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# Reduction of Additive White Gaussian Noise from Computed Tomography Images using Adaptive Wavelet Thresholding

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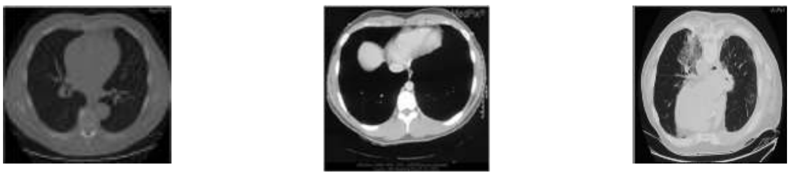
**ABSTRACT:**

Computed Tomography (CT) is one of the most important imaging technique used in medicine for the diagnosis of internal abnormalities. Due to some phenomenon; noise signals are introduced which are modelled as additive White Gaussian Noise (AWGN). Introduction of noise deteriorates the quality of images by suppressing the anatomical information. This information is useful for correct interpretation of CT images. Hence it required to remove Gaussian noise from CT images. In this research paper, reduction of white Gaussian noise from Computed tomography is done. For this purpose; adaptive wavelet thresholding technique is used. Bivariate thresholding is example of soft thresholding technique and its performance is better than other thresholding techniques. To evaluate the performance of de-noising technique three most important parameters have been calculated viz. Power Signal to Noise Ratio (PSNR), Signal to Noise Ratio (SNR) and Structural Similarity Index Measure (SSIM).

***Keywords:*** AWGN, CT image, Image De-noising, PSNR, SNR, SSIM.

## Introduction:

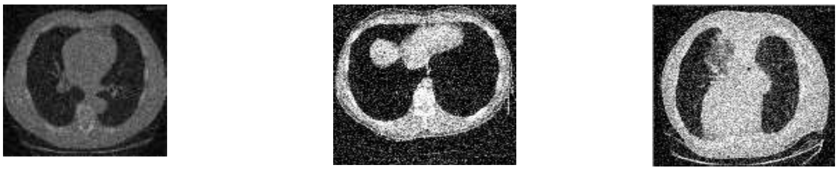
Computed tomography is one of the most important medical images[1] which is used to diagnose any abnormality or disease in the internal body [2,3]. To take CT image; patient must take a radioactive liquid which is basically a dose of Barium Sulphate [4,5]. Its small amount is give orally because it may be harmful for the body. It is spread in the patient’s body and when X-ray radiation is imposed; it helps to create clear image of the internal organs. This type of CT images is called Low Dose CT (LDCT) images [6-8]. Due to low dose the quality of CT images is degraded as a result various types of noise signals are introduced which are modelled as AWGN[9-10]. It is additive in nature; i. e. noise signals are direct added to the pixel values of the original images. It affects almost all the frequency spectrum hence referred as white noise [11]. Its Probability Density Function (PDF) is Gaussian and bell shaped. Introduction of noise deteriorates the quality of CT images; as a result image information is affected; due to which false interpretation may be done by the radiologists [12-13]. Hence it is essential to remove AWGN from LDCT images [14-15]. However; it is almost impossible to remove noise from any images completely but it should be reduce to such an extent so that diagnosis purpose must be done properly[16]. Various techniques have been proposed to reduce AWGN from images; in which Wavelet based techniques give better results among many traditional techniques [17]. Fig. 1.1 illustrates some noise free CT images taken from the public database.



(a) (b) (c)

Fig. 1.1 CT images

If some amount of AWGN is introduced in these images than their quality are deteriorate. Noisy images are shown in fig. 1.2



(a) (b) (c)

Fig. 1.2 Noisy CT images affected by AWGN

## Methodology:

In this section; CT image de-noising using DTCWT is explained. Discrete wavelet transform (DWT) is one of the best tool for image processing. However DWT suffers from two major drawbacks viz. directionality and shift variance. These two problems can be overcome in DTCWT[12,17]. In this work; it is used for image de-noising. Image de-noising can be done simply by applying three simple steps:

- In the first stage; DTCWT of the image is calculated which is affected by AWGN which gives signal coefficients and noisy coefficients.
- In the second stage; noisy wavelet coefficients are modified by applying adaptive thresholding. In this work bivariate thresholding is used.
- In third stage; inverse wavelet transform i.e. IDTCWT is calculated. As a result; de-noised image is obtained.

Fig. 1.3 illustrates simplified block diagram of DTCWT based image de-noising method.

