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Fault-Tolerant Control of Quadcopter Systems using LMI-optimized $\mathcal{L}_1$-Adaptive Controller

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A B S T R A C T

Quadcopters are a type of unmanned aerial vehicles (UAVs) with four fixed-pitch propellers that have become part of the aviation industry. They are multivariable, highly coupled underactuated mechanical systems, and exhibit open loop instability. They are used in both military and civilian applications, including search and rescue, photography, reconnaissance, agriculture and so on. The use of quadcopter systems requires that they carry some payload to be able to carry out operations. Several control techniques including L_1adaptive controllers have been developed and deployed in order to meet certain performance and robustness requirements, however, in the event of loss of effectiveness (LOE) of the actuators, the stability of the system can no longer be guaranteed, leading to catastrophic failures. This is due to the nominal design approach of the L_1adaptive controller, which is not optimal. Although, several optimization techniques such as genetic algorithms and local best harmony search (lbest-HS) algorithms have been used to optimize the L_1adaptive controller, they do not guarantee global convergence of the solutions. This work investigated the use of linear matrix inequality (LMI)- based optimization to determine the controller parameters of the L_1adaptive controller with a view to increase performance and robustness of the controller to external disturbances and internal disturbances caused by LOE of the actuators. The results indicate that the quadcopter system can be able to maintain stability in the event of a 10% LOE in one of the actuators. Furthermore, the LMI-optimized controller has better performance compared to the nominal approach.

Keywords: LMI, $\mathscr{L}1$ Adaptive Control, Optimization, Robustness, Fault-tolerant control

1. Introduction

The quadcopter (or quadrotor) is an UAV with four fixed-pitched rotors, and has six degrees of freedom, making it an under-actuated system. It also exhibits open loop instability, making its control a challenging task. There are two basic configurations of the quadcopter; the '+' configuration, and the 'x' configuration. The 'x' configuration is considered to be more stable in terms of operations(Thu & Gavrilov, 2017). Due to its under-actuation, the control of the quadcopter becomes increasingly difficult in the event of a failure of any of the actuators. Quadcopters rely on four rotors for stability and manoeuvrability. However, if one of the rotors experiences a partial failure, it leads to a compromise of one of the degrees of freedom of the already under-actuated system which disrupts the stability and control of the quadcopter system. $\mathcal{L}_1$ adaptive controller has the potential to increase stability and robustness of quadcopter systems because of the decoupled nature of the controller which allows for a trade-off between performance and robustness, this enables the realization of fast adaptation while maintaining robustness. $\mathcal{L}_1$ adaptive control has been designed for fault-tolerant control of quadcopter systems with actuator loss, but can only maintain stability when the partial loss is not more than 24% (Xu et al., 2016), however, in the event of a total loss of a single actuator, the quadcopter system suffers catastrophic stability and robustness failure. This is because the parameters of the $\mathcal{L}_1$ adaptive controller obtained from mathematical calculations using the $\mathcal{L}_1$-norm conditions do not provide optimal performance of the controller (Maiti et al., 2022), hence, the need to provide optimization techniques, which will increase its performance and robustness in the event of actuator faults of the quadcopter system. Linear Matrix Inequality (LMI)-based optimization has been applied to the optimization of the filter parameters only of the $\mathcal{L}_1$ adaptive controller (Hashim et al., 2017; Kim & Hovakimyan, 2014), however, the relaxation of non-linear constraints carried out in the formulation of the LMIs compromises the robustness of the controller.

$\mathcal{L}_1$ adaptive controller has been designed and developed for stabilization of rotocrafts and quadcopter systems (Ahmed et al., 2009)(Michini & How, 2009), however, the robustness of the control algorithm was not investigated under wind disturbances. (Michini & How, 2009) proposed an optimization approach linking both the performance and robustness of the controller to the design of the underlying filter, however, the non-convexity of the cost function acted to limit the complexity of the assumed form of the filter. The output feedback architecture of the $\mathcal{L}_1$ adaptive controller was developed for quadcopter systems by (Thu & Gavrilov, 2017), and it was shown that the time delay margins were better compared to a model reference adaptive controller. However, the methodology presented was based on intuition, furthermore, the performance of the controller was not investigated under faulty conditions. (Gasparyan & Darbinyan, 2019) developed a framework for the development of a fault-tolerant control system for multirotor UAVs using $\mathcal{L}_1$ adaptive control.

$\mathcal{L}_1$ adaptive controller based on nominal and degraded models, focusing on stability and performance was developed in (Souanef et al., 2023; Souanef & Fichter, 2015). The method employed a multiple-model $\mathcal{L}_1$ adaptive control, which includes a minimal reference model and degraded models to maintain robustness against critical failures. The fault-tolerant properties of an $\mathcal{L}_1$ adaptive controller for quadrotor vehicles were investigated, focusing on actuator faults only. The structure of the controller adopted included an inner-loop LQR controller for stability, and an outer-loop $\mathcal{L}_1$ adaptive controller for robustness against actuator faults (Xu et al., 2016). The results obtained were compared to that of a fixed gain LQR with integral action controller based on the recovery performance to partial actuator failures in the rotors due to voltage loss, and the results indicated that the designed $\mathcal{L}_1$ adaptive controller performed better. However, the nominal controller design approach can be made to increase robustness and performance by introducing an optimization approach. This is because according to (Maiti et al., 2022), the nominal approach to the controller design is not optimal, hence its performance can be improved by introducing optimization algorithms.

