WWW.IJRPR.COM

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Learning Fractions through Visual Imageries across Learner Levels: Impact to the Proposed Strategic Numeracy Program

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ABSTRACT –

This experimental study investigated the effectiveness of the Visual Imageries Strategy as an intervention teaching strategy in improving the mathematical skills of grade four learner in Mathematics in terms of conceptual understanding; strategies and reasoning; computation and execution; and communication. One section of 25 learners participated in this study, the group that is not use to have knowledge about Visual Imageries. Data were collected through a researcher made a 25 items pre-test and post-test based on most essential learning competencies. While frequency and percentage were used to analyze the pre-test and post-test results, mean and standard deviation were use to analyze the respondents' perception of Visual Imageries.

The group was insignificant prior to implementing the strategies, and once the technique was introduced to them, the group became significant. The results of the pre-test and post-test showed a significant difference. The development of Strategic Numeracy Program in Mathematics 4 and to develop specific grade 4 skills, including recognizing, understanding, writing, comparing and problem-solving skills is facilitated using visual imageries teaching strategy. These findings suggest that this strategy can be effective intervention strategy for improving the academic performance of grade 4 learners in fractions. The study recommends the wider implementation of Visual Imageries based on its positive impact of the learners' academic achievement.

Keywords – visual imageries strategy, mathematical skills, strategic numeracy program

Introduction

Students understand mathematical ideas by making connections between language, symbols, pictures and real-life situations (Linatoc, 2016). Mathematics seeks out patterns and formulates new conjectures. Mathematicians resolve the truth or falsity of conjectures by mathematical proof. Mathematics is often thought of as a subject that a student either understands or doesn't with little in between. Mathematics encompasses a wide variety of skills and concepts.

Fractions are parts of daily life. For example, when cooking, ingredients will be divided into portions. In financial transactions, the discount or interest rate will require understanding in terms of fraction. And in time management, hours are divided to smaller units. All applications depend on the ability to interpret and manipulate parts of a whole. For example, Siegler and Lortie-Forgues (2015) note that knowing fractions is essential for tasks involving estimation, measurement, and reasoning about quantities.

Fractions are one of the most significant parts of elementary mathematics but very essential in understanding complex subjects such as ratios, proportions, and algebra. However, it is one of the hardest things to learn for students all over the world. Recent studies show that this struggle continues because many students

score below proficiency in fraction-related tasks, even in international mathematics assessments (Siegler & Lortie-Forgues, 2015; Namkung et al., 2018). These results of the Department of Education national assessments in the Philippines rank fractions as one of the worst-understood topics in the elementary school curriculum. It cannot even learn the very central ideas of fraction equivalence and magnitude (Bautista et al., 2023). In 5 years of teaching and being a Mathematics Coordinator among observed results of different assessments in Mathematics like Rapid Assessment in Mathematics (RMA) and quarterly examination, Grade 4 pupils in the locale do not meet the passing rate with fraction relevant concepts being the least mastered in most cases.

Objectives of the Study

This study aimed to explore the effects of a teaching strategy on visual imagery for Grade 4 students in the Philippines who show varying levels of learning in dispelling misconceptions about fractions. Evidence-based recommendations are hope that possible improvements may be made towards mathematics education in the Philippines within not-so-distant future.

Methodology

The researcher used a quasi-experimental design as a framework for conducting the study. Like a true experiment, a quasi-experimental design aims to establish a cause-and-effect relationship between an independent and dependent variable. However, unlike a true experiment, a quasi-experiment does not rely on random assignments. Instead, subjects are assigned to groups based on non-random criteria. The pre-test and post-test were used in gathering information about student's performance before and after the lesson discuss by the researcher.

Quasi-experimental design with pre-test and post-test non-equivalent groups framework is utilized in the study for determining the effectiveness of utilizing visual imagery in teaching fractions at different learning levels from Grade 4 students, and this design is acceptable to assess changes in performances (for example, difference in scores between pre-tests and post-tests) or comparing groups with different characters that are not through randomization (Creswell & Creswell, 2018).

