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Econophysics: Analytical Modelling of Wealth Distribution in a Closed Economy

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Abstract

Econophysics applies statistical physics to formulate mathematical models for complex economic systems, offering insights into wealth distribution dynamics. This study presents an analytical model describing wealth distribution in a closed economy by applying a binomial distribution framework that incorporates saving propensity and group stratification. The model demonstrates that as saving propensity increases, wealth distribution gradually shifts from a Boltzmann-Gibbs form to an asymmetric Gaussian-like form, indicating a reduction in the number of agents with near-zero wealth. Additionally, the Lorenz curve and Gini coefficient are derived to quantify income inequality, highlighting the influence of parameters such as saving propensity, group structure, and success probability on economic disparity. The analytical results align closely with those obtained through simulation approaches, confirming the robustness of the model. This approach elucidates how macroeconomic patterns and inequality measures emerge from micro-level transactional interactions within closed systems. The model provides a quantitative framework for understanding the underlying dynamics of wealth accumulation and distribution, offering potential applications in evaluating economic policies aimed at reducing inequality and promoting equitable wealth distribution in society.

Keywords: Econophysics, Wealth distribution, Closed economy, Binomial distribution, Saving propensity, Lorenz curve, Gini coefficient, Income inequality, Statistical mechanics, Boltzmann-Gibbs distribution, Economics

1. Introduction

Econophysics, an interdisciplinary field, leverages statistical physics to analyse economic systems, treating economic agents as interacting particles akin to those in thermodynamic systems [1]. This approach, pioneered by researchers like Eugene Stanley and Victor Yakovenko, models phenomena such as wealth distribution, market fluctuations, and economic inequality using tools like entropy maximization and kinetic exchange models [2]. Their work has been extended by studies incorporating behavioural factors, such as saving propensity, which significantly alters the wealth distribution by introducing deviations from the exponential form observed in idealized models [3]. In a closed economy, where total money and the number of agents are conserved, wealth distribution often exhibits patterns analogous to energy distributions in statistical mechanics, notably the Boltzmann-Gibbs distribution [4].
This study proposes an analytical model to describe wealth distribution in a closed economy, focusing on the impact of saving propensity and group stratification. Unlike previous simulation-based approaches, such as those by Chakraborti and Chakrabarti [3], our model derives the wealth distribution analytically, confirming their findings that zero saving-propensity yields a Boltzmann-Gibbs distribution, while non-zero saving propensity shifts the distribution to a Gaussian-like form. We also compute the Lorenz curve and Gini coefficient to assess income inequality, providing a quantitative measure of economic disparity [5]. Our results align with real-world observations and offer a framework for policy analysis to mitigate inequality.

Econophysics has been shaped by seminal contributions from physicists applying statistical mechanics to economics. Stanley introduced scaling laws to analyse financial market fluctuations, establishing econophysics as a distinct field [1]. Yakovenko and Dragulescu demonstrated that money distribution in a closed economy follows the Boltzmann-Gibbs distribution, where the average money per agent acts as an effective temperature [2]. Their model assumes random pairwise transactions with local money conservation, analogous to energy conservation in particle collisions.

Chakrabarti explored wealth distribution using kinetic exchange models, noting that money distribution in markets with zero saving propensity resembles the Maxwell-Boltzmann distribution, with most agents holding minimal wealth [6]. Chakraborti extended this by incorporating saving propensity, showing that non-zero savings lead to a Gamma-like distribution, reducing the number of agents with near-zero wealth [3]. These findings highlight the role of individual behaviours, such as saving, in shaping macroeconomic outcomes.

Our model builds on these works, introducing a binomial distribution to represent wealth acquisition through random transactions, allowing for analytical derivation of the wealth distribution and its dependence on parameters like saving propensity and group structure.