Fault-tolerant control of quadcopter systems was carried out in (Beikzadeh & Liu, 2018; Fernández et al., 2017; Jafarnejadsani et al., 2017) using $\mathcal{L}_1$ adaptive control and the results compared to LQR and PID controllers. Optimization techniques were introduced but for only the filter design. The work of (Nguyen et al., 2020) focused on fault-tolerant control of quadcopter systems using H_∞ synthesis, which tracks the desired trajectory subject to actuator faults, and an adaptive augmentation controller. (Wu et al., 2023) developed an $\mathcal{L}_1$ adaptive controller for quadcopter systems for both the rotational and translational dynamics. The uncertainties and disturbances ae lumped together as unknown, non-linear forces. The results indicate that the developed controller can accurately estimate unknown forces, and it outperforms baseline controllers with small tracking errors recorded. Adaptive sliding mode observer was also used to fault-tolerant control of quadcopter systems in (Dhahri & Naifar, 2023) (Chnib et al., 2023) (Khattab et al., 2024). Sufficient conditions for stability of the state estimation errors were developed using Lyapunov stability and H_∞ techniques. Those conditions were then articulated as linear matrix inequality (LMI) problems to determine optimal values of the controller parameters. The results obtained show the practical applicability of the proposed controller, however, the method has not been extended to $\mathcal{L}_1$ adaptive control synthesis, which gives better robustness and performance. (Mao et al., 2024; Zhou et al., 2024) also designed an $\mathcal{L}_1$ adaptive controller with a fault-tolerant mode for quadrotor system. However, this method of using the $\mathcal{L}_1$ adaptation law is only applicable if the damage is within the actuator constraints. furthermore, the methodology adopted for the design is not optimal, and does not lead to an optimized controller.

The aim of this research is to design and develop an LMI-based optimized fault-tolerant robust $\mathcal{L}_1$ adaptive controller for quadcopter systems subjected to actuator faults

2. Materials and Methods

2.1 Quadcopter Dynamic Model

Consider the quadcopter system presented in figure 1. The system is described by its inertial frame ($\dot{\varphi}, \dot{\theta}, \dot{\psi}$) and the earth frame ($x, y, z$).

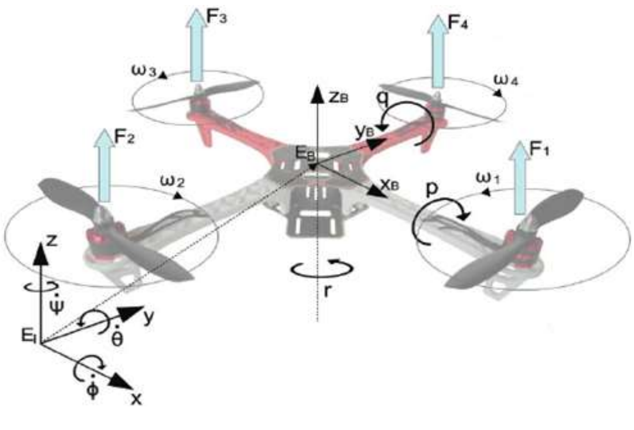

Figure 1: Quadcopter F450 showing the Earth Frame and the Inertial Frame

The dynamic model equations of the quadcopter are presented in three sections; the translational motion dynamics, the rotational motion dynamics and the actuator faults dynamics (Nguyen & Hong, 2018). The translational and rotational motions of the quadcopter are described in equations (1) – (6).

$$\ddot{x} = (U_z(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) - K_x\dot{x}) / m \quad (1)$$

$$\ddot{y} = (U_z(\cos\varphi\sin\theta\sin\psi - \sin\varphi\sin\psi) - K_y\dot{y}) / m \quad (2)$$

$$\ddot{z} = -g + (U_z(\cos\varphi\cos\theta) - K_z\dot{z}) / m \quad (3)$$

$$\ddot{\varphi} = (U_\varphi + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi} - J_T\dot{\theta}\omega - K_\varphi\dot{\varphi}) / I_{xx} \quad (4)$$

$$\ddot{\theta} = (U_\theta + (I_{zz} - I_{xx})\dot{\varphi}\dot{\psi} - J_T\dot{\varphi}\omega - K_\theta\dot{\theta}) / I_{yy} \quad (5)$$

$$\ddot{\psi} = (U_\psi + (I_{xx} - I_{yy})\dot{\varphi}\dot{\theta} - K_\psi\dot{\psi}) / I_{zz} \quad (6)$$

where x, y, z represent the three positions of the quadcopter, I_{xx}, I_{yy}, I_{zz} represent the moments of inertia along the x, y, z directions, $K_\varphi, K_\theta, K_\psi, K_x, K_y, K_z$ represent the drag coefficients, which depend on the flight conditions. J_T is the moment of inertia of each motor, while ω represents the angular velocity of the motors. m is the total mass of the quadcopter system, φ, θ, ψ are the roll, pitch and yaw Euler angles respectively. The four control inputs of the quadcopter according to (Nguyen & Hong, 2018) are presented in equation (7).

$$\begin{aligned} U_z &= F_1 + F_2 + F_3 + F_4 \\ U_\varphi &= (F_4 - F_2)L \\ U_\theta &= (F_3 - F_1)L \\ U_\psi &= \tau_1 - \tau_2 + \tau_3 - \tau_4 \end{aligned} \quad (7)$$

where $\tau_i = d\omega_i^2$ and $F_i = b\omega_i^2$ are the torque and thrust forces produced by the *i*-th motor, while *b*, *d* are positive constants depending on the density of air, radius of the propeller, number of blades and geometry. ω_i is the angular velocity of the *i*-th motor, and U_z is the total thrust generated by the motors, $U_\varphi, U_\theta, U_\psi$ are the torques in the roll, pitch and yaw Euler angles respectively. *L* is the length of the arm of the quadcopter from the centre.