This research will categorize the respondents according to their learning levels by prior academic performance or results of a diagnostic test. The groups will be given instruction through visual imagery techniques, and the learning results will be measured through a pre-test and post-test. The approach will make it possible to compare the learning results within and between different groups, thus satisfying the specific objectives of this study.

In addition to the statistical analysis, the effect size of the difference between pre-tests and post-tests as well as at various levels of learning will be calculated in terms of the Cohen's d. It is a measure for the practical significance of an intervention (Fazio & Siegler, 2014).

These statistical methods shall provide a sound framework for the analysis of the efficacies of visual imagery in learning fractions at different levels of the participants' learning difference in the computed t-value was tested at 0.05 levels of significance.

Results and Discussion

Table 1 ***Distribution of respondents based on learning levels.***

Learning Level	***Frequency***	***Percentage (%)***
Low	*8*	*32*
Average	*9*	*36*
High	*8*	*32*
Total	*25*	*100*

Table 1 presents the distribution of respondents based on their learning levels. Out of 25 respondents, 32% (8 students) are classified under the low learning level with 76-80 Mathematics Grades, indicating they may need additional support and intervention. The majority, comprising 36% (9 students), fall under the average learning level with 81-85 Mathematics Grades, suggesting a moderate understanding of the subject matter. Meanwhile, 32% (8 students) belong to the high learning level with 86 and above Mathematics Grades, demonstrating strong academic performance and proficiency.

The distribution suggests a fairly balance d spread across the three learning levels, with a slight majority in the average category. However, the equal percentages of low and high learners indicate a diverse range of learning capabilities within the group. This variation highlights the importance of differentiated instruction to cater to the unique learning needs of each student. This can ensure the fair assessment of varying skills level among the students (Tomlinson, 2014).

Table 2. The mean pre-test and post-test scores of the respondents based on the mathematical skills assessed.

Mathematical Skills	Pre-Test			Post-test		
	Mean	SD	VI	Mean	SD	VI
Recognizing	3.40	0.71	Proficient	4.76	0.52	Advanced
Understanding	2.00	1.23	Developing	3.84	1.25	Proficient
Writing	2.36	1.15	Developing	3.80	1.19	Proficient
Comparing	1.92	1.32	Emerging	2.88	1.42	Developing
Problem-solving	1.84	1.21	Emerging	2.28	1.43	Developing
Overall	**2.30**	**1.12**	**Developing**	**3.51**	**1.16**	**Proficient**

Legend: 0.00-1.00- No skill- The student is unable to perform the skill; 1.01-2.00-Emerging skill- The student has minimal ability but struggles with application; 2.01-3.00-Developing skill- The student shows partial understanding but makes noticeable errors; 3.01- 4.00-Proficient skill- The student *demonstrates good understanding and applies the skill with some errors; 4.01-5.00-Adnaced The student has strong understanding and applies the skill accurately (DepEd's Assessment Guidelines (DO No. 8, s. 2015).*

Table 2 displays the mean and standard deviation (SD) of students' pre-test and post-test scores across five mathematical skills: Recognizing, Understanding, Writing, Comparing, and Problem-Solving.

Before the intervention, students generally showed low proficiency in all the skills, which was interpreted as "developing." This suggests that while students demonstrated a partial understanding, they also made noticeable errors. Among the skills, Recognizing Fractions appeared to be the area where students performed relatively better, whereas Problem-Solving was identified as the most challenging, particularly in applying mathematical concepts to real-life situations.

Students showed emerging skills in comparing and problem-solving, meaning they possessed only minimal abilities and struggled with real-life application. In understanding and writing, students were at the "developing" level, reflecting partial grasp of fractions but with consistent errors. As for recognizing fractions, students were classified as "proficient," showing a good understanding and ability to apply the concept, although some errors were still evident.

After the intervention, there was a marked improvement in students' mathematical skills. Their performance in Recognizing Fractions significantly increased and was interpreted as "advanced," suggesting a strong understanding and accurate application of the skill. In both Understanding and Writing Fractions, students reached the "proficient" level, indicating that they grasped the concepts well and made only minimal errors in application. Meanwhile, students remained at the "developing" level in Comparing and Problem-Solving. This means they showed partial understanding of the concepts but continued to commit noticeable errors.