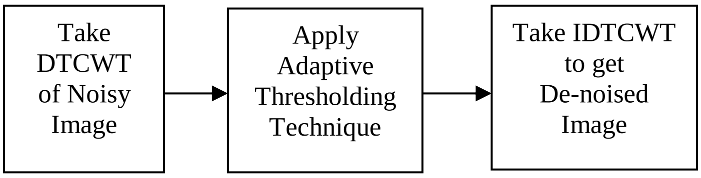



Fig. 1.3 DTCWT based image de-noising method

Below is the description of bivariate thresholding proposed by Sendur and Selesnick. It is an adaptive thresholding which exploits the statistical dependencies among wavelet coefficients[18]. Let; a CT image is taken from the database; represented by $x(i, j)$. If noise $n(i, j)$ is introduced in it then noisy image is represented by $y(i, j)$. Since AWGN is additive in nature then; mathematically,

$$y(i, j) = x(i, j) + n(i, j) \qquad (3.1)$$

If wavelet transform of noisy image is taken then it contains both signal and noisy coefficients. Mathematically;

$$W=X+N \qquad (3.2)$$

In the wavelet domain; noise problem may be considered as:

$$y_{1k} = w_{1k} + n_{1k} \qquad (3.3)$$

$$y_{1k} = w_{1k} + n_{1k} \qquad (3.4)$$

By taking into account the statistical dependency between the coefficients and its parents. $y_{1k}$ & $y_{2k}$ are the noisy observations of $w_{1k}$ & $y_{2k}$ and $n_{1k}$ & $y_{2k}$ are noisy sample. In general; it can be written as:

$$y_k= w_k +n_k \qquad (3.5)$$

$k=1$ ...number of wavelet coefficients

where $w_k=(w_{1k}, w_{2k})$, $y_k =(y_{1k}, y_{2k})$, $nk=(n_{1k}, n_{2k})$

The standard MAP estimator for $w$ given the corrupted observation $y$ is:

$$\hat{W}(y) = \underset{w}{argmax}\ p_{w|y}(w|y) \qquad (3.6)$$

With some manipulations eq. (3.6) can be written as:

$$\hat{W}(y) = \underset{w}{argmax}\ p_n(y-w).p_w(w) \qquad (3.7)$$

For the coefficients and its parents proposed a non-Gaussian bivariate PDF given by:

$$p_w(w) = \frac{3}{2x\sigma^2}\, exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{w_1^2 + w_2^2}\right) \qquad (3.8)$$

The marginal variance $\sigma^2$ depends on coefficient index $k$ .

Using above two equations, MAP estimator of $w_1$ can be given as:

$$\hat{W}_1 = \frac{\left(\sqrt{y_1^2 + y_2^2} - \sqrt{3}\,\frac{\sigma_n^2}{\sigma}\right)_+}{\sqrt{y_1^2 + y_2^2}} \,.\, y_1 \qquad (3.9)$$

Bivariate thresholding function. $(g)_+$ is defined as:

$$(g)_+ = \begin{cases} 0 \; for \; g < 0 \\ g \; otherwise \end{cases} \qquad (3.10)$$

This estimator requires prior knowledge of $\sigma_n^2$ and marginal variance $\sigma^2$ for each coefficient. $\sigma^2$ can be estimated using neighbouring coefficients while $\sigma_n^2$ can be estimated from noisy coefficients using robust median estimator[9].

$$\sigma_n^2 = \frac{median(|y_i)}{0.6745}\; y_i \; \in subband \; HH \qquad (3.11)$$

Fig. 3.1 illustrates the flow diagram of CT image de-noising.