2.2 Quadcopter Model with Rotor Faults

The dynamic model of the quadcopter with rotor faults are presented in equations (8) – (13).

$$\ddot{x} = (U_{zf}(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) - K_x\dot{x}) / m \quad (8)$$

$$\ddot{y} = (U_{zf}(\cos\varphi\sin\theta\sin\psi - \sin\varphi\sin\psi) - K_y\dot{y}) / m \quad (9)$$

$$\ddot{z} = -g + (U_{zf}(\cos\varphi\cos\theta) - K_z\dot{z}) / m \quad (10)$$

$$\ddot{\varphi} = (U_{\varphi f} + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi} - J_T\dot{\theta}\omega - K_\varphi\dot{\varphi}) / I_{xx} \quad (11)$$

$$\ddot{\theta} = (U_{\theta f} + (I_{zz} - I_{xx})\dot{\varphi}\dot{\psi} - J_T\dot{\varphi}\omega - K_\theta\dot{\theta}) / I_{yy} \quad (12)$$

$$\ddot{\psi} = (U_{\psi f} + (I_{xx} - I_{yy})\dot{\varphi}\dot{\theta} - K_\psi\dot{\psi}) / I_{zz} \quad (13)$$

where $U_{zf}, U_{\varphi f}, U_{\theta f}, U_{\psi f}$ are the control inputs to the quadcopter system during faulty operations, which are described by equation (14).

$$\begin{aligned} U_{zf} &= F_{1f} + F_{2f} + F_{3f} + F_{4f} \\ U_{\varphi f} &= L(F_{4f} - F_{2f}) \\ U_{\theta f} &= L(F_{3f} - F_{1f}) \\ U_{\psi f} &= d(F_{1f} - F_{2f} + F_{3f} - F_{4f})b \end{aligned} \quad (14)$$

The fault model of the actuator can be presented in terms of the thrust generated due to partial or complete loss of effectiveness of the i-th rotor by equation (15).

$$F_{if} = (1 - \gamma_i)F_i \quad (15)$$

where $0 \le \gamma_i \le 1$ indicates that the *i*-th actuator's level of loss of effectiveness.

Substituting equation (14) into equations (8) – (13) yields equations (16) – (21).

$$\ddot{x} = (U_z(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi) - K_x\dot{x}) / m + \delta_x \quad (16)$$

$$\ddot{y} = (U_z(\cos\varphi\sin\theta\sin\psi - \sin\varphi\sin\psi) - K_y\dot{y}) / m + \delta_y \quad (17)$$

$$\ddot{z} = -g + (U_z(\cos\varphi\cos\theta) - K_z\dot{z}) / m + \delta_z \quad (18)$$

$$\ddot{\varphi} = (U_{\varphi} + (I_{yy} - I_{zz})\dot{\theta}\dot{\psi} - J_T\dot{\theta}\omega - K_{\varphi}\dot{\varphi}) / I_{xx} + \delta_{\varphi} \quad (19)$$

$$\ddot{\theta} = (U_{\theta} + (I_{zz} - I_{xx})\dot{\varphi}\dot{\psi} - J_T\dot{\varphi}\omega - K_{\theta}\dot{\theta}) / I_{yy} + \delta_{\theta} \quad (20)$$

$$\ddot{\psi} = (U_{\psi} + (I_{xx} - I_{yy})\dot{\varphi}\dot{\theta} - K_{\psi}\dot{\psi}) / I_{zz} + \delta_{\psi} \quad (21)$$

where the unknown terms are given by equations (22) – (27).

$$\delta_x = -(\cos\varphi\sin\theta\cos\psi + \sin\varphi\sin\psi)(\gamma_1F_1 + \gamma_2F_2 + \gamma_3F_3 + \gamma_4F_4) / m \quad (22)$$

$$\delta_y = -(\cos\varphi\sin\theta\cos\psi - \sin\varphi\sin\psi)(\gamma_1F_1 + \gamma_2F_2 + \gamma_3F_3 + \gamma_4F_4) / m \quad (23)$$

$$\delta_z = -\cos\varphi\cos\theta(\gamma_1F_1 + \gamma_2F_2 + \gamma_3F_3 + \gamma_4F_4) / m \quad (24)$$

$$\delta_{\varphi} = -L(\gamma_4F_4 - \gamma_2F_2) / I_{xx} \quad (25)$$

$$\delta_{\theta} = -L(\gamma_3F_3 - \gamma_1F_1) / I_{yy} \quad (26)$$

$$\delta_{\psi} = -d(\gamma_1F_1 - \gamma_2F_2 + \gamma_3F_3 - \gamma_4F_4) / bI_{zz} \quad (27)$$

2. $\mathcal{L}_1$ Adaptive Controller Design for Quadcopter System

$\mathcal{L}_1$ adaptive control concept evolved from indirect model reference adaptive control to ensure robustness without compromising performance of a controlled system subjected to unmodelled dynamics, external disturbances and time-varying uncertainties (N. Hovakimyan & Cao, 2010). Figure 2 shows the structure of the $\mathcal{L}_1$ adaptive controller. It comprises of a low pass filter, which filters out oscillations from the control signal that may arise as a result of high adaptation gains, thus ensuring that the controlled system remains stable. It comprises of a predictor and adaptation laws to adapt unknown constants, and time varying uncertainties.

