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More Functionsin Topological Spaces

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ABSTRACT:

The determination of this paper is to introduce newfunction,namely strongly$\widehat{S_P}^*$- continuous function, perfectly$\widehat{S_P}^*$- continuous function. Additionally some properties and theorems of these functions are investigated.

AMS Subject Classification : 54A05

Key words :$\widehat{S_P}^*$- closed sets, $\widehat{S_P}^*$- open sets, strongly $\widehat{S_P}^*$- continuous function, perfectly$\widehat{S_P}^*$- continuous function.

Introduction

In 1960,N.Levine [9] introduced the concept of strong continuity in topological spaces. J.ArulJesti et al[17] has introduced and studied $S_g{}^*$-closed sets, $S_g{}^*$-open sets, $S_g{}^*$-continuous function , $S_g{}^*$- irresolute function. Recently, S.PiousMissier and A. Siluvai[16] have introduced the concept of $\widehat{S_P}^*$-open sets, $\widehat{S_P}^*$ - closed sets and studied their properties in topological spaces.In this direction, we introduce newcontinuous functions called, strongly$\widehat{S_P}^*$continuous function, perfectly $\widehat{S_P}^*$- continuous functionin topological spaces. In addition to this, we discussed some of its properties.

Preliminaries

Throughout this paper, X, Y and$Z$ always denote topological spaces (X, τ), (Y, σ) and (Z, η) on which no separation axioms are assumed, unless explicitly stated.

Definition 2.1[18] Let A be a subset of a topological space (X, τ). Then

(a) A is called a semi generalized star open set (briefly $S_g{}^*$-open) if there is an open set U in X such that $U \subseteq A \subseteq Scl^*(U)$.

(a) A is called a semi generalizedstar closed set (briefly $S_g{}^*$-closed) if its complement is a semi generalizedstar open set in (X, τ).

Definition 2.2 [16]A subset A of a topological space (X, τ) is called a $\widehat{S_P}^*$-open set, if there is an open set U such that $U \subseteq A \subseteq PCl^*(U)$. The collection of all $\widehat{S_P}^*$-open sets in (X, τ) is denoted by $\widehat{S_P}^*O(X, \tau)$ or $\widehat{S_P}^*O(X)$.

Theorem 2.3 [16] Arbitrary union of $\widehat{S_P}^*$-open sets is $\widehat{S_P}^*$-open

Definition 2.4[16] A subset A of a Space X is called $\widehat{S_P}^*$-closed set if its complement $(X \backslash A)$ is $\widehat{S_P}^*$-open in X.The class of all $\widehat{S_P}^*$closed sets in (X, τ) is denoted by $\widehat{S_P}^* \subseteq (X, \tau)$ or simply $\widehat{S_P}^*$ is a collection of X in (X, τ)

Theorem. 2.5[16] :

(i) Every open set is a $\widehat{S_P}^*$-open set and Every closed set is $\widehat{S_P}^*$-closed.

(ii) Every α-open set in X is a $\widehat{S_P}^*$- open set and Every α-closed set is$\widehat{S_P}^*$- closed.

(iii) Every semi α-open set in X is a $\widehat{S_P}^*$- open set.

(iv) Every $semi^*$-open set is $\widehat{S_P}^*$-open and Every$semi^*$-closed set is $\widehat{S_P}^*$-closed.

(v) Every S_g^*-open set is a $\widehat{S_P}^*$-open set and EverySg^*-closed set is $\widehat{S_P}^*$-closed.

Theorem 2.6 [16] IfA is a subset of a topological space X, the following statements are equivalent

(i) A is $\widehat{S_P}^*$closed

(ii) There is a pre-closed F in X such that $PInt^*(F) \subseteq A \subseteq F$

Theorem 2.7 [16] If A is any subset of a topological space X,A is $\widehat{S_P}^*$-closed if and only if $\widehat{S_P}^* Cl(A) = A$.

Theorem 2.8 [16]IfA is a subset of a topological space (X, τ), then $PCl^*\big(Int(A)\big) = PCl^*(A)$

Theorem 2.9 [15] Let$f: X \to Y$ be a function. Then

(i) $Int^*\Big(PCl\big(f^{-1}(F)\big)\Big) = Int^*\big(f^{-1}(F)\big)$ for every closed set F in Y

(ii) $PCl^*(Int\big(f^{-1}(V)\big) = PCl^*\big(f^{-1}(V)\big)$ for every open set V in Y.