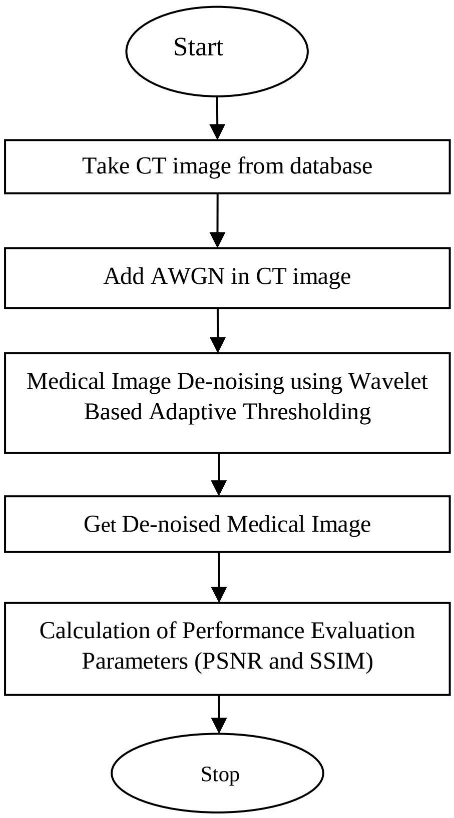



Fig. 3.1 flow diagram of CT image de-noising

In this technique; first of all CT images are taken from database. This image is noise free. Now some amount of AWGN is added in the noise free image; as a result noisy image is obtained. Now Wavelet based adaptive thresholding technique is applied. Dual Tree Complex Wavelet Transform (DTCWT) performs better than the conventional Discrete Wavelet Transform (DWT) hence it is used to take wavelet transform of noisy image[17]. After taking DTCWT of noisy image; adaptive thresholding technique is applied on the noisy images. In this project bivariate transform is used to remove noisy coefficients from the image. Now inverse DTCWT is taken to get de-noised image. To evaluate the performance of proposed technique; some parameters are calculated as discussed in result section.

**Results & Discussion:** This section contains the results of proposed technique. Results are illustrated in the form of images, calculated parameters values and graphs. Three important parameters have been calculated viz. PSNRand SSIM. Fig 4(a) to 6(a) are noise free images, fig. 4(b) to 6(b) are noisy images and 4(c) to 6(c) are de-noised images of CT image 1, 2 and 3 respectively.

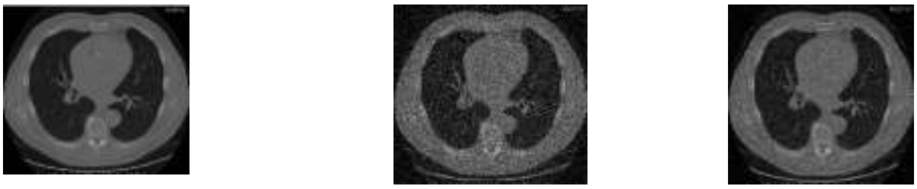


Fig. 4.1 (a) Test image CT 1 (b) noisy image with $\sigma^2_{AWGN}$=0.01 (c) de-noised image

Fig. 4.2 (a) Test image CT 1 (b) noisy image with $\sigma^2_{AWGN}$=0.03 (c) de-noised image

Fig. 4.3 (a) Test image CT 1 (b) noisy image with $\sigma^2_{AWGN}$=0.05 (c) de-noised image

Table 4.1 PSNR values for CT test images for different values of AWGN

| **Test Image** | **PSNR** | **AWGN Noise Variance ($\sigma^2_{AWGN}$)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **CT 1** | Noisy PSNR | 23.21 | 20.45 | 18.82 | 17.29 | 15.86 | 14.86 | 13.64 | 13.16 | 12.72 |
| | **Denoised PSNR** | **34.09** | **33.43** | **33.12** | **32.82** | **32.25** | **31.58** | **30.28** | **29.64** | **28.67** |
| **CT 2** | Noisy PSNR | 22.73 | 20.57 | 18.67 | 16.78 | 15.95 | 14.95 | 13.74 | 13.18 | 12.74 |
| | **Denoised PSNR** | **34.64** | **34.15** | **33.75** | **33.51** | **32.77** | **31.54** | **30.86** | **29.57** | **27.89** |
| **CT 3** | Noisy PSNR | 23.14 | 21.16 | 18.67 | 16.89 | 15.91 | 14.79 | 13.87 | 13.25 | 12.85 |
| | **Denoised PSNR** | **34.49** | **34.24** | **33.47** | **33.08** | **32.25** | **31.21** | **30.59** | **29.12** | **27.83** |

Table 4.2 SSIM values for CT test images for different values of AWGN

| **Test Image** | **SSIM** | **AWGN Noise Variance ($\sigma^2_{AWGN}$)** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | **0.01** | **0.02** | **0.03** | **0.04** | **0.05** | **0.06** | **0.07** | **0.08** | **0.09** |
| **CT 1** | Noisy SSIM | 0.67 | 0.52 | 0.45 | 0.42 | 0.36 | 0.35 | 0.31 | 0.33 | 0.28 |
| | **De-noised SSIM** | **0.91** | **0.89** | **0.87** | **0.83** | **0.82** | **0.78** | **0.76** | **0.76** | **0.74** |
| **CT 2** | Noisy SSIM | 0.66 | 0.60 | 0.35 | 0.35 | 0.33 | 0.32 | 0.32 | 0.25 | 0.25 |
| | **De-noised SSIM** | **0.92** | **0.87** | **0.82** | **0.83** | **0.81** | **0.77** | **0.77** | **0.75** | **0.72** |
| **CT 3** | Noisy SSIM | 0.62 | 0.43 | 0.35 | 0.32 | 0.28 | 0.22 | 0.23 | 0.24 | 0.20 |
| | **De-noised SSIM** | **0.91** | **0.81** | **0.84** | **0.82** | **0.79** | **0.74** | **0.73** | **0.74** | **0.59** |

Plots between noise variance and PSNR and SSIM are plotted for CT 1 image which are illustrated in fig. 4 and 6 respectively.