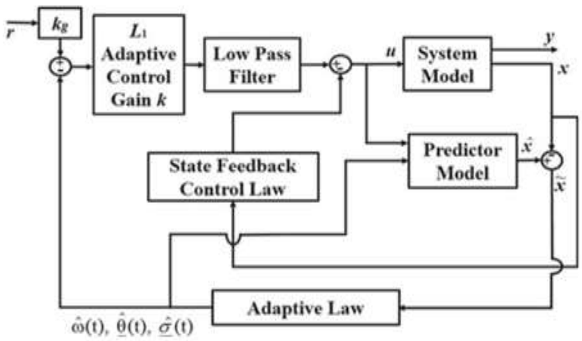

Figure 2: $\mathcal{L}_1$ Adaptive Controller Architecture

For the quadcopter system described by the state equations given in equation (27) with open loop dynamics containing unknown constants, time varying uncertainties and time varying disturbances (Maiti et al., 2022)

$$\dot{x}(t) = A_0x(t) + B(\omega u_1(t) + \theta(t)\|x\|_{L\infty} + \sigma(t) + Bu_2(t)$$
$$y(t) = C^T x(t) \quad (28)$$

Where $A_0 \in \Re^{n\times n}$ is the open loop matrix for the system, $B \in \Re^{n\times p}$ is the input matrix, $C \in \Re^{m\times n}$.

According (N. Hovakimyan & Cao, 2010), the $\mathcal{L}_1$ adaptive control signal is formulated from the adaptive estimates of the unknown constant, time varying uncertainties and time varying disturbances, and is given by equation (29).

$$u_2(s) = -kD(s)\left(\hat{r}_{ud}(s) - k_g r(s)\right) \quad (29)$$

Where $\hat{r}_{ud}(s)$ is the Laplace transform of equation (30).

$$\hat{r}_{ud}(t) = \hat{\omega}(t)u_1(t) + \hat{\bar{\theta}}(t)\|x\|_{L\infty} + \hat{\sigma}(t) \quad (30)$$

And k and k_g are the pre-filter and feedforward gains respectively, while *r(s)* is the input to the system, and *D(s)* is a strictly proper stable transfer function. The *D(s)* is designed in such a way that it incurs a low-pass filter of strictly proper stable transfer function as presented in equation (31).

$$C(s) = \frac{\omega k D(s)}{\left(1 + \omega k D(s)\right)} \quad (31)$$

The state feedback control law considered for the inner control loop of the quadcopter system is given by equation (32).

$$u_1(t) = -K^T x(t) \quad (32)$$

Where $K \in \Re^{n \times m}$ is the state feedback gain matrix which ensures that the quadcopter closed loop system matrix given in equation (33) is Hurwitz.

$$A_m = A_0 - BK^T \quad (33)$$

The overall control law for the control system is given by equation (34)

$$u(t) = u_1(t) + u_2(t), \in \Re^p \quad (34)$$

Substituting (29) and (32) into (34) gives equation

$$u(s) = -kD(s)\left(\hat{r}_{ud}(s) - k_g r(s) - K^T x(s)\right) \quad (35)$$

Therefore, the model of the quadcopter system in closed loop with the $\mathcal{L}_1$ adaptive control law can be written as presented in equation (36).

$$\dot{x}(t) = A_m x(t) + B\left(\omega u_1(t) + \bar{\theta}(t)\|x\|_{L\infty}\right) + \sigma(t) \quad (36)$$

And the predictor model for the controller can be written as presented in equation (37).

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + B\left(\hat{\omega} u_1(t) + \hat{\bar{\theta}}(t)\|x\|_{L\infty} + \hat{\sigma}(t)\right) \quad (37)$$

$$\hat{y}(t) = C^T \hat{x}(t) \quad (38)$$

4. LMI-based Optimization of Fault-Tolerant $\mathcal{L}_1$ Adaptive Controller

Figure 3 presents the block diagram of the $\mathcal{L}_1$ adaptive control structure, which includes the inner loop stabilizing controller, and the outer loop robust controller. The low pass filter and the controller gain parameters are the static parameters tuned using the LMI-based parameter tuning, while the adaptive control law parameters are tuned using the concurrent LMI-based parameter tuning.

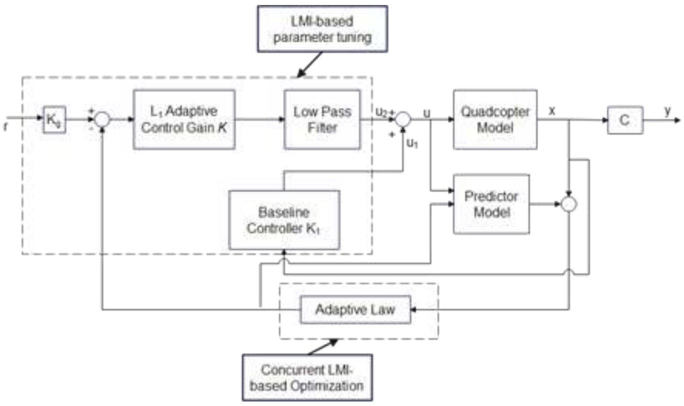

Figure 3: LMI-based tuning of $\mathcal{L}_1$ adaptive controller

The objective of the optimization problem is to minimize the tracking errors between the desired trajectory and the actual outputs of the quadcopter system. It is also aimed at determining the optimal feedback gain parameters of the $\mathcal{L}_1$ adaptive controller while improving the robustness and tracking capability of the quadcopter in the event of the loss of effectiveness of the actuators. Furthermore, the objective is also to minimize the control signal to prevent aggressive responses that can lead to instability of the system in the event of the loss of effectiveness. The optimization problem is a convex problem with LMI constraints, seeking the optimal values of the controller parameters and fault accommodation thresholds that satisfy the objectives and constraints.

The candidate solution vector (CSV) for the LMI-based optimization is formed as presented in equation (39)

$$Z = \left[k \,|\, k_g \,|\, \Gamma_c \,|\, \hat{\omega} \,|\, \hat{\theta} \,|\, \hat{\sigma} \right] \quad (39)$$

where $k, k_g, \Gamma_c, \hat{\omega}, \hat{\theta}, \hat{\sigma}$ are the pre-filter gain, feed-forward gain, adaptation gain, unknown constant, time-varying uncertainties and time varying disturbances respectively.