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Theorem 2.10 [15] Let$f: (X, \tau) \to (Y, \sigma)$ be a function. Then the following are equivalent

(i) f is contra $\widehat{S_P}^*$irresolute

(ii) $f^{-1}(F)$ is $\widehat{S_P}^*$open in X for each $\widehat{S_P}^*$closed set F in Y.

Definition 2.11[ ]A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-continuous if $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X for every open set V in Y.

Definition 2.12[ ] A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-continuous at a point x in X if for each open set V in Y containing $f(x)$, there is $\widehat{S_P}^*$-open set U in X such that $x \in U$ and $f(U) \subset V$.

Definition 2.13[ ] A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-irresolute at a point $x \in X$ if for each $\widehat{S_P}^*$-open set V in Y containing $f(x)$, there is a $\widehat{S_P}^*$-open set U of X such that $x \in U$ and $f(U) \subseteq V$.

Definition 2.14[ ]A function $f: X \to Y$ is said to be $\widehat{S_P}^*$-irresolute if $f^{-1}(V)$ is $\widehat{S_P}^*$-open in X for every $\widehat{S_P}^*$-open set V in Y.

3. Strongly $\widehat{S_P}^*$Continuous Functions

In this section, we define strongly$\widehat{S_P}^*$-continuous function and study its properties.

Definition 3.1 A mapping $f: X \to Y$ is said to be strongly $\widehat{S_P}^*$ continuous if the image of every $\widehat{S_P}^*$open set in Y is open in X.

Theorem 3.2 If$f: X \to Y$ is strongly $\widehat{S_P}^*$ then f is a continuous function.

Proof :Let G be any open set in Y. Since every open set is $\widehat{S_P}^*$open, G is $\widehat{S_P}^*$ open in Y. Since $f: X \to Y$ is strongly $\widehat{S_P}^*$ continuous, $f: X \to Y$ is strongly $\widehat{S_P}^*$continuous, $f^{-1}(G)$ is open in X. Hence f is continuous.

Remark 3.3 The converse of the above theorem need not be true as can be seen from the following example.

Example 3.4 Let $X = Y = \{a, b, c, d\}$, $\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}$ and $\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$. Then $\widehat{S_P}^*$ O(X, τ) = { X, ϕ , {b},{c},{b,d},{c,d},{a,b,c},{b,c,d},{a,c,d},{a,b,d}}. $\widehat{S_P}^*$ O(Y, σ) = {Y, ϕ, {a},{a,b},{c,d},{a,c},{a,d},{a,b,c},{a,c,d},{a,b,d}}. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = b, f(b) = c, f(c) = d, f(d) = a$.Here f^{-1}{a,b} = {a.d} is not open in X. Hence the function f is continuous but not strongly $\widehat{S_P}^*$continuous function.

Theorem 3.5 A map $f: X \to Y$ is strongly $\widehat{S_P}^*$ if and only if the inverse image of every $\widehat{S_P}^*$ closed set in Y is closed in X.

Proof: Suppose that f is strongly $\widehat{S_P}^*$continuous. Let B be any $\widehat{S_P}^*$closed set in Y. Then B^c is $\widehat{S_P}^*$open in Y. Since f is strongly $\widehat{S_P}^*$continuous, $f^{-1}(B^c)$ is open in X. But $f^{-1}(G^c) = f^{-1}(Y - B) = X - f^{-1}(G)$. Hence $f^{-1}(B)$ is closed in X.

Conversely, suppose that the inverse image of every $\widehat{S_P}^*$closed set in Y is closed in X. Let G be any $\widehat{S_P}^*$closed set in Y. Then $f^{-1}(G^c)$ is closed in X. But $f^{-1}(G^c) = X - f^{-1}(G)$. Hence $f^{-1}(G)$ is open in X. Therefore, f is strongly $\widehat{S_P}^*$continuous.

Theorem 3.6 If $f: X \to Y$ is strongly $\widehat{S_P}^*$continuous and $g: Y \to Z$ is $\widehat{S_P}^*$ continuous, then $g \circ f: X \to Z$ is continuous.

Proof: Let V be an open set in Z. Since $g: Y \to Z$ is $\widehat{S_P}^*$ continuous, $g^{-1}(V)$ is $\widehat{S_P}^*$ open in Y. Also since f is strongly $\widehat{S_P}^*$continuous, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is open in X. Hence $g \circ f$ is continuous.

Theorem 3.7 If $f: X \to Y$ is strongly $\widehat{S_P}^*$ continuous and $g: Y \to Z$ is $\widehat{S_P}^*$irresolute then $g \circ f: X \to Y$ is strongly $\widehat{S_P}^*$continuous.

