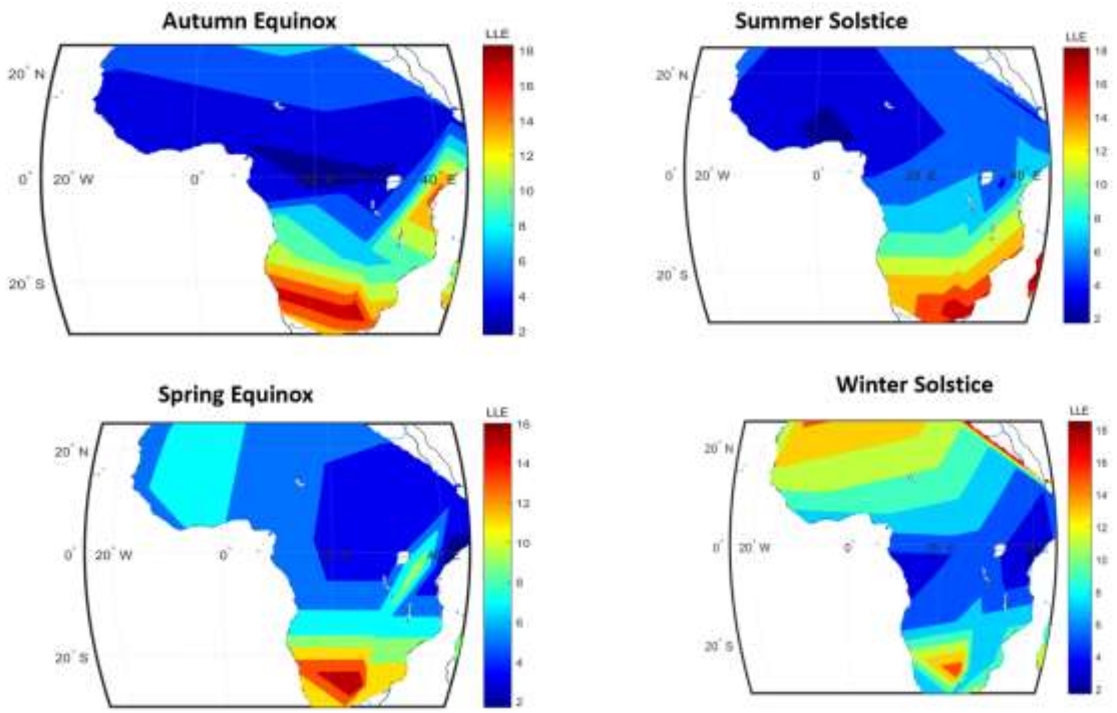

Figure 2b(i): Geographical Variation of LLE for Low Solar Activity Year 2019

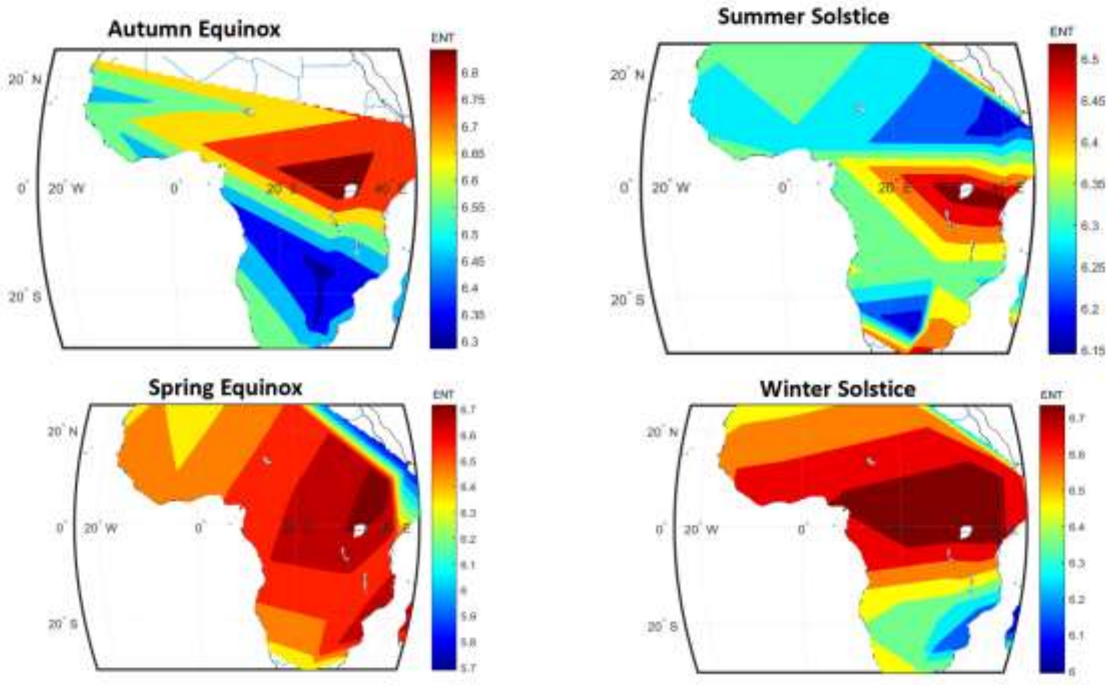

Figure 2b(ii): Geographical Variation of ENT for Low Solar Activity Year 2019