2. Proposed Model

We consider a closed economy with a fixed population of D agents and total money M. The population is divided into X groups based on economic conditions, indexed by n (from 1 to X), where higher n corresponds to wealthier groups. The fraction of the population in the n^{th} group, Y_n, follows a geometric distribution:

$$Y_n = ar^{n-1} \tag{01}$$

where $0 < r < 1$. Thus, Y_n is smaller for higher n, as observed in every society. In the above equation, 'a' is a constant ensuring $\sum_{n=1}^{X} Y_n = a \sum_{n=1}^{X} r^{n-1} = 1$, yielding:

$$a = \frac{1-r}{1-r^X} \tag{02}$$

According to this model, the wealth acquisition by agents is based on N Bernoulli trials conducted hypothetically for each agent. Each Bernoulli trial has two outcomes: *success* and *failure*. The probability of success in each trial for an agent in the n^{th} group is p_n. The money m possessed by an agent is proportional to the number of successes x:

$$m = kx \tag{03}$$

where k is the constant of proportionality. The minimum and maximum values of m are therefore 0 and kN respectively. No monetary gain or loss is associated with *failure* in a Bernoulli trial.
For an agent in the n^{th} group, the probability of x successes in N trials (according to Binomial distribution of probability) can be expressed as,

$$P(n,x) = {}^{N}_{x}C\ (p_n)^x\ (1-p_n)^{1-x} \tag{04}$$

Here x can be any integer between 0 and N.
Using equation (03) in equation (04), the probability of an agent in the n^{th} group having m money can be expressed as,

$$P(n,m) = {}^{N}_{m/k}C\ (p_n)^{m/k}\ (1-p_n)^{N-(m/k)} \tag{05}$$

To calculate $P(n,m)$, using equation (05), one must use integral values of the ratio m/k ranging from 0 to N.
The probability that an agent (chosen randomly from the population) has m amount of money:

$$P(m) = \sum_{n=1}^{X} Y_n\, P(n,m) \tag{06}$$

Here, Y_n is regarded as the probability that an agent belongs to the n^{th} group.
It can be shown that, $\sum_{m=0}^{kN} P(m) = 1$, because $\sum_{m=0}^{kN} P(n,m) = 1$ and $\sum_{n=1}^{X} Y_n = 1$ by the definitions of Y_n and $P(n,m)$.
For a linear variation of the success probability p_n, with group number n, one may choose the following expression:

$$p_n = p_1 + (n-1)\frac{p_X - p_1}{X-1} \tag{07}$$

where p_1 and p_X are the success probabilities of the hypothetical Bernoulli trials conducted for the poorest and richest groups, respectively.
For non-linear variations of p_n, with group number n, we generalize p_n as:

$$p_n = p_1 + \left(\frac{n-1}{X-1}\right)^j (p_X - p_1) \tag{08}$$

where the parameter j controls the non-linearity. For $j = 1$, equation (08) reduces to equation (07).
We introduce saving propensity λ $(0 < \lambda < 1)$, which reduces the effective money available for transactions:

$$p_X = (1-\lambda)\epsilon \tag{09}$$

where ϵ is a positive fractional constant. As λ increases, p_X decreases, causing a reduction in the success probability (p_n) in the N hypothetical Bernoulli trials conducted for each agent.

In conventional kinetic exchange models, saving propensity (λ) denotes the fraction of money each agent retains before participating in transactions, thereby reducing the amount exchanged. However, in our analytical framework based on Bernoulli trials, we model saving propensity by scaling down the effective success probability in each trial by a factor of $(1-\lambda)$. This approach ensures that agents with higher saving propensity engage less actively in transactions, effectively reflecting wealth retention behaviour within the probabilistic structure of the model. This formulation is mathematically equivalent in outcome to reducing the wealth available for transactions, while remaining analytically tractable within our binomial framework.

The average money per agent, T, is:

$$T = \sum_{m=0}^{kN} m\, P(m) \tag{10}$$

The total money (M) involved in the transactions among D agents is therefore,

$$M = DT = D\sum_{m=0}^{kN} m\, P(m) \tag{11}$$

This model allows us to compute $P(m)$ and study its variation with parameters like λ, r, p_1, X, and j. During the variation of any of these parameters, the average money per agent (T) (eqn. 10) must be kept constant because this model has been formulated for a closed economy. To keep T constant, as one changes any of these parameters, the parameter k can be adjusted.

