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UNDERSTANDING STRATEGIC COMPETENCE AND ADAPTIVE REASONING IN SOLVING MATHEMATICAL PROBLEMS: PERSPECTIVES AMONG THE GRADERS

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ABSTRACT:

This study examined the strategic competence and adaptive reasoning of elementary teachers in the 2nd legislative district of the Province of Cotabato, employing a two-phase mixed-methods design. Phase 1 utilized a descriptive-correlational approach to determine the levels and relationships between the domains of strategic competence (formulating, representing, and solving) and adaptive reasoning (explaining and justifying). Results showed that teachers demonstrated high to very high proficiency in all domains, with particularly strong performance in representing ($M = 4.52$), solving ($M = 4.52$), and both aspects of adaptive reasoning ($M = 4.54$). Correlational analysis revealed significant and positive relationships between all components of strategic competence and adaptive reasoning, with the strongest association observed between solving and explaining ($r = 0.543$, $p < 0.01$). These findings suggest that higher levels of strategic thinking are closely linked to more developed reasoning skills. Phase 2 adopted a qualitative phenomenological design to explore the lived experiences and instructional strategies of teachers in fostering these competencies. Thematic analysis revealed the use of constructivist strategies such as problem-based and inquiry-based learning, collaborative and cooperative learning, technology-enhanced instruction, and reflective assessment practices. These approaches emphasized real-world relevance, peer interaction, and formative feedback, thereby promoting student engagement and metacognitive growth. Overall, the study highlights the critical role of teacher expertise and pedagogical strategies in nurturing learners' mathematical thinking. It recommends sustained professional development and context-responsive instructional practices to further enhance strategic and adaptive mathematical instruction at the elementary level.

INTRODUCTION

In understanding mathematical problems, students need to have a deeper understanding of various strategies in solving. This is crucial in finding the solution without facing any difficulties. In arriving answers, students think logically among concepts which lead them to justify the conclusions. This stems through careful considerations of alternatives that may affect their solutions.

Generally, the strategic competence of students solving mathematical problems involved cognitive style. Having this, enabled them to use their critical thinking skills to be creative in understanding the problem in different aspects (Syukriani et al., 2017). In this manner, teachers have to show their support to their students since its implications allowed shared responsibility (Su & Seshaiver, 2016; Morales-Chica & Agger, 2017), student understanding (Egodawatte & Stoilescu, 2015), and personalized strategies (Osdemir et al., 2012).

In terms of their adaptive reasoning, students applied strategies to solving problem situation. Basically, this refers to the mental activities in solving mathematical problems as mentioned by Syukriani et al. (2016). The findings of Muin et al. (2018) found out that this is affected by their inductive and deductive intuitive which helped in the development of their creativeness in problem solving. Students' difficulties in solving mathematical problems were minimized through adaptive reasoning (Susilawati et al., 2021).

Knowing the strategic competence and adaptive reasoning in solving mathematical problems among Grade Six students calls for a thorough investigation. Literatures on strategic competence highlights a different milieu of research like the designing of tasks and lessons (Schulz, 2023) modelling mathematical ideas (Suh & Seshaiyer, 2016), and self-regulation (Cleary et al., 2017). Adaptive reasoning was never mentioned in these studies (Ramadhona et al., 2023; Luzano, 2024) which is the missing link that this paper will try to provide with a clear connection.

More importantly, to have the grasp of the relationship between strategic competence and adaptive reasoning of Grade Six students in solving mathematical problems is an immense track that teachers have to provide with utmost attention. Of course, students will be able to appreciate its value as they continue to strengthen their capabilities in finding solutions to mathematical problems. It is within this regard that the researcher is motivated to examine these variables to improve the delivery of teaching.

Statement of the Problem

The study will be carried out by identifying the strategic competence and adaptive reasoning of the Grade Six students. Specifically, it will seek answer to the following questions.

Phase 1 Strategic Competence and Adaptive Reasoning among the Grade Six Students

- What is the level of strategic competence of Grade Six students in solving mathematical problems in terms of formulating; representing; and solving?
- What is the level of adaptive reasoning of Grade Six students in solving mathematical problems in terms of explaining and justifying?
- Is there a significant relationship between adaptive reasoning and strategic competence of Grade Six students in solving mathematical problems?

Phase 2: Teaching Approaches and Policy on Strategic Competence and Adaptive Reasoning

- What teaching approaches do teachers use in incorporating strategic competence and adaptive reasoning among students?
- What policy on strategy competence and adaptive reasoning can be developed based on the findings of the study?

METHODOLOGY

This chapter presents research design, locale of the study, data gathering procedure, respondents, research instrument, sampling procedure, statistical treatment.