Overall, the results indicate that the intervention had a positive impact on students' mathematical skills, with the most significant improvements observed in Recognizing, Understanding, and Writing Fractions. However, further support may be needed to strengthen students' abilities in Comparing and Problem-Solving.

The results suggest that the use of Visual Imagery in learning fractions has a positive impact on students' comprehension as evident to the increase in the mean scores from the pre-test to post-test. This is aligned with research indicating that visual representations enhance students' ability to conceptualize abstract mathematical ideas (Booth & Koedinger, 2012). The increase in scores suggests that visualization techniques foster better retention and engagement in fraction-related problem-solving.

Table 3. ***Summary of the difference among the post-test scores of the Grade 4 pupils across the different learning levels upon the use of Visual Imagery in learning fractions.***

	Pre-Test		**Post-test**				
Mathematical Skills	Mean	SD	Mean	SD	**t-value**	**df**	**p-value**
RECOGNIZING	3.40	0.71	4.76	0.52	-10.66	24	< .001
UNDERSTANDING	2.00	1.23	3.84	1.25	-7.58	24	< .001
WRITING	2.36	1.15	3.80	1.19	-5.31	24	< .001
COMPARING	1.92	1.32	2.88	1.42	-3.59	24	0.001
PROBLEM SOLVING	1.84	1.21	2.28	1.43	-1.22	24	0.235
Overall	2.30	1.12	3.51	1.16	-8.56	24	< .001

Table 3 presents the summary of the paired t-test results comparing students' pre-test and post-test scores. The results show that Recognizing (t-value=-10.66), Understanding (t-value=-7.58), Writing (t-value=-5.31), and Comparing (-3.59) fractions all had significant differences as evident to their computed value being less that 0.001. This confirms the effectiveness of the intervention in these areas. However, Problem-Solving ($t = -1.22$, $p = 0.235$) did not show a statistically significant improvement, indicating that further instructional support may be needed. These support the hypothesis that Visual Imagery significantly enhances students' mathematical skills. Studies have shown that visual representations help bridge the gap between symbolic and conceptual understanding in mathematics (Wright et al., 2019). These findings align with Piaget's theory, which suggests that concrete visual aids support the transition to abstract thinking (Piaget, 1952).

The findings reveal that there were significant improvements in Recognizing, Understanding, Writing, and Comparing Fractions. These results confirm that the intervention was effective in enhancing students' skills in these areas. On the other hand, Problem-Solving did not show a statistically significant improvement, suggesting that additional instructional support may be necessary for this skill.

Overall, the findings support the hypothesis that Visual Imagery plays a significant role in improving students' mathematical abilities. Prior research highlights that visual representations help connect symbolic expressions with conceptual understanding in mathematics (Wright et al., 2019). These results also align with Piaget's theory, which emphasizes that concrete visual aids are essential in helping learners move from concrete experiences to abstract thinking (Piaget, 1952).

Table 4. **Post test results by learning level**

Mathematical Skills	F	df1	df2	p
RECOGNIZING	0.894	2	13.3	0.432
UNDERSTANDING	10.063	2	11.4	0.003
WRITING	4.729	2	13.7	0.027
COMPARING	9.066	2	13.4	0.003
PROBLEM SOLVING	2.258	2	14.3	0.140
OVERALL	11.773	2	13.6	0.001

Table 4 summarizes the results of the one-way ANOVA. Given the values computed, the mathematical skills, specifically understanding, writing, and comparing fractions, show variations across different learning levels. This suggests that students' performance in these areas differs depending on their level of learning. Furthermore, the overall findings indicate that learning levels have a significant influence on students' overall performance in the topic.

On the other hand, no significant differences were found in Recognizing and Problem-Solving across the various learning levels. This implies that students' abilities in these particular skills remained relatively stable regardless of their learning group.

The existence of evidence for the assertion that mathematical competence differentiates according to cognitive load and teaching methods comes from a 2024 study by Ngu and Phan. This work analyzes teaching methods based on the theories of comparison-based learning and cognitive load and hypothesizes that such kinds of teaching methods can effectively differentiate different aspects of mathematical competence. Moreover, Thompson's 2008 study examines mathematics teachers' notions of higher-order thinking within Bloom's Taxonomy, where difficulties are observed in applying the concepts to mark items. This is consistent with the hypothesis that understanding and comparison tasks can lead to higher-order thinking skills, potentially explaining big differences in learning outcomes.