3. Lorenz Curve and Gini Coefficient

Based on the values of m and $P(m)$, we can calculate the quantities required for drawing Lorenz curve, which represents the inequality of income/wealth distribution in a population. By definition, $P(m)$ is proportional to the number of people possessing m amount of money each. Thus, the cumulative share of the population (denoted by $L(m)$ here) corresponding to a value of m, which is essentially the percentage of population having m or less amount of money each, is given by,

$$L(m) = \frac{\sum_{y=0}^{m} P(y)}{\sum_{y=0}^{kN} P(y)} \times 100 \quad (12)$$

Since $\sum_{y=0}^{kN} P(y) = 1$, we get,

$$L(m) = 100 \times \sum_{y=0}^{m} P(y) \quad (13)$$

By definition, the product, $mP(m)$, is proportional to the total amount of money in possession of people having m amount of money each. Thus, the cumulative percentage of money (denoted by $W(m)$ here) corresponding to a value of m, which is essentially the percentage of total wealth in possession of people having m or less amount of money each, is given by,

$$W(m) = \frac{\sum_{y=0}^{m} yP(y)}{\sum_{y=0}^{kN} yP(y)} \times 100 \quad (14)$$

Using equation (10) in equation (14), we get,

$$W(m) = \frac{\sum_{y=0}^{m} yP(y)}{T} \times 100 \quad (15)$$

The minimum and maximum values of both $L(m)$ and $W(m)$ are 0 and 100 respectively, by definition. Lorenz curve can be obtained by plotting $W(m)$ graphically as a function of $L(m)$. If the money is distributed equally among the population, this graph would be a straight line passing through the origin with $slope = +1$. This line may be called the *line of equality*. The greater the inequality in wealth distribution, the farther would be the Lorenz curve from the line of equality. Gini coefficient is defined as the area between the *line of equality* and the Lorenz curve divided by the area under the *line of equality*. It is thus measured on a scale of 0 to 1. Its value is 0 if the money is equally distributed among all members of population. Its value is closer to 1 for populations with smaller fractions of people possessing larger shares of wealth.

4. Results and Discussion

We have used the model to generate wealth distribution curves and compute the Lorenz curve and Gini coefficient. The following figures illustrate key findings.

Figure 1 shows the population distribution across groups, consistent with real-world observations where wealthier groups have fewer members [7]. It is evident that higher r values reduce economic disparity. Quantitatively, for example, when the parameter r is set to 0.6, the fraction of population Y_n decreases from approximately 0.4 for the first group to around 0.02 for the tenth group. This rapid decrease illustrates how higher r values create a sharper stratification in society, with wealthier groups having proportionally fewer members. Conversely, when r is closer to 1, Y_n values become more uniform across groups, indicating reduced disparity in group sizes and a more equitable population distribution.

Figure 2 illustrates p_n vs. n variations for different j values ($j = 1,2,3$). Higher j enhances the success probability for richer groups, reflecting real-world income disparities [8].

Figure 3 analytically reproduces the Boltzmann-Gibbs distribution for $\lambda = 0$, where most agents hold minimal wealth, aligning with Chakraborti's simulations [3]. This validates our model's ability to capture equilibrium distributions without extensive computational simulations.

Figure 4 shows that increasing saving propensity shifts the wealth distribution to a Gaussian-like form, reducing inequality as more agents retain finite wealth, consistent with Chakraborti's findings [3]. The shift of the distribution from exponential (Boltzmann-Gibbs-like) to Gaussian-like forms with increasing saving propensity λ arises because higher λ values reduce effective transactional participation. Consequently, money accumulation becomes more clustered around the mean wealth rather than being widely spread, implying greater economic stability and reduced likelihood of extreme poverty or extreme wealth among agents.

Figure 5 shows Lorenz curves, showing that higher saving propensity reduces the Gini coefficient, indicating a more equitable wealth distribution [5]. This aligns with economic theories suggesting that savings stabilize income distributions [9].

For figures 4 & 5, the average money T has been kept fixed (at 113.08), for plots with different values of λ, by adjusting the parameter k. It has been done only to meet the requirement that the average money should not change in a closed economy.