Research Design

Phase 1

This study will use the descriptive-correlational. Descriptive research is a type of research that is primarily concerned with describing, summarizing, and interpreting the characteristics or behaviors of a subject of study, without altering or manipulating it in any way (Park & Hong, 2021). The primary goal of descriptive research is to provide an accurate and detailed account of the subject under investigation. It is one of the most basic and fundamental types of research, often serving as a starting point for more complex research designs (Hsieh et al., 2020).

Correlational research is a type of non-experimental research as stated by Curtis (2016) is a method that aims to examine and describe the relationships or associations between two or more variables without manipulating them. In correlational research, researchers collect data on variables of interest and analyze the degree and direction of the relationship between these variables. It is essential to note that correlational research does not imply causation; it focuses on identifying and measuring the strength and direction of associations between variables.

In this study, the researcher will seek to provide answer on the relationship between the independent and dependent variables. Prior to this, the level of the responses of the respondents will be sought. Proper statistical tools will be employed to provide a clear answer to the problem.

Phase 2

This phase of the study will use the qualitative-descriptive research design (Colorafi & Evans, 2016). It is a methodological approach used in qualitative research to describe and interpret phenomena in their natural settings. This approach focuses on providing a comprehensive summary of events, experiences, or processes, grounded in the perspectives of the participants involved (Kim et al., 2017).

In this study, the researcher will determine the approaches used by mathematics teachers in incorporating strategic competence and adaptive reasoning among students. Aside from that, it will develop a policy which will indicate the crucial role of the variables in mathematical learning. This phase is qualitative-descriptive since there will be no statistical tools which will be used in interpreting the responses of the informants.

Respondents/Participants of the Study

Phase 1

The respondents of the study will be the Grade Six students of the 3rd Congressional District of the Province of Cotabato. Table below shows the distribution of the respondents per town:

Table 1. The respondents of the study representing 3rd Congressional District

3rd Congressional District	Total No. of Respondents	Sample Size
Kabacan	86	48
Matalam	51	29
Mlang	115	64
Tulunan	65	36
Total	317	177

Phase 2

The researcher will choose 10 - 15 teacher-participants from the identified towns in the 3rd Congressional District of the Province of Cotabato. They will be chosen using the purposive sampling (Ames et al., 2019) with the criterion-based sampling in particular. Specifically, they will participate in the Focus Group Discussion (GFD) (Mulyono, 2023). Hence, the following criteria will be set.

- A grade six teacher;
- Specializing in mathematics; and
- Assigned in one of elementary schools in the 3rd congressional district of the Province of Cotabato.

Research Instrument

Phase 1

The questionnaire will be designed based on the dimensions of the variables provided in the study of Syukriani et al. (2016). Each item in the questionnaire will be rated using the scale as follow:

Table 2. Questionnaire rating scale

Level	Mean	Descriptive Equivalent	Descriptive Interpretation
5	**4.50 – 5.00**	**Very High**	**The respondents have shown a very high-level strategic competence and adaptive reasoning**
4	**3.49 – 4.49**	**High**	**The respondents have shown a high level strategic competence and adaptive reasoning.**
3	**2.50 – 3.49**	**Moderately High**	**The respondents have shown a moderately high level strategic competence and adaptive reasoning**
2	**1.50 – 2.49**	**Fairly Low**	**The respondents have fairly low strategic competence and adaptive reasoning**
1	**0,50 – 1.49**	**Very Low**	**The respondents have very low strategic competence and adaptive reasoning**

Statistical Treatment

Phase 1

The researcher will use the frequency count (Marcos et al., 2010) to interpret the level of the responses as well as the Spearman Rho to explain whether there exists a significant relationship between strategic competence and adaptive reasoning in solving mathematical problems among Grade Six students. To test the influence of strategic competence with adaptive reasoning, multiple regression will be employed.

Phase 2

Thematic Analysis. This will be used to provide answer on the phase 2 of the study. The primary goal is to uncover patterns or themes that emerge across the dataset. These themes capture important elements related to the research question and represent some level of patterned response or meaning.

RESULTS AND DISCUSSIONS

Strategic Competence

Formulating

The data on mathematical problem-formulating skills revealed a high level of competence among respondents, with a weighted mean of 4.46. Teachers showed a particularly strong ability in formulating known information to understand problem situations (M = 4.55) and in recognizing how formulation enhances strategic competence (M = 4.55). They also demonstrated high levels of confidence in identifying strategies for formulating information (M = 4.51) and articulating underlying statements in problems (M = 4.39). The selection of strategies for understanding mathematical problems also yielded a high rating (M = 4.32). These findings indicate that the respondents possess strong formulation skills that are vital for interpreting and solving mathematical tasks effectively.