The objection function based on the performance of the quadcopter system is expressed in equation (40). (Mousakazemi, 2021) showed that the ITAE is the most suitable for computing the performance index.

$$_{\min} J = \int_0^{\infty} \left(e(t) \right)^2 dt \quad (40)$$

where $e(t) = r(t) - y(t)$ is the error between the reference inputs and the outputs of the system, the objective function represents the cumulative squared tracking error over the time to infinity (Hashim et al., 2015, 2017).

The goal of the optimization problem formulation is to minimize the upper bounds of the estimation error of the $\mathcal{L}_1$ adaptive control parameters subject to the constraints stated below;

1. Constraints of the $\mathcal{L}_1$ adaptive control law, which is designed following the $\mathcal{L}_1$ – norm conditions, to ensure fast adaptation and robustness against disturbances, unmodelled dynamics and time varying uncertainties. For the $\mathcal{L}_1$ adaptive control, it is to be ensured that the adaptive law $\hat{\theta}(t)$ is bounded, and the overall system satisfies the $\mathcal{L}_1$-norm bounded adaptive gain given in equation (41)

$$\left\| \hat{\theta}(t) \right\|_{l1} \leq \mu \quad (41)$$

where μ is the adaptation gain bound.

This can be approximated using a dissipation inequality (Yin *et al.* 2020), and expressed in equation (42).

$$\int_0^{\infty} \hat{\theta}(t)^T Q \hat{\theta}(t) dt \leq \mu \quad (42)$$

This is expressed as an LMI as presented in equation (43).

$$Q > 0$$
$$\begin{bmatrix} -A^T P - PA & PB \\ B^T P & -\mu I \end{bmatrix} < 0 \quad (43)$$

2. The actuator saturation which constrains the control signal $u(t)$ by ensuring that the it is bounded by the maximum specified value, this is expressed in equation (44).

$$\| u(t) \| \leq u_{\max} \quad (44)$$

This can be enforced and expressed as an LMI using an auxiliary matrix W as expressed in equation (45).

$$\begin{bmatrix} -W & B \\ B^T & -u_{\max}^2 I \end{bmatrix} \leq 0 \quad (45)$$

3. The stability constraint is derived using the Lyapunov stability. Consider the system given by equation (46),

$$\dot{x} = Ax \quad (46)$$

where *A* is the system matrix and *x* are the system states. Consider the Lyapunov function expressed in equation (47),

$$V(x) = x^T P x \quad (47)$$

$V(x)$, which is real, continuous and has continuous first partial derivatives with $V(x) > 0$ for $x \neq 0$, then the derivative of $V(x)$ along its trajectories should satisfy the expression in equation

$$\dot{V}(x) = (Ax)^T Px + x^T PAx = x^T A^T Px + x^T PAx = x^T (A^T P + PA) x \quad (48)$$

This can be expressed as presented in equation (49)

$$\dot{V}(x) = x^T Q x \quad (49)$$

where $-Q=(A^TP+PA)$.

This can be expressed as an LMI as presented in equation (50).

$$P>0$$
$$Q>0$$
$$\begin{bmatrix} A^TP+PA+Q & PB \\ B^TP & -\gamma I \end{bmatrix} \le 0 \quad (50)$$

Where γ is the stability bound parameter to trade-off between performance and robustness.

4. The low-pass filter is the key component of the $\mathcal{L}_1$ adaptive controller as it separates the performance of the controller from its robustness by limiting the bandwidth of the control signal. Hence, the filter is defined by equation (51).

$$\|C(s)\|_{L1} \le \kappa \quad (51)$$

This can be formulated as an LMI as expressed in equation (52). Note that the detailed LMI design for the filter can be found in (Hovakimyan, 2013).

$$\begin{bmatrix} -P & C^T \\ C & -\kappa^2 I \end{bmatrix} \le 0 \quad (52)$$

5. Results and Discussions

The $\mathcal{L}_1$ adaptive controller was designed to provide adaptation to failure and modelling uncertainties of the quadcopter system (Xu et al., 2016). The responses of the quadcopter under 10% loss of effectiveness in motor 2 were simulated, and the results were compared with that of the nominally designed $\mathcal{L}_1$ adaptive controller.