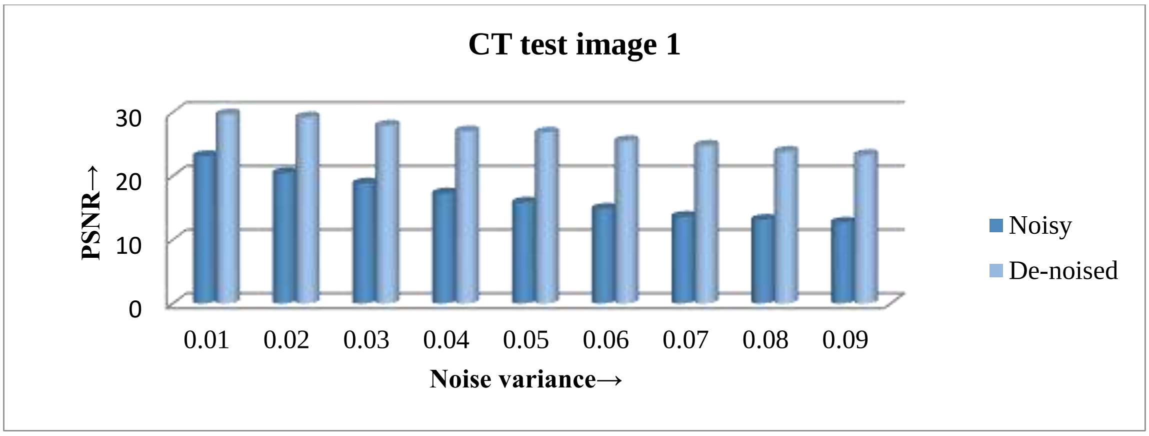



Fig 4.4 PSNR values for noisy and de-noised test image CT 1

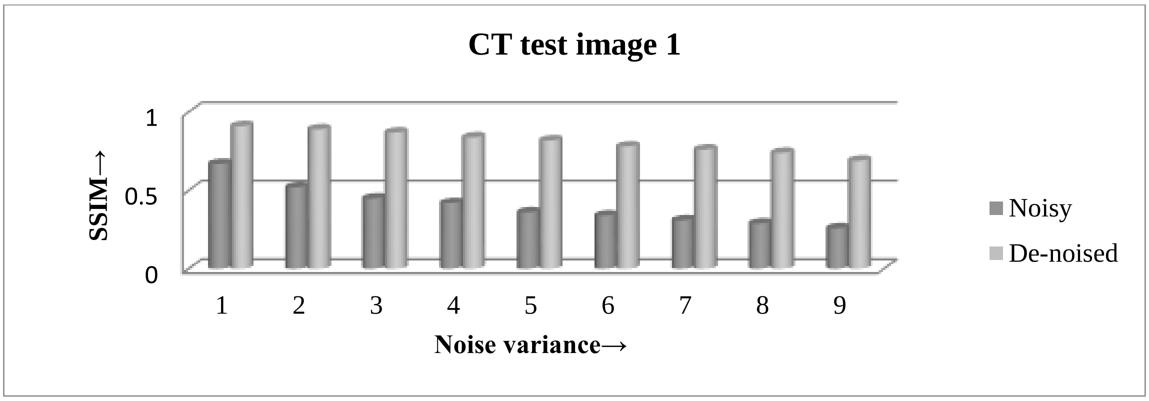



Fig. 4.5 SSIM values for noisy and de-noised test image CT 1

From these two graphs it is clear that Adaptive wavelet technique gives higher PSNR and SSIM values. This technique reduces not only effect of AWGN but retains image information also.

## Conclusion:

From the above discussion and performance evaluation parameters it is clear that; adaptive wavelet thresholding technique suppresses AWGN from CT images. It also retains image information which is very important feature to diagnose diseases or internal body structures. CT images are very useful in medicine for the study of internal organs. Introduction of noise may affect the diagnosis process done by the radiologists. Any misinterpretation may mislead the doctors and further treatments. This technique can be used to de-noise other medical images also but it is important to study about those imaging systems and noise introduced in those images; then only image de-noising may be done by the proposed technique.

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