Table 5. **Overall analysis across learning levels.**

		low	average	high
low	Mean difference	—	-0.986	-6.50 ***
	p-value	—	0.788	< .001
average	Mean difference		—	-5.51 **
	p-value		—	0.003
high	Mean difference			—
	p-value			—

Note. * p < .05, ** p < .01, *** p < .001

The overall post-hoc analysis, as shown in Table 9, confirms the earlier findings. A significant difference is observed between high- and low-level learners, indicating that higher-level learners perform notably better. Similarly, a significant difference is found between high and average learners. However, no significant difference is observed between the average and low-level learners, suggesting that both groups may require similar levels of support.

These results support Vygotsky's (1978) Zone of Proximal Development (ZPD), which supports that students with enhanced past knowledge and scaffolding of thinking have enhanced conceptual understanding. In addition, these results agree with Hattie (2009) and his work, where high-scoring students in self-regulated learning skills perform better in subject areas that require conceptual elaboration.

Moreover, students develop mathematical understanding through progressive cognitive structures, with advanced learners benefiting from more established schemas in fraction writing as per the Constructivist Learning Theory (Piaget, 1952).

The findings suggest the need for targeted interventions to support students in the lower learning levels. Differentiated instruction and scaffolding (Bruner, 1966) may help bridge the gap by providing structured guidance and incremental learning support. Additionally, incorporating visual aids and real-world applications of fractions could enhance comprehension among students with lower proficiency.

A quasi-experiment by Odicta (2017) demonstrated that Grade 10 students administered with differentiated instruction exhibited significant improvements in mathematics achievement and critical thinking skills. The study emphasized the necessity for educators to design lessons that cater to diverse learner needs, to bridge gaps in terms of the learning of the students.

Similarly, a study published in Learning and Instruction examined the effects of an interactive eBook designed with scaffolding features on sixth graders learning fractions. The results highlight how low level learners are impacted by the adaptive scaffolds thus, the effectiveness of integrating this technology to support learners who struggle in various mathematical concepts, specifically in the primary levels.

Table 6. **Strategic Numeracy Program**

Goal:

To improve students' numeracy skills across all grade levels by implementing structured programs, providing necessary resources, and fostering a supportive learning environment.

Programs and Activities:

Programs/Objectives	Strategies/Activities	Implementation Date(s)	Source of Funds	Persons Involved	Expected Output
1. Numeracy Assessment and Benchmarking Establish baseline data to identify students' numeracy levels.	- Administer diagnostic tests. - Analyze results to identify learning gaps.	June 15–19, 2025	School MOOE	Math Teachers' Assessment Coordinator	Diagnostic Report, Identified Intervention Areas
2. Numeracy Enhancement Program Provide targeted interventions for students needing support.	- Implement remedial classes. - Utilize peer tutoring sessions.	July 1–August 31, 2025	School MOOE, Donations	Math Teachers, Trained Peer Tutors	Improved Assessment Scores, Progress Reports
3. Math Club Activities Promote interest and excellence in mathematics.	- Organize math quizzes and competitions. - Conduct math-related workshops.	Monthly (Every 3rd Friday)	Club Funds, Sponsorships	Math Club Adviser, Student Officers	Event Reports, Increased Student Participation
4. Parent Engagement Sessions Involve parents in supporting numeracy	- Conduct workshops on assisting with homework. - Share strategies for	Quarterly (Starting July 2025)	PTA Funds	Teachers, Parent Leaders	Attendance Records, Feedback Forms

Conclusion

Based on the gathered data in the study the following conclusions are formulated:

1. The hypothesis that there is no significant difference between the pre-test and post-test scores of the learners is rejected. This means that in terms of Recognizing, Understanding, Writing, and Comparing Fractions, intervention was effective in improving these mathematical skills.

2. The results on the one-way ANOVA, supported by the post-hoc analysis confirms that there is significant difference the learners scores upon the use of visual imageries in fractions in terms of understanding, writing and comparing consistently across the learning levels. one-way ANOVA.