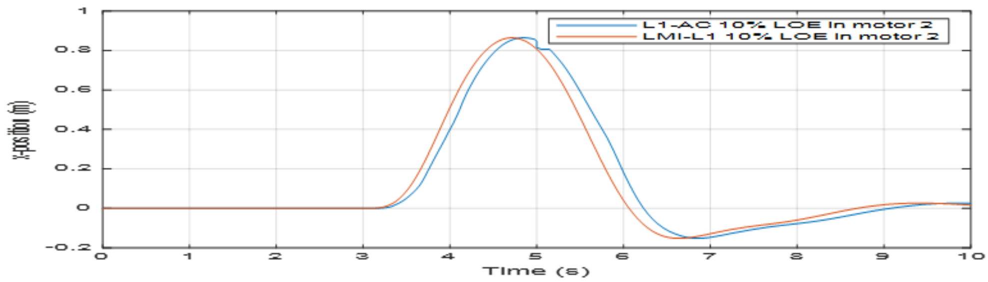

Figure 4: x-position Response to 10% LOE in M-2

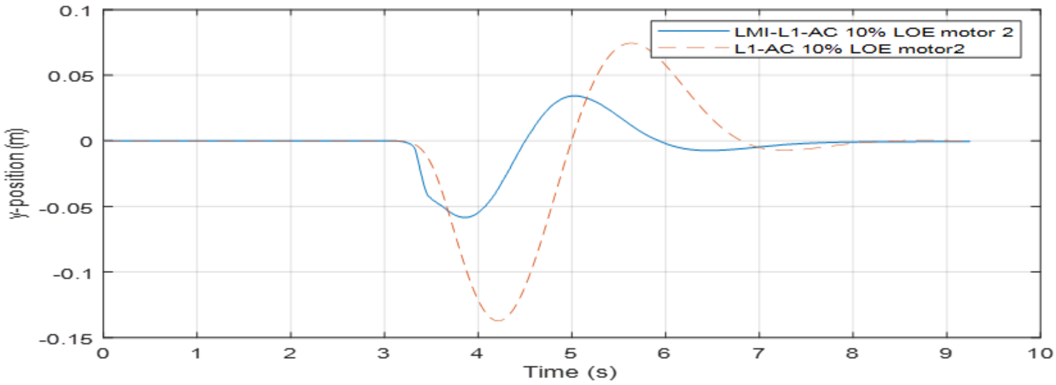

Figure 5: y-position Response to 10% LOE in M-2

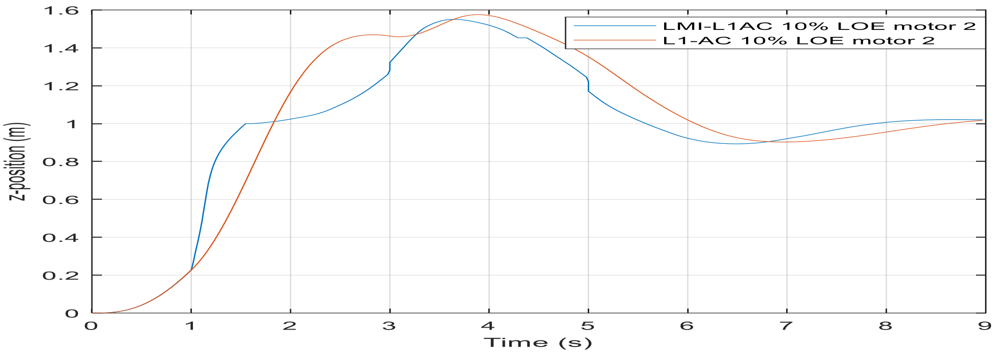

Figure 6: z-position Response to 10% LOE in M-2

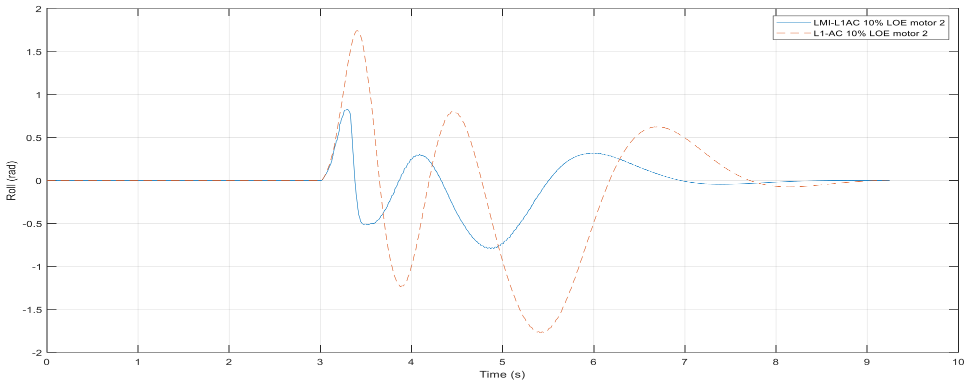

Figure 7: Roll Response to 10% LOE in M-2

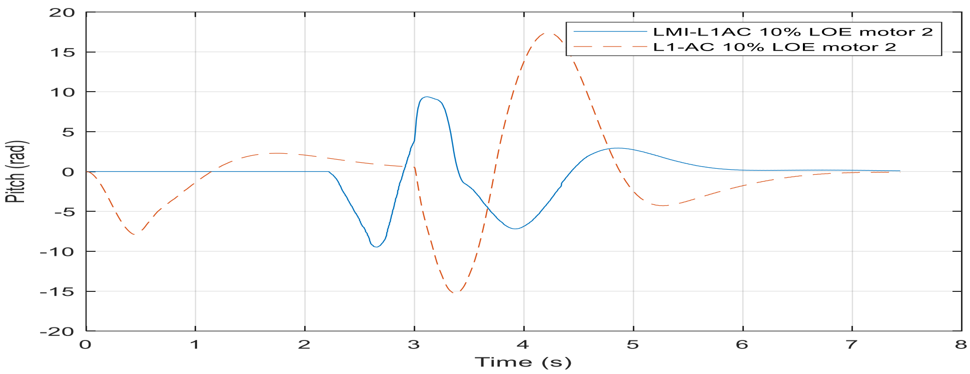

Figure 8: Pitch Response to 10% LOE in M-2

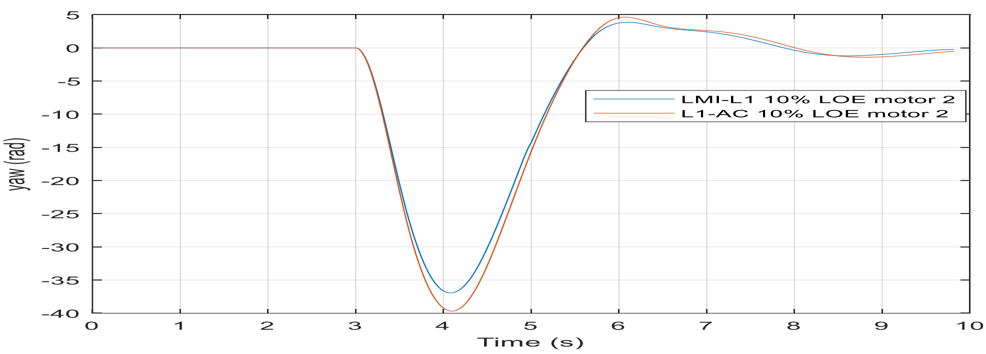

Figure 9: Yaw Response to 10% LOE in M-2

It can be seen from figure 4 – 6 that the position response of the LMI-optimized controllers when a fault is introduced at 3s, the quadcopter hovers at a steady state until the introduction of the fault, in which case, the displacement is observed in the positions. However, the LMI-optimized controller provides a better response in terms of the tracking error and the robustness to the loss of effectiveness of the actuators. Furthermore, the performance in terms of the rise time, settling time and overshoot is improved in all cases. The actual trajectories of the *x* and *y* positions converge quickly to the desired positions even in the presence of actuator faults in one of the motors of up to 10% loss.

The attitude response of the quadcopter to actuator fault of 10% loss of effectiveness in motor 2 is presented in figures 7 – 9. It can be seen that the percentage overshoot in the roll response is reduced by 23%, while the settling time remains the same. This is as a result of the reduction in control effort due to the LMI-based optimization of the control parameters of the controller. Furthermore, the percentage overshoot of the pitch response is reduced by 52%, but the settling time remains the same. The yaw response shows a reduction in the undershoot by 7.4%. The stability of the quadcopter system is maintained in all cases even with the occurrence of a fault in one of the actuators.

The LMI-based optimization of the $\mathcal{L}_1$-adaptive controller improves efficiency of the control, and reduces energy consumption, however, solving LMIs can be computationally intensive, especially for multivariable, MIMO systems such as the quadcopter. This can lead to longer solution times and higher resource consumption. Furthermore, LMI-based optimization can be sometimes conservative due to the relaxation of non-linear constraints, which means that sometimes the solutions provided may not be the most optimum in terms of performance.

6. Conclusions

The quadcopter system, being a multivariable, multi-input-multi-output system with open loop instability requires a robust control system in the event of loss of effectiveness (LOE) of its actuators. $\boldsymbol{\mathcal{L}_1}$ adaptive controller design based on LMI optimization techniques provide an optimal control structure in terms of performance and robustness for the quadcopter system. The performance of the quadcopter system attitude and position responses can be improved by up to 20% in terms of the overshoot, and the system can be able to withstand up to 10% loss of effectiveness in any of the actuators. Further research will look at the performance of the controller due to sensor faults and estimation errors of the system dynamics. It will also consider the performance analysis of the controller when implemented on a physical quadcopter system.