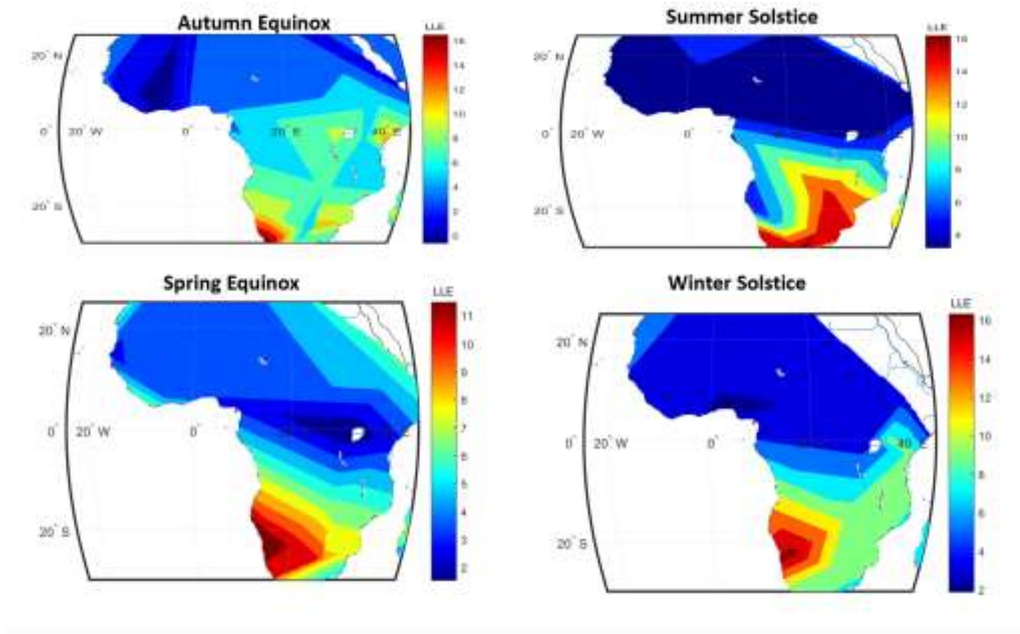

Figure 2c(i): Geographical Variation of LLE for Medium Solar Activity Year 2017

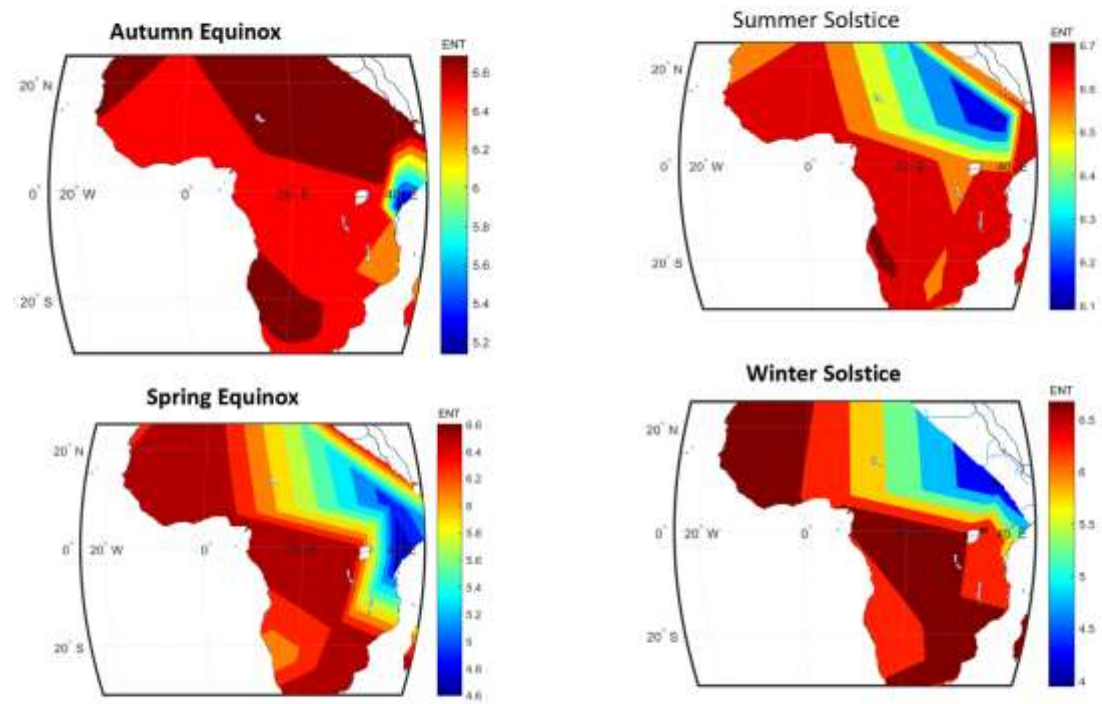

Figure 2c(ii): Geographical Variation of ENT for Medium Solar Activity Year 2017

Limitations and Future Work:

