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The Dose Response Regarding Microbial Disease: A Mathematical View

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A B S T R A C T :

In the society, we have seen the incidence of both communicable and non-communicable disease, we are also familiar with the events of chronic diseases like T. B. which takes longer duration to be cured as compared to cold, cough or flu etc. The chances of disease development in an individual basically depend upon the growth rate of micro-organism causing that disease. The drug given to infected individual after reaching to the target organ/s kill the pathogens and help to cure against disease through pharmaco kinetics and pharmaco dynamics (PKPD). In this way the rate of curing of disease in infected individual depends upon the rate of killing of causing microbes by that particular drug. But how much drug one should be given, otherwise its overdose may cause deleterious side effect or direct effect to the infected persons and may result into development of symptoms of other disease. Beside growth of microbes is also influenced by resisting ability of body, unfavorable/favorable environment, nutrition but in the present paper we have neglected these all factors and have focused on relation of growth rate of microbes in individual given and dose. In the present paper we have proved our above said effect through mathematical modeling which involves various facts.

Keywords: *Communicable vs. Non-communicable diseases, Pharmacokinetics and Pharmacodynamics (PKPD), Microbe Growth and Drug Efficacy, Drug Overdose.*

1. Introduction

First of all Robert Koch(1876) find a beautiful result in our laboratory that- "An essential activity of fermentation technology is the quantitation of both the amount and the rate of change of microbial cell mass." This can often be achieved by direct measurement of either cell mass or number. In many cases, direct methods of measurement are not applicable, and the physiological activity, which is partially related to the amount of biomass, must be measured. Since the main point of interest for many practical applications is the activity of microorganisms, many people make a virtue of necessity.

Problem: What is the dose response regarding to bacterial disease in infected person?

Variables: For given problem, time is independent variable and number of viable microbes (V_n) after time't' is dependent variable. Here 'n' represent number of generation in time t.

Parameters : Birth rate μ and death rate λ of microbes are parameters for given problem.

Mathematical Model : Assume Δt is time interval between two continuous generations. Then the generation of microbial after time t is $n = \frac{t}{\Delta t}$. Thus $\Delta t \to 0 \; as \; n \to \infty$ and population of viable microbial after time t is $V_n = 2^n V_0 = 2^{\frac{t}{\Delta t}} V_0$, when death rate is zero. Thus the population of microbial increase as $\Delta t \to 0$.

Construct a model for the total viable microbes with respect to time such as: $V_0 - V_1 - V_2 - V_3 - - - - V_n - V_{n+1} - - - -$

This model represent a sequence of viable microbes $< V_n >$.

Mathematical Analysis: Assuming viable microbes in infected person are V_0 as well as total microbial population at initial level. If μ is the specific growth rate and λ is specific death rate (due to dose or immunity) of microbial in a time t, then the net growth rate of viable cells is given by

$$\frac{dV_n}{dt} = (\mu - \lambda)V_n \quad \text{...............................} \quad (1)$$

Integrating equation **(1)**, we find

$$\log\frac{V_n}{V_0} = (\mu - \lambda)t \quad \text{...............................} \quad (2)$$

$$V_n = V_0 e^{(\mu - \lambda)t} \quad \text{...............................} \quad (3)$$

We are considering some cases as follows:

CASE 1: If specific death rate of microbial $\lambda = 0$.

This is the case before the dose. From equation **(3)**

$$V_n = V_0 e^{\mu t}$$

$$V_{n+1} - V_n = V_0(e^{\mu(t+1)} - e^{\mu t})$$

$$V_{n+1} - V_n = V_0 e^{\mu t}(e - 1) \geq 0$$

$$V_{n+1} \geq V_n \text{ and } V_n \to \infty \text{ as } t \to \infty.$$

Thus the sequence of viable microbes is monotonic increasing and upper bound of the sequence not exist so, sequence is divergent.

Conclusion: Conclude that the microbial population as well as diseases day by day exponentially increased.

CASE 2: If specific death rate of microbial due to dose, equal to specific growth rate s.t. $\lambda = \mu$.

This is the case when dose quantity is such as $\lambda = \mu$. From equation (3) $V_n = V_0$ this shows that total viable microbial are constant or equal to V_0 at every generation or time interval.