Proof: Let G be an $\widehat{S_P}^*$ open set in Z. Since $g: Y \to Z$ is $\widehat{S_P}^*$ irresolute, $g^{-1}(G)$ is $\widehat{S_P}^*$ open in Y. Also, since f is strongly $\widehat{S_P}^*$ continuous, $f^{-1}(g^{-1}(G)) = (g \circ f)^{-1}(G)$ is open in X. Hence $g \circ f: X \to Z$ is strongly $\widehat{S_P}^*$continuous.

Theorem 3.8 If $f: X \to Y$ is $\widehat{S_P}^*$continuous and $g: Y \to Z$ is strongly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is $\widehat{S_P}^*$irresolute.

Proof: Let U be an $\widehat{S_P}^*$open set in Z. Since g is strongly $\widehat{S_P}^*$continuous, $g^{-1}(U)$ is open in Y. Also, since f is $\widehat{S_P}^*$ continuous, $f^{-1}(g^{-1}(U)) = (g \circ f)^{-1}(U)$. Hence, $g \circ f: X \to Y$ is $\widehat{S_P}^*$irresolute.

Theorem 3.9 Every strongly $\widehat{S_P}^*$continuous function is $\widehat{S_P}^*$continuous function.

Proof: Let $f: X \to Y$ be strongly $\widehat{S_P}^*$continuous. Let A be any open set in Y. Since every open set is $\widehat{S_P}^*$open set in Y. Since every open set is $\widehat{S_P}^*$open, A is $\widehat{S_P}^*$open in Y. Therefore, $f^{-1}(A)$ is open in X which implies $f^{-1}(A)$ is $\widehat{S_P}^*$open in X. Hence f is $\widehat{S_P}^*$continuous.

Remark 3.10 The converse of the above theorem need not be true as can be seen from the following example.

Example3.11 Let$X = Y = \{a, b, c, d\}$,$\tau = \{X, \phi, \{b\}, \{c\}, \{b, c\}\}and\sigma = \{Y, \phi, \{a\}, \{a, b\}\}$.Then$\widehat{S_P}^*O(X, \tau)$={X,$\phi$,{b},{c},{b,d},{c,d},{a,b,c},{b,c,d},{a,c,d},{a,b,d}}.$\widehat{S_P}^*O(Y, \sigma)$={Y,$\phi$,{a},{a,b},{c,d},{a,c},{a,d},{a,b,c},{a,c,d},{a,b,d}}. Define a function $f: (X, \tau) \to (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = d, f(d) = b$.Here {a,c} is $\widehat{S_P}^*$ open in Y. f^{-1}{a,c} = {a.b} is not open in X. Hence the function f is $\widehat{S_P}^*$continuous but not strongly $\widehat{S_P}^*$continuous function.

Theorem 3.12 Every strongly continuous function is strongly $\widehat{S_P}^*$continuous function.

Proof: Let $f: X \to Y$ be strongly continuous. Let A be any $\widehat{S_P}^*$open set in Y. Since$f$ is strongly continuous, $f^{-1}(A)$ is open and closed in X. Hence f is strongly $\widehat{S_P}^*$continuous.

Remark 3.13 The converse of the above theorem need not be true as can be seen from the following example.

Example3.14 Let$X = Y = \{a, b, c, d\}$,$\tau = \{X, \phi, \{a\}, \{a, b\}\}and\sigma = \{Y, \phi, \{a\}\}$.Then$\widehat{S_P}^*O(X, \tau)$={X,$\phi$,{a},{a,b},{a,c},{a,d},{a,b,c},{a,c,d},{a,b,d}}.$\widehat{S_P}^*O(Y, \sigma)$={Y,$\phi$,{a}}. Define the identity map of f .Here. f^{-1}{a} = {a} is open in(X, τ) but not closed in (X, τ). Hence the function f is strongly $\widehat{S_P}^*$continuous but not strongly continuous function.

Theorem 3.15 If $f: X \to Y$ is strongly $\widehat{S_P}^*$continuous and $g: Y \to Z$ is strongly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is strongly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$open set in Z. Since g is strongly $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is open in Y. By Theorem (every open set is $\widehat{S_P}^*$open), $g^{-1}(A)$ is $\widehat{S_P}^*$ open in Y. Since f is strongly $\widehat{S_P}^*$continuous, $f^{-1}(g^{-1}(A))$ is open in X. Hence $g \circ f$ is strongly $\widehat{S_P}^*$continuous.