3. This strategic plan is designed to integrate a practical, classroom-based approach that can help students of varying abilities develop a strong foundation in fractions. By focusing on visual imagery and hands-on learning, this plan ensures that every student can grasp and apply mathematical concepts in a way that is engaging, effective, and meaningful.

Recommendation

Based on the findings of the study the following recommendations are offered:

1. Improve and encourage the use of Visual Imagery. With the ability of visual imagery to enhance students' mathematics skills in fractions, teachers should include more organized and varied visual tools. Teachers may use interactive and digital visual representations to enhance conceptual understanding, most importantly in problem-solving where improvement was lacking. More so, the inclusion of interactive learning tools such as manipulatives, fraction strips, and technological tools (e.g., educational software and gamified learning systems) may have an additional impact on conceptualization and participation. There is evidence that digital scaffolding and interactive e-books significantly facilitate students' comprehension of mathematical concepts (Authors Unknown, 2019).

2. Targeted Instruction for Problem-Solving Skills. Since the problem-solving skills showed the lowest gain as per the results, a targeted instructional approach with real-world applications, step-by-step visual aids, and guided practice needs to be implemented. Scaffolding strategies (Bruner, 1966) can be used to support learners, especially lower-level learners, in offering increasing levels of learning support until independent problem-solving skills are developed.

3. 3. Use of differentiated Instruction for Varied Learning Levels. The study confirms that high-ability students performed better than their peers. To close this gap, instructors should adopt differentiated instruction (Tomlinson, 2017), varying lesson presentation and activities to address varied learning needs of students. This can involve tiered assignments, flexible grouping, and targeted interventions to enable all students to benefit from visual imagery strategies.

References

Abdullah, N., Zakaria, E., & Halim, L. (2012). The effect of a thinking strategy approach through visual representation on achievement and conceptual understanding in solving mathematical word problems. Asian Social Science, 8(16), 30. https://doi.org/10.5539/ass.v8n16p30

Albers, A. M., Kok, P., Toni, I., Dijkerman, H. C., & de Lange, F. P. (2013). Shared representations for working memory and mental imagery in early visual cortex. Current Biology, 23(15), 1427-1431. https://doi.org/10.1016/j.cub.2013.05.065

Anderson, J. R. (2005). Cognitive Psychology and Its Implications (6th ed.). Worth Publishers.

Atit K. (2021) Examining the relations between spatial skills and mathematical performance: A meta-analysis, Volume 29, pages 699–720, (2022)

Bailey, D. H., Siegler, R. S., & Geary, D. C. (2017). Early predictors of middle school fraction knowledge. Developmental Science, 20(5), e12322.

Baron, R. M., & Kenny, D. A. (1986). The moderator-mediator variable distinction in social psychological research: Conceptual, strategic, and statistical considerations. Journal of Personality and Social Psychology, 51(6), 1173-1182.

Bautista J.A., Salompao L.C., Pingot P.S. (2023), Teaching Fractions Through Fraczones Among Grade 5 Pupils, Vol-9 Issue-4

Bloom, B. S. (1956). Taxonomy of Educational Objectives: The Classification of Educational Goals. Longmans.

Booth, J. L., & Koedinger, K. R. (2012). Key Misconceptions in Mathematical Learning. Journal of Educational Psychology.

Chapman, L. R. (Ed.). (1972). The process of learning mathematics. Oxford: Pergamonic Press Ltd.

Charalambous, C. Y., & Pitta-Pantazi, D. (2007). Drawing on a theoretical model to study students’ understandings of fractions. Educational Studies in Mathematics, 64(3), 293–316.

Chen, J. Q. (2020). Multiple intelligences theory: Its application in mathematics education. Journal of Educational Research, 37(4), 453–470.

Cohen, R. J., & Swerdlik, M. E. (2009). Psychological Testing and Assessment: An Introduction to Tests and Measurement (7th ed.). McGraw-Hill.

Creswell, J. W., & Creswell, J. D. (2018). Research design: Qualitative, quantitative, and mixed methods approaches (5th ed.). Thousand Oaks, CA: SAGE Publications.