Conclusion: Conclude that the microbial population as well as diseases remains constant. From modal $<V_n>$ as a constant sequence of viable microbes so it is convergent therefore disease will be under controlled.

CASE 3: If specific death rate of microbial $\lambda > \mu, \lambda - \mu > 0$.

From equation **(3)**

$$V_n = V_0 e^{-kt} \text{ where } \mu - \lambda = -k < 0 ; \quad k \text{ is positive.}$$

$$V_{n+1} - V_n = V_0(e^{-k(t+1)} - e^{-kt})$$

$$V_{n+1} - V_n = V_0 e^{-kt}(e^{-k} - 1) < 0$$

$$V_{n+1} < V_n \text{ and } V_n \to 0 \text{ as } t \to \infty.$$

Thus the sequence of viable microbes is monotonic decreasing and lower bound of the sequence exist so, sequence is convergent.

CASE 3I: If k is small. This is the case after dose but quantity is low. In this case the microbial population as well as diseases slowly decreased.

CASE 3II: If k is large. This is the case after dose but quantity is high. In this case the microbial population as well as diseases rapidly decreased and patient cure early. But, here k depend on patient capacity. So k is restricted to larger.

Conclusion: From Above both cases, conclude that the microbial population as well as diseases day by day decreased and patient cure.

CASE 4: If specific death rate of microbial $\lambda < \mu, \lambda - \mu < 0$.

This is the case proper dose quantity. From equation **(3)**

$$V_n = V_0 e^{kt} \quad \text{where } \mu - \lambda = k > 0.$$

$$V_{n+1} - V_n = V_0 (e^{k(t+1)} - e^{kt})$$

$$V_{n+1} - V_n = V_0 e^{kt} (e - 1) > 0$$

$$V_{n+1} > V_n \text{ and } V_n \to \infty \text{ as } t \to \infty.$$

Thus the sequence of viable microbes is monotonic increasing and upper bound of the sequence is not exist so, sequence is divergent.

Conclusion: Conclude that the microbial population as well as diseases day by day increased.

Strength of Paper: This paper represents dose response batter than [Rajan Singh, B. K. Singh, Mukesh Chandra].

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REFERENCES

[1] D. W. Tempest: "Dynamics of Microbial Growth", Laboratoriumvoor Microbiologie, Universiteit van Amsterdam, Plantage Muidergracht14, Amsterdam-C, The Netherlands.
[2] Rajan Singh, B. K. Singh, Mukesh Chandra: "Mathematical Modelling in effect of drugs on diseases", World Journal of Pharmacy and Pharmaceutical Sciences, Volume 4, Issue 05 May 2015, 2031-2034 ISSN 2278 – 4357
[3] D. Herbertd, R. Elswortrh & R. C. Tellingr: "The Continuous Culture of Bacteria- A Theoretical and Experimental Study". J. gen. *Mictobiol.* 14, 601, 1956
[4] David H. Anderson: "Compartmental Modeling and Tracer Kinetics". Springer- Verlag, Berlin, Heidelberg, New York, Tokyo, 1983.
[5] Yonathan Bard: "Nonlinear Parameter Estimation". Academic Press New York and London, 1974.
[6] Peter L. Bonate and Danny R. Doward: "Pharmacokinetics in Drug Development- Advances and Applications", volume 3. Springer, 2011.
[7] P. L. Bonate: "Pharmacokinetic-Pharmacodynamic Modeling and Simulation". Springer, 2006.
[8] Aristides Dokoumetzidis and Panos Macheras: "Fractional kinetics in drug absorption and disposition processes". Journal of Pharmacokinetics and Pharmacodynamics, 2009; 36:165–178.
[9] R. D. Driver: "Ordinary and Delay Differential Equations". Volume Applied Mathematical Science 20. Springer-Verlag, New York, Heidelberg, Berlin, 1977.
[10] P. Magni, M. Simeoni, I. Poggesi, M. Rocchetti, and G. De Nicolao: "A mathematical model to study the effects of drugs administration on tumor growth dynamics". Mathematical Biosciences, 2006; 200(2):127 – 151.