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References

Ahmed, F., Zafar, M. N., Mohanta, J. C., Gasparyan, O., Darbinyan, H., Guerreiro, B. J., Silvestre, C., Cunha, R., Cao, C., Hovakimyan, N., Toolbox, C., Zulu, A., John, S., Mo, O., Mobayen, S., Bejarano, G., Rodr, D., Bejarano, G., Rodr, D., … Karimi, H. R. (2009). L1 adaptive control for autonomous rotorcraft. *Proceedings of the American Control Conference*, *43*(1), 3250–3255. https://doi.org/10.1109/ACC.2009.5159940

Beikzadeh, H., & Liu, G. (2018). Trajectory tracking of quadrotor flying manipulators using L1 adaptive control. *Journal of the Franklin Institute*, *355*(14), 6239–6261. https://doi.org/10.1016/j.jfranklin.2018.06.011

Chnib, E., Bagnerini, P., & Zemouche, A. (2023). LMI based H∞ Observer Design for a Quadcopter Model Operating in an Adaptive Vertical Farm. *IFAC-PapersOnLine*, *56*(2), 10837–10842. https://doi.org/10.1016/j.ifacol.2023.10.757

Dhahri, S., & Naifar, O. (2023). Fault-Tolerant Tracking Control for Linear Parameter-Varying Systems under Actuator and Sensor Faults. *Mathematics*, *11*(23), 1–30. https://doi.org/10.3390/math11234738

Fernández, R. A. S., Dominguez, S., & Campoy, P. (2017). L1 adaptive control for Wind gust rejection in quad-rotor UAV wind turbine inspection. *2017 International Conference on Unmanned Aircraft Systems, ICUAS 2017*, 1840–1849. https://doi.org/10.1109/ICUAS.2017.7991485

Gasparyan, O., & Darbinyan, H. (2019). L1 Adaptive Control of Quadcopters. *12th International Conference on Computer Science and Information Technologies, CSIT 2019*, *1*, 96–100. https://doi.org/10.1109/CSITechnol.2019.8895217

Hashim, H. A., El-Ferik, S., & Abido, M. A. (2015). A fuzzy logic feedback filter design tuned with PSO for L1 adaptive controller. *Expert Systems with Applications*, *42*(23), 9077–9085. https://doi.org/10.1016/j.eswa.2015.08.026

Hashim, H. A., El-Ferik, S., Ayinde, B. O., & Abido, M. A. (2017). *Optimal Tuning of Fuzzy Feedback filter for L1 Adaptive Controller Using Multi-Objective Particle Swarm Optimization for Uncertain Nonlinear MIMO Systems*. http://arxiv.org/abs/1710.05423

Hovakimyan, K. K. K. N. (2013). Multi-Criteria Optimization for Filter Design of L1 Adaptive Control. *Journal of Optimization Theory and Applications*. https://doi.org/10.1007/s10957-013-0412-7

Hovakimyan, N., & Cao, C. (2010). L1 Adaptive Control Theory: Guaranteed Robustness with Fast Adaptation. In *Экономика Региона* (First). Society for Industrial and Applied Mathematics.