Theorem 3.16 If $f: X \to Y$ is continuous and $g: Y \to Z$ is strongly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is strongly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$open in Z. Since $g: Y \to Z$ is strongly $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is open in Y. Also, since f is continuous, $f^{-1}(g^{-1}(A))$ is open in X. Hence $g \circ f$ is strongly $\widehat{S_P}^*$continuous.

Theorem 3.17 Let (X, τ) be any topological space and Y be a $\widehat{S_P}^*$-$T_{1/2}$ space and $f: X \to Y$ be a map then the following are equivalent

(i) f is strongly $\widehat{S_P}^*$continuous

(ii) f is continuous

Proof: (i) → (ii) Let A be any open set in X. By Theorem (Every open set is $\widehat{S_P}^*$open), A is $\widehat{S_P}^*$open in Y. Then by (i), $f^{-1}(A)$ is open in X. Hence f is continuous.

(ii) → (i) Let A be any $\widehat{S_P}^*$open in Y. Since Y is a $\widehat{S_P}^*$-$T_{1/2}$ space, A is open in Y. Then by (ii), $f^{-1}(A)$ is open in X. Hence f is strongly $\widehat{S_P}^*$continuous.

Theorem 3.18 Let (X, τ) be any topological space and Y be a $\widehat{S_P}^*$-$T_{1/2}$ space and $f: X \to Y$ be a map. Then the following are equivalent

(i) f is $\widehat{S_P}^*$irresolute

(ii) f is strongly $\widehat{S_P}^*$continuous

(iii) f is continuous

(iv) f is $\widehat{S_P}^*$continuous

Proof: The proof is straight forward.

4. Perfectly $\widehat{S_P}^*$Continuous Functions

Definition 4.1 A mapping $f: X \to Y$ is said to be perfectly $\widehat{S_P}^*$ continuous if the inverse of every $\widehat{S_P}^*$open set in Y is open and closed in X.

Theorem 4.2 If a map $f: X \to Y$ is perfectly $\widehat{S_P}^*$continuous then, it is strongly $\widehat{S_P}^*$continuous.

Proof: Let G be any $\widehat{S_P}^*$open set in Y. Since $f: X \to Y$ is perfectly $\widehat{S_P}^*$continuous, $f^{-1}(G)$ is open in X. Hence, f is strongly $\widehat{S_P}^*$continuous.

Remark 4.3 The converse of the above theorem is not true as seen from the following example.

Example 4.4 Let$X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a, b\}\}and\sigma = \{Y, \phi, \{a\}, \{b\}\}$.Then$\widehat{S_P}^*O(X, \tau)$={X,$\phi$,{a,b}}.$\widehat{S_P}^*O(Y, \sigma)$={Y,$\phi$,{a},{b},{a,b},{a,c},{b,c}}. Define f as an identity map. Here. f^{-1}{a,b} = {a,b} is open in X but not closed in X. Hence the function f is strongly $\widehat{S_P}^*$continuous but not perfectly $\widehat{S_P}^*$continuous function.

Theorem 4.5 A map $f: X \to Y$ is perfectly $\widehat{S_P}^*$continuous if and only if $f^{-1}(G)$ is both open and closed in X for every $\widehat{S_P}^*$ closed set G in Y.

Proof: Assume that f is perfectly $\widehat{S_P}^*$continuous. Let F be any $\widehat{S_P}^*$closed in Y. Then F^c is $\widehat{S_P}^*$open in Y. Since f is perfectly $\widehat{S_P}^*$continuous, $f^{-1}(F^c)$ is both open and closed in X. But $f^{-1}(F^c) = X - f^{-1}(F)$. Hence $f^{-1}(F)$ is both open and closed in X.

Conversely, assume that the inverse image of every $\widehat{S_P}^*$ closed set in Y is both open and closed in X. Let G be any $\widehat{S_P}^*$open in Y. Then G^c is $\widehat{S_P}^*$ closed in Y. By assumption, $f^{-1}(G^c)$ is both open and closed in X. But $f^{-1}(G^c) = X - f^{-1}(G)$ and so $f^{-1}(G)$ is both open and closed in X. Therefore, f is perfectly $\widehat{S_P}^*$continuous.

Theorem 4.6 Every perfectly $\widehat{S_P}^*$continuous function is perfectly continuous.

Proof: Let $f: X \to Y$ be perfectly $\widehat{S_P}^*$continuous and O be any open set in Y. Since every open set is $\widehat{S_P}^*$ open, O is $\widehat{S_P}^*$open in Y. Therefore, $f^{-1}(O)$ is both open and closed in X. Hence, f is perfectly continuous.