Danzi (2021) Impact of Visual Aids in Enhancing the Learning Process Case Research: District Dera Ghazi Khan, Journal of Education and Practice https://files.eric.ed.gov/fulltext/EJ1079541.pdf

DeWolf, T., & Vosniadou, S. (2015). The difficulty of fractions for learners of all ages. Educational Studies in Mathematics, 90(2), 1-24.

Fazio, L. K., & Siegler, R. S. (2014). Improving children's knowledge of fraction magnitudes. PLoS ONE, 9(3), e89257. https://doi.org/10.1371/journal.pone.0089257

Fazio, L. K., Kennedy, C. A., & Siegler, R. S. (2016). Improving fraction knowledge in struggling students. Journal of Educational Psychology, 108(4), 45–62.

Fischbein, E. (1987). Intuition in science and mathematics: An educational approach. Dordrecht, Holland: Kluwer.

Frontiers (2020) A Study on Congruency Effects and Numerical Distance in Fraction Comparison by Expert Undergraduate Students

Fuchs, L. S., Schumacher, R. F., Sterba, S. K., & Malone, A. S. (2013). The effects of tutoring with and without conceptual scaffolding on fraction learning by fourth graders with mathematics difficulties. Learning and Instruction, 23, 1–11. https://doi.org/10.1016/j.learninstruc.2012.10.002

Gray, E. M., & Tall, D. O. (1994). Duality, ambiguity, and flexibility: A proceptual view of simple arithmetic. Journal for Research in Mathematics Education, 25(2), 115-141. Retrieved from http://wrap.warwick.ac.uk/470/

Hariyanti, U., & Hwang, W. Y. (2020). Design and implementation of a ubiquitous fraction app for fraction learning in authentic contexts. International Conference on Learning and Teaching, 6(2), 1–16. https://doi.org/10.17501/24246700.2020.6201

Kunnan, A. J. (2014). Fairness in Language Testing. Cambridge University Pres

Lamberg, T., & Moss, B. (2023). Exploring the impact of hands-on learning tools in mathematics education. Mathematics Education Review, 29(4), 207–221.

Lee, X. (2004). The problems of traditional math notation. Retrieved from http://www.xahlee.org/cmaci/notation/trad_math_notation.html

Lemonidis, C., & Kaiafa, I. (2019). The effect of using storytelling strategy on students' performance in fractions. Journal of Education and Learning, 8(2), 165. https://doi.org/10.5539/jel.v8n2p165

Linatoc (2016). The use of symbolism strategy in teaching Grade 3 Mathematics at San Benito Elementary School Malone, A., & Fuchs, L. S. (2019). Effective fractions intervention for students with mathematics learning difficulties. Learning Disabilities: A Multidisciplinary Journal, 24(2), 39–50.

McMillan, J. H. (2017). Classroom Assessment: Principles and Practice for Effective Standards-Based Instruction (7th ed.). Pearson Education.Miller, S. P., & Mercer, C. D. (1993). Mnemonics: Enhancing the math performance of students with learning difficulties. Intervention in School and Clinic, 29, 78-82.

Namkung, J. M., & Fuchs, L. S. (2018). Understanding the magnitude of fractions. Journal of Educational Psychology, 110(5), 725–738.

Ngu, S. F., & Phan, H. P. (2024). Instructional approaches grounded in learning by comparison and cognitive load theories: Implications for mathematical proficiency. International Journal of Mathematical Education in Science and Technology, 55(4), 527-544. https://doi.org/10.1177/27527263241266765

Nitsch K., Renkl A., Berthold K., (2015) Making better use of multiple representations: how fostering metacognition can help, International Handbook of Metacognition and Learning Technologies (pp.397-408)

Okunuga L., Olena K., Iryna K. (2019) The impact of visualization techniques on student's learning vocabulary 34(7) 227-229 https://ijoness.com/article/107

Paivio, A. (1991). Dual coding theory: Retrospect and current status. Canadian Journal of Psychology / Revue canadienne de psychologie, 45(3), 255–287. https://doi.org/10.1037/h0084295

Piaget, J. (1985). The equilibration of cognitive structures. Cambridge, MA: Harvard University Press. Retrieved from http://www.personal.psu.edu/wxh139/construct.htm

Piaget, J. (1952). The Origins of Intelligence in Children. Norton. Prayitno, S., Lu'luilmaknunn, U., Sridana, N., & Subarinah, S. (2021). Analyzing the ability of mathematics students as prospective mathematics teachers on multiple mathematical representation. Journal of Mathematics Education, 24(3), 196–208.