Jafarnejadsani, H., Sun, D., Lee, H., & Hovakimyan, N. (2017). Optimized L1 adaptive controller for trajectory tracking of an indoor quadrotor. *Journal of Guidance, Control, and Dynamics*, *40*(6), 1415–1427. https://doi.org/10.2514/1.G000566

Khattab, A., Mizrak, I., & Alwi, H. (2024). Fault tolerant control of an octorotor UAV using sliding mode for applications in challenging environments. *Annual Reviews in Control*, *57*(March). https://doi.org/10.1016/j.arcontrol.2024.100952

Kim, K. K. K., & Hovakimyan, N. (2014). Multi-Criteria Optimization for Filter Design of L1 Adaptive Control. *Journal of Optimization Theory and Applications*, *161*(2), 557–581. https://doi.org/10.1007/s10957-013-0412-7

Maiti, R., Sharma, K. Das, & Sarkar, G. (2022). *Hybrid L1 Adaptive Control. Applications of Fuzzy Modeling, Stochastic Optimization and Metaheuristics* (Volume 422). Springer. https://doi.org/https://doi.org/10.1007/978-3-030-97102-1

Mao, J., Yeom, J., Nair, S., & Loianno, G. (2024). From Propeller Damage Estimation and Adaptation to Fault Tolerant Control: Enhancing Quadrotor Resilience. *IEEE Robotics and Automation Letters*, *9*(5), 4297–4304. https://doi.org/10.1109/LRA.2024.3380923

Michini, B., & How, J. P. (2009). $\mathscr{L}$1 adaptive control for indoor autonomous vehicles: Design process and flight testing. *AIAA Guidance, Navigation, and Control Conference and Exhibit*, *August*, 1–15.

Mousakazemi, S. M. H. (2021). Comparison of the error-integral performance indexes in a GA-tuned PID controlling system of a PWR-type nuclear reactor point-kinetics model. *Progress in Nuclear Energy*, *132*(December 2020), 103604. https://doi.org/10.1016/j.pnucene.2020.103604

Nguyen, N. P., & Hong, S. K. (2018). Sliding mode Thau observer for actuator fault diagnosis of quadcopter UAVs. *Applied Sciences (Switzerland)*, *8*(10). https://doi.org/10.3390/app8101893

Nguyen, N. P., Mung, N. X., Thanh Ha, L. N. N., Huynh, T. T., & Hong, S. K. (2020). Finite-Time Attitude Fault Tolerant Control of Quadcopter System via Neural Networks. *Mathematics*, *8*(9), 1541. https://doi.org/10.3390/math8091541

Souanef, T., & Fichter, W. (2015). Fault Tolerant L1 Adaptive Control based on Degraded Models. *Advances in Aerospace Guidance, Navigation and Control*. https://doi.org/10.1007/978-3-319-17518-8

Souanef, T., Whidborne, J., & Boubakir, A. (2023). $\mathscr{L}$1 Adaptive Control of Quadrotor UAVs in Case of Inversion of the Torque Direction. *2023 IEEE International Conference on Control and Automation (ICCA) Edinburgh, UK*. https://doi.org/https://doi.org/10.35534/dav.2023.10004

Thu, K. M., & Gavrilov, A. I. (2017). Designing and Modeling of Quadcopter Control System Using L1 Adaptive Control. *Procedia Computer Science*, *103*(October 2016), 528–535. https://doi.org/10.1016/j.procs.2017.01.046

Wu, Z., Cheng, S., Zhao, P., Gahlawat, A., Ackerman, K. A., Lakshmanan, A., Yang, C., Yu, J., & Hovakimyan, N. (2023). *$\mathcal{L}_1$Quad: $\mathcal{L}_1$ Adaptive Augmentation of Geometric Control for Agile Quadrotors with Performance Guarantees*. https://arxiv.org/abs/2302.07208v1

Xu, D., Whidborne, J. F., & Cooke, A. (2016). Fault tolerant control of a quadrotor using C 1 adaptive control. *International Journal of Intelligent Unmanned Systems*, *4*(1), 43–66. https://doi.org/10.1108/IJIUS-08-2015-0011

Yin, H., Packard, A., Arcak, M., & Seiler, P. (2020). Reachability analysis using dissipation inequalities for uncertain nonlinear systems. *Systems & Control Letters*, *142*, 104736. https://doi.org/10.1016/j.sysconle.2020.104736

Zhou, Y., Liu, H., & Guo, H. (2024). L1 Adaptive Fault-Tolerant Control for Nonlinear Systems Subject to Input Constraint and Multiple Faults. *Actuators*, *13*(7), 1–21. https://doi.org/10.3390/act13070258

Van der Geer, J., Hanraads, J. A. J., & Lupton, R. A. (2000). The art of writing a scientific article.*Journal of Science Communication, 163*, 51–59.

Strunk, W., Jr., & White, E. B. (1979).*The elements of style* (3rd ed.). New York: MacMillan.

Mettam, G. R., & Adams, L. B. (1999).How to prepare an electronic version of your article. In B. S. Jones & R. Z. Smith (Eds.), *Introduction to the electronic age* (pp. 281–304). New York: E-Publishing Inc.

Fachinger, J., den Exter, M., Grambow, B., Holgerson, S., Landesmann, C., Titov, M., et al. (2004).Behavior of spent HTR fuel elements in aquatic phases of repository host rock formations, 2nd International Topical Meeting on High Temperature Reactor Technology. Beijing, China, paper #B08.

Fachinger, J. (2006). Behavior of HTR fuel elements in aquatic phases of repository host rock formations.*Nuclear Engineering & Design*,*236*, 54.