- This analysis used single-frequency TEC, which has inherent limitations compared to dual-frequency methods. Future work could employ dual-frequency TEC estimation for improved accuracy.
- The study focused on a specific timeframe and limited stations. Expanding the data set and incorporating more stations that cover the whole African region and beyond could provide a more comprehensive picture of regional ionospheric dynamics.
- Investigating the mechanisms responsible for observed differences in chaotic behavior between equatorial and low-latitude regions is a potential area for future research.

Conclusion

The equatorial and low-latitude regions of Africa are known for their unique ionospheric characteristics influenced by solar and geomagnetic activity. This study leverages GNSS-derived TEC data to assess the complex dynamics of the ionosphere in this geographical area. Understanding the behavior of the ionosphere in these regions is crucial for applications such as satellite communication, navigation, and space weather monitoring. This study demonstrates the distinct complex dynamics of the ionosphere over Africa's equatorial and low-latitude regions using chaos assessment techniques. This study shows that the ionosphere is a complex medium, and its complexity depends on locations and solar activity. The findings here could be used for measuring ionospheric stability for space-based applications. Further research using advanced techniques and longer data sets can deepen our understanding of the complex phenomena and their impact on GNSS applications in Africa.

REFERENCES

Abarbanel H. D., Brown R., Sidorowich J. J., and. Tsimring L. S, (1993): The analysis of observed chaotic data in physical systems. Reviews of modern physics, vol. 65, no. 4, 1331.

Bagiya, S. Mala, Joshi, H. P., Iyer, K. N., Aggarwal, M., Ravindran, S., & Pathan, B. M. (2009). TEC variations during low solar activity period (2005-2007) near the Equatorial Ionospheric Anomaly Crest Region in India. Annales Geophysicae, 27, pp.1047-1057. http://dx.doi.org/10.5194/angeo-27-1047-2009

Boeing, G. (2016): Visual Analysis of Nonlinear Dynamical Systems: Chaos, Fractals, Self-similarity and the Limits of Prediction. doi:10.3390/systems4040037.

Ciraolo, L., Azpilicueta, F., Brunini, C., Meza, A., Radicella, S.M., Azpilicueta, F., Brunini, C., Meza, A., Radicella, S.M., 2007. Calibration errors on experimental slant total electron content (TEC) determined with GPS. J Geod 81, 111–120. https://doi.org/10.1007/s00190-006-0093-1

Klobuchar, J.A. [1985], "Ionospheric Total Electron Content (TEC)" in A. S. Jursa (ed.), Handbook Of Geophysics and the Space Environment, Bedford (Mass.), Air Force Geophysical Laboratory, pp.10-89:10-96.

Marwan, N., (2010): Cross recurrence plot toolbox 5.21 (R31), http://www.agnld.unipotsdam.de/_marwan/toolbox.

Marwan, N., (2010): Cross recurrence plot toolbox 5.21 (R31), http://www.agnld.unipotsdam.de/_marwan/toolbox.

Ogunsua, B. O., Ojo, J. S., Adediji, A.T., (2018): Atmospheric Chaoticity and Complexity from Radio Refractivity Derived from Akure Station, Advances in Space Research, doi: https://doi.org/10.1016/j.asr.2018.06.035

Oladipo, O. A.,& Schueler, T., 2012. GNSS single frequency ionospheric range delay corrections: NeQuick data ingestion technique. Advances in Space Research, 50(9):1204-1212. https://doi.org/10.1016/j.asr.2012.06.035.

Olwendo, O. J., Baki, P., Mito, C., et al., 2012. Characterization of ionospheric GPS Total Electron Content (GPS–TEC) in low latitude zone over the Kenyan region during a very low solar activity phase. Journal of Atmospheric and Solar-Terrestrial Physics, 84-85(none):52-61. https://doi.org/10.1016/j.jastp.2012.06.003.

O.E. Abe, C. Paparini, R.H. Ngaya, S.M. Radicella, B. Nava, A. Kashcheyev, 2019. Assessment study of ionosphere correction model using single- and multi-shell algorithms approach over sub-Saharan African region. Advances in Space Research; https://doi.org/10.1016/j.asr.2019.01.044

Unnikrishnan, K., (2010): Comparative study of chaoticity of Equatorial/low latitude ionosphere over Indian subcontinent during geomagnetically quiet and disturbed periods. Non-Linear Processes in Geophys, 26, 941-953.