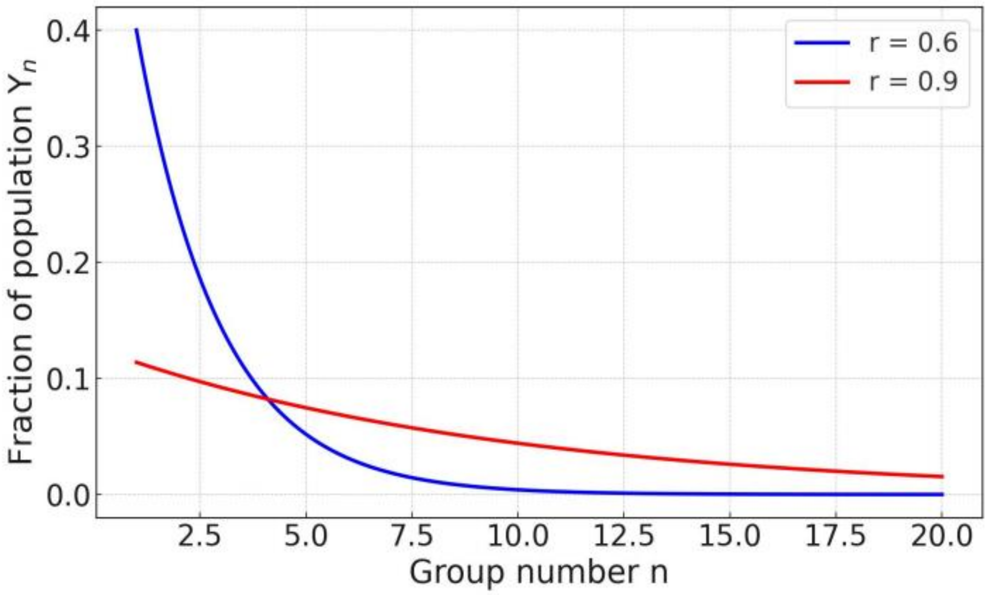

Figure 1: Fraction of population Y_n vs. group number n, for $r = 0.6$ and 0.9, with $X = 20$. Higher r reduces disparity between group populations, approaching uniform distribution as r approaches 1.

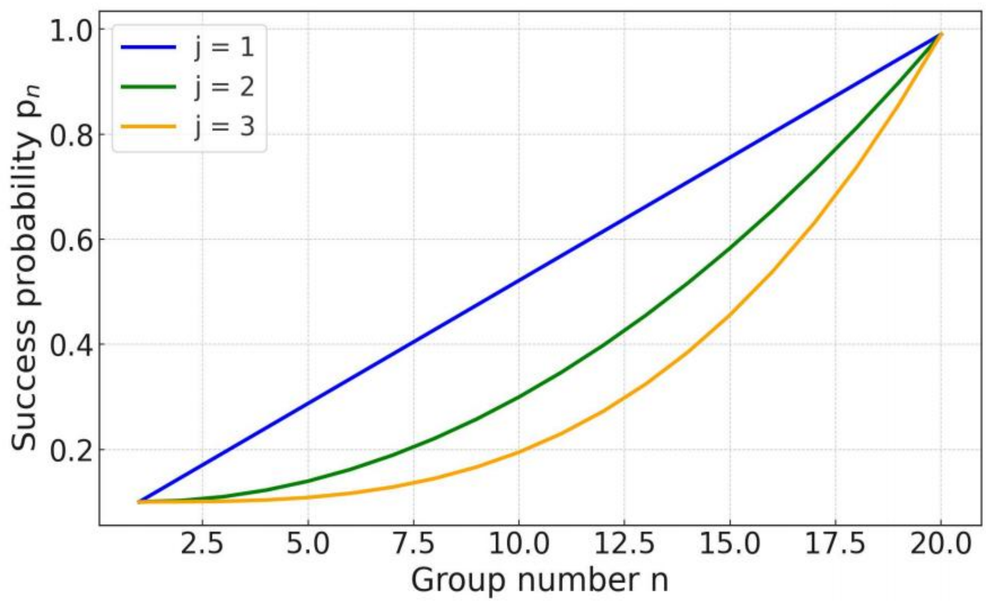

Figure 2: Success probability p_n vs. group number n, for $j = 1,2,3$, with $X = 20$, $p_1 = 0.1$, $\varepsilon = 0.99$, $\lambda = 0$. Higher j introduces more non-linear variations, increasing p_n for wealthier groups.