Remark 4.7 The converse of the above theorem need not be true as can be seen from the following example.

Example 4.8 Let$X = Y = \{a, b, c\}$,$\tau = \{X, \phi, \{a\}, \{a, b\}\}$and $\sigma = \{Y, \phi, \{a, b\}\}$.Then$\widehat{S_P}^*O(X, \tau)$={X,$\phi$,{a},{a,b},{a,c}}.$\widehat{S_P}^*O(Y, \sigma)$={Y,$\phi$,{a,b}}. Define a map f by f(a) = a, f(b) = c, f(c) = b.Here. f^{-1}{a,b} = {a,c} is not open and closed in X. Hence the function f is not perfectly continuous function.

Theorem 4.9 Let $f: (X, \tau) \to (Y, \sigma)$ be strongly $\widehat{S_P}^*$ continuous. Then f is perfectly $\widehat{S_P}^*$continuous if (X, τ) is a discrete topology.

Proof: Let U be any $\widehat{S_P}^*$open set in (Y, σ). By hypothesis, $f^{-1}(U)$ is open in (X, τ). Since (X, τ) is a discrete topology, $f^{-1}(U)$ is closed in (X, τ), i.e., $f^{-1}(U)$ is both open and closed in (X, τ). Hence f is perfectly $\widehat{S_P}^*$continuous.

Theorem 4.10 If $f: X \to Y$ is perfectly $\widehat{S_P}^*$continuous and $g: Y \to Z$ is perfectly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is perfectly continuous.

Proof: Let A be any $\widehat{S_P}^*$open in Z. Since g is perfectly $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is both open and closed in Y. By theorem (Every open set is $\widehat{S_P}^*$open), $g^{-1}(A)$ is both $\widehat{S_P}^*$open and $\widehat{S_P}^*$closed in Y. since f is perfectly $\widehat{S_P}^*$continuous $f^{-1}(g^{-1}(A))$ is open and closed in X. Hence, $g \circ f$ is perfectly $\widehat{S_P}^*$continuous.

Theorem 4.11 If $f: X \to Y$ is continuous and $g: Y \to Z$ is perfectly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is perfectly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$ open set in Z. Since g is perfectly $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is both open and closed in Y. Since f is continuous $f^{-1}(g^{-1}(A))$ is open and closed in X. Hence, $g \circ f$ is perfectly $\widehat{S_P}^*$continous.

Theorem 4.12 If $f: X \to Y$ is perfectly continuous and $g: Y \to Z$ is $\widehat{S_P}^*$ irresolute, then $g \circ f: X \to Z$ is perfectly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$open set in Z. Since g is $\widehat{S_P}^*$irresolute $g^{-1}(A)$ is $\widehat{S_P}^*$open in Y. Since f is perfectly $\widehat{S_P}^*$continuous, $f^{-1}\big(g^{-1}(A)\big)$ is both open and closed in X. Hence, $g \circ f$ is perfectly $\widehat{S_P}^*$continuous.

Theorem 4.13 If $f: X \to Y$ is contra-continuous and $g: Y \to Z$ is perfectly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is perfectly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$open in Z. Since g is perfectly $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is both open and closed in Y. Since f is contra-continuous, $f^{-1}\big(g^{-1}(G)\big)$ is closed and open in X. Hence $g \circ f$ is perfectly continuous.

Theorem 4.14 If $f: X \to Y$ is perfectly $\widehat{S_P}^*$continuous and $g: Y \to Z$ is $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is perfectly continuous.

Proof: Let A be any open set in Z. Since g is $\widehat{S_P}^*$continuous, $g^{-1}(A)$ is $\widehat{S_P}^*$open in Y. Since f is perfectly $\widehat{S_P}^*$continuous, $f^{-1}\big(g^{-1}(A)\big)$ is both open and closed in X. Hence, $g \circ f$ is perfectly continuous.

Theorem 4.15 If $f: X \to Y$ is perfectly continuous and $g: Y \to Z$ is strongly $\widehat{S_P}^*$continuous, then $g \circ f: X \to Z$ is perfectly $\widehat{S_P}^*$continuous.

Proof: Let A be any $\widehat{S_P}^*$open set in Z. Since f is perfectly continuous, $g^{-1}(A)$ is open in Y. Since f is perfectly continuous, $f^{-1}\big(g^{-1}(A)\big)$ is both open and closed in X. Hence, $g \circ f$ is perfectly $\widehat{S_P}^*$continuous.

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