Rau, M., Aleven, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. Journal of Educational Psychology, 107(1), 30–46. https://doi.org/10.1037/a0037211

Raiyn J. (2016). The Role of Visual Learning in Improving Students' High-Order Thinking Skills (7), 111-116.

Real, E., Ventura, F., & Martinez, A. (2015). Bridging the gap between abstract concepts and tangible learning: Visual representations in math education. Mathematics and Education Research Journal, 19(4), 145–168.

Rittle-Johnson, B., & Schneider, M. (2015). Developing conceptual and procedural knowledge of mathematics. In R. C. Kadosh & A. Dowker (Eds.), The Oxford handbook of numerical cognition (pp. 1118–1134). Oxford University Press. https://doi.org/10.1093/oxfordhb/9780199642342.013.014

Rubenstein, R. N., & Thompson, D. R. (2001). Learning mathematical symbolism: Challenges and instructional strategies. In P. M. Van Hiele (Ed.), Structure and insight (pp. 1–38). Academic Press.

Roberts, J. (2024, March 25). Teaching from the front: Explicit teaching methods help struggling students catch up. The Australian. https://www.theaustralian.com.au/inquirer/teaching-from-the-front-explicit-teaching-methods-help-struggling-students-catch-up/news-story/a29543d1e7b0aa4ee1f

Siegler, R. S., & Braithwaite, D. W. (2017). Numerical development. Annual Review of Psychology, 68, 187–213.

Siegler, R. S., & Lortie-Forgues, H. (2015). Fraction learning and the mathematics achievement gap. Current Directions in Psychological Science, 24(5), 379-385.

Siegler, R. S., Fazio, L., Bailey, D. H., & Zhou, J. (2011). Fractions: The new frontier for theories of numerical development. Columbia University

Skemp, R. R. (1979). Intelligence, learning, and action. London: Wiley.

Sfard, A. (1991). On the dual nature of mathematical conceptions: Reflections on processes and objects as different sides of the same coin. Educational Studies in Mathematics, 22, 1-36. https://doi.org/10.1007/BF00302715

Stelzer F., Andres M., Canet L., Urzuijo S., Influence of domain-general abilities and prior division competence on fifth-graders' fraction understanding

Sweller, J. (1988). Cognitive load during problem solving: Effects on learning. Cognitive Science, 12(2), 257–285.

Sweller, J. (1988). Cognitive Load During Problem Solving: Effects on Learning. Cognitive Science.

Sweller, J. (2011). Cognitive load theory. In J. P. Mestre & B. H. Ross (Eds.), The psychology of learning and motivation: Cognition in education (pp. 37–76). Elsevier Academic Press. https://doi.org/10.1016/B978-0-12-387691-

Thompson, I. (2008). Mathematics teachers' interpretation of higher-order thinking in Bloom's taxonomy. International Journal of Mathematical Education in Science and Technology, 39(5),

Tomlinson, C. A. (2014). The Differentiated Classroom: Responding to the Needs of All Learners. ASCD.

Thurlings, M., Koopman, M., Brok, P., & Pepin, B. (2019). Portraying primary fraction teaching: A variety of mathematical richness, pedagogic strategies, and use of curriculum materials. International Journal of Education in Mathematics, Science and Technology, 7(1), 55–70.

Tzur R. (1999) Teacher and students' joint production of a reversible fraction conception, The Journal of Mathematical Behavior 23(1):93-114

Vygotsky, L. S. (1978). Mind in society: The development of higher psychological processes. Cambridge, MA: Harvard University Press

Wright, D., McTigue, E. M., Wang, C., & Staley, R. (2019). Visual Representations and Learning in Mathematics. Journal of Educational Research.

Zhang, J., & Likova, L. T. (2014). Motor imagery learning and functional connectivity across multiple brain systems. NeuroImage, 89(3), 104–116. https://doi.org/10.1016/j.neuroimage.2014.01.062