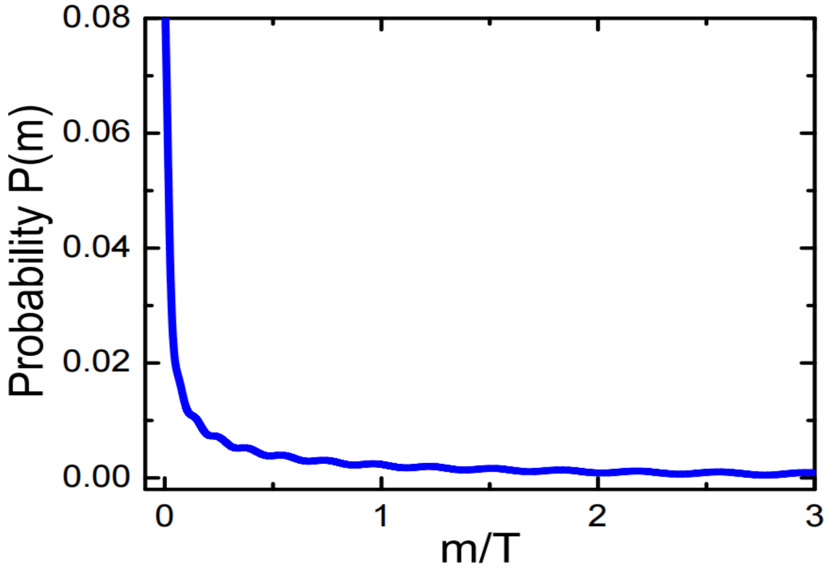

Figure 3: Probability $P(m)$ vs. m/T, for $\lambda = 0$, with $N = 500$, $X = 20$, $r = 0.9$, $p_1 = 0.001$, $\varepsilon = 0.99$, $j = 2$, $k = 1$. The curve matches the Boltzmann-Gibbs distribution, confirming Chakraborti's simulation results [3].

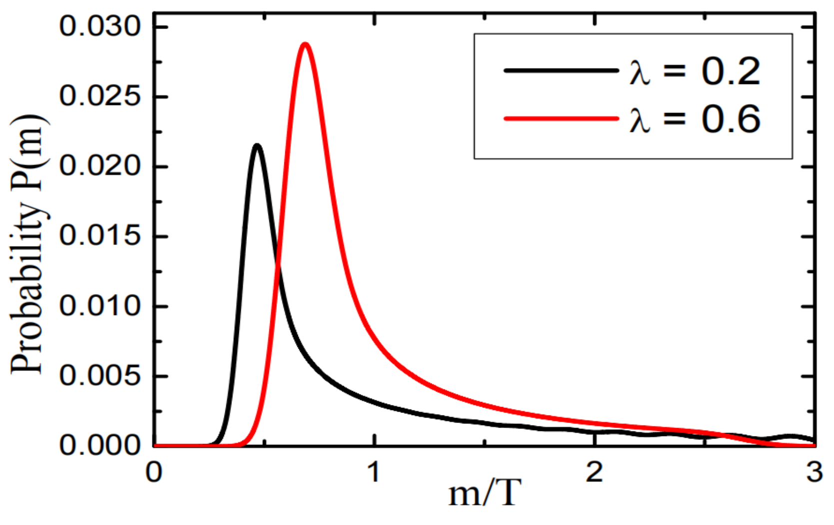

Figure 4: $P(m)$ vs. m/T for λ =0.2, 0.6, with $N = 500$, $X = 20$, $r = 0.9$, $p_1 = 0.1$, $\varepsilon = 0.99$, $j = 2$. Average money, T, was kept constant (at 113.08) as λ was varied. Higher λ shifts the distribution to a Gaussian-like form, reducing the number of agents with near-zero wealth.

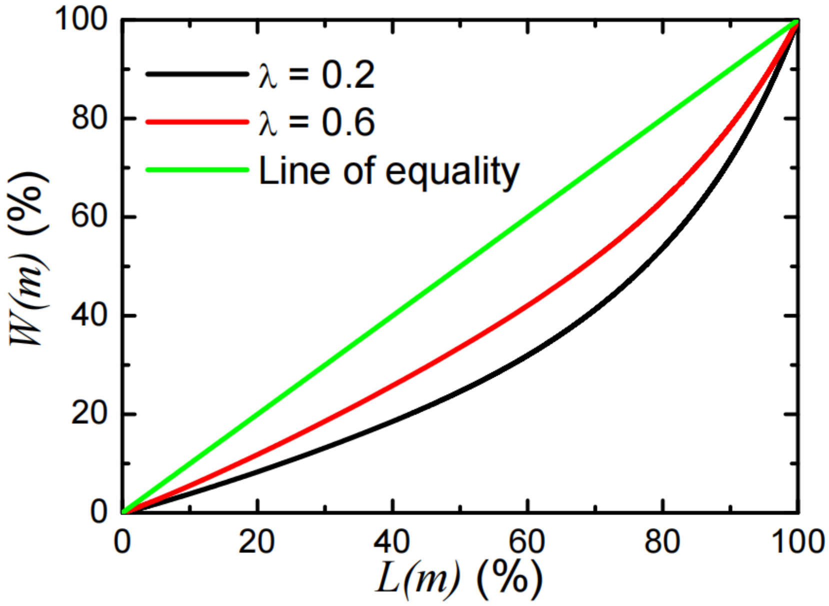

Figure 5: Lorenz curves for $\lambda = 0.2,\ 0.6$, with $N = 500$, $X = 20$, $r = 0.9$, $p_1 = 0.1$, $\varepsilon = 0.99$, $j = 2$, $T = 113.08$. Gini coefficients are 0.37 and 0.24 respectively, indicating reduced inequality with higher λ.

Specifically, to maintain an average money (T) of 113.08 when saving propensity λ increases from 0.2 to 0.6, the parameter k was adjusted from approximately 1 to 1.469. This reflects the model's property that, as saving propensity increases (reducing effective success probability), the proportionality constant k must be increased to preserve the overall average money in the closed economy.

The proposed analytical model effectively captures the dynamics of wealth distribution in a closed economy, demonstrating its utility in quantifying economic inequality through the Lorenz curve and Gini coefficient. The model's strength lies in its analytical derivation of wealth distributions, transitioning from a Boltzmann-Gibbs form to a Gaussian-like form as saving propensity increases, consistent with prior simulation-based studies [3]. This framework allows policymakers to evaluate interventions, such as progressive taxation or savings incentives, to mitigate inequality. The model's incorporation of group stratification and saving propensity mirrors real-world economic structures, such as India's stratified society with distinct income groups and high savings rates, making it relevant for analysing policy impacts in similar contexts [11]. However, the model's closed-economy assumption limits its applicability to open economies with global trade and capital flows. It also simplifies agent interactions by using a binomial distribution, potentially overlooking complex behavioural factors like risk aversion, speculative investments, or social networks. Future enhancements could integrate dynamic agent behaviours, time-varying parameters, or network effects to better reflect real-world complexities, improving predictive accuracy and policy relevance [12].

In the context of the Indian economy, the model's structure aligns closely with India's socioeconomic framework, characterized by a stratified society with significant income disparities and a large informal sector. The computed Gini coefficient, which quantifies income inequality, reflects India's high disparity, with a Gini coefficient of approximately 0.35 as reported in recent studies [11]. Higher saving propensity, reflective of India's household savings rate of around 20% of GDP [13], shifts wealth distributions toward a Gaussian-like form, reducing the proportion of agents with near-zero wealth. This supports government policies promoting financial inclusion, which encourages savings among low-income groups to reduce income disparity [14]. However, the closed-economy assumption may not fully account for India's integration into global markets through trade and remittances, which significantly influence wealth dynamics. Tailoring the model to incorporate these external factors could enhance its applicability, providing a robust tool to assess policies aimed at reducing India's income disparity and fostering equitable wealth distribution.

5. Analogies with Statistical Mechanics

Our model draws parallels with statistical mechanics, where the Boltzmann-Gibbs distribution describes energy states [4]. In our framework, money m is analogous to energy (E), and the average money T corresponds to temperature. The probability $P(m)$ mirrors the probability of a system occupying a state with energy E, given by $P(E) \propto g(E)exp(-E/k\tau)$ where k stands for Boltzmann constant and τ stands for the absolute temperature. For $\lambda = 0$, $P(m)$ follows the Boltzmann-Gibbs form, with most agents at low wealth states, akin to low-energy states in a cold system. Non-zero λ introduces a cooperative effect, shifting $P(m)$ to a Gaussian-like distribution, similar to modified energy distributions in interacting particle systems [10].

6. Conclusion

This study presents an analytical model for wealth distribution in a closed economy, confirming the simulation-based results obtained by Chakraborti and others [3]. By modelling wealth acquisition as Bernoulli trials and incorporating saving propensity, we derive a distribution that transitions from Boltzmann-Gibbs to Gaussian-like forms as saving propensity increases, reflecting real-world economic behaviours. The computed Lorenz curves and Gini coefficients quantify inequality, showing that higher saving propensity reduces disparity. These findings underscore the power of econophysics in bridging micro-level interactions with macroeconomic outcomes, offering a robust framework for analysing economic policies aimed at reducing inequality.

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