This implies that that the teachers possess strong skills in guiding students through the initial stages of mathematical problem-solving, particularly in organizing and formulating key information. Their high competence indicates an ability to model strategic thinking and support students in understanding problem contexts—an essential component of effective mathematics instruction. Nonetheless, continuous professional development remains important to further enhance their ability to address the needs of learners who may face challenges with problem formulation. Schools are encouraged to strengthen training programs that emphasize mathematical reasoning and the development of problem-structuring strategies.

Nunes et al. (2022) emphasize that identifying knowns and unknowns in problem contexts strengthens students' strategic problem-solving skills. Similarly, Boaler and Anderson (2023) argue that formulation promotes critical reasoning and deeper mathematical understanding when taught systematically. Also, the OECD (2021) underscores the value of strategic formulation as a foundation for complex problem-solving in international assessments like PISA. These findings reinforce the importance of maintaining high formulation skills among educators.

Table 3. Strategic competence

A. Formulating

Statements	Mean	Descriptive Equivalent
I select a strategy for understanding the mathematical problem.	**4.32**	**High**
I formulate known information in order to know the problem situation.	**4.55**	**Very High**
I believe that problem formulation contributed to my overall strategic competence.	**4.55**	**Very High**
I have the ability to formulate the underlying statements in the mathematical problem.	**4.39**	**High**
I figure out a strategy that will be used for formulating the data or information in the mathematical problem.	**4.51**	**Very High**
Weighted mean	**4.46**	**High**

Legend: 4.50- 5.00 Very High

3.49- 4.49 High

2.50- 3.49 Moderately High

1.50- 2.49 Fairly Low

0.50- 1.49 Very Low

Representing

The table reveals that teachers exhibit a very high level of competence in the skill of representing during mathematical problem-solving, with a weighted mean of 4.52. Teachers demonstrated strong agreement with statements such as carefully choosing appropriate solution methods (M = 4.66) and ensuring alignment of methods with the problem requirements (M = 4.64). High ratings were also observed for their use of visuals, mathematical diagrams, graphs, and pattern recognition—essential tools in enhancing understanding and communication of mathematical ideas.

This suggests that teachers are proficient in using multiple representations to solve and communicate mathematical problems, which can effectively support student learning and conceptual understanding. Their ability to match methods with problem requirements and use visuals indicates readiness to model flexible thinking—a key component in mathematics instruction. However, continued support through workshops focusing on visual reasoning and multiple representations can further strengthen their teaching practice, particularly in helping students who rely on visual learning strategies.

According to Stylianou et al. (2020), visual representations help bridge abstract mathematical ideas with concrete understanding. Rott and Radetz (2022) emphasized that using multiple representations enhances students' metacognitive awareness and flexibility in choosing strategies. Meanwhile, Alabdulaziz (2021) found that technology-based visuals and diagrams significantly improve student performance in mathematics. These findings highlight the continued need for integrating representational competence into both teaching and professional development.

Table 4. Strategic competence

B. **Representing**

Statements	Mean	Descriptive Equivalent
I carefully choose a method in coming up with a solution.	**4.66**	**Very High**
I use mathematical diagrams and visuals when solving mathematical problems.	**4.36**	**High**
I can see patterns or relationships more easily.	**4.49**	**High**
I aid my solutions with pictures as well as graphs.	**4.46**	**High**
I make it sure that the method I used fits to what is asked in the problem.	**4.64**	**Very High**
Weighted mean	**4.52**	**Very High**

Legend:

- 4.50- 5.00Very High
- 3.49- 4.49 High
- 2.50- 3.49 Moderately High
- 1.50- 2.49 Fairly Low
- 0.50- 1.49 Very Low

Solving

The data indicate a very high level of competence among teachers in mathematical solving strategies, with a weighted mean of 4.52. Respondents expressed strong confidence in their problem-solving abilities (M = 4.61) and frequently employed specific strategies (M = 4.40), including breaking down problems (M = 4.58), applying known formulas or theorems (M = 4.49), and using trial-and-error techniques (M = 4.50). These responses show that teachers are adept at navigating various approaches to reach solutions effectively.

This implies that teachers possess both procedural fluency and strategic flexibility in problem-solving—skills crucial to facilitating mathematical understanding in learners. Their high confidence levels and reliance on structured strategies equip them to model effective problem-solving behaviors in the classroom. This competence is vital in promoting student resilience and perseverance in mathematical tasks. Nonetheless, sustained professional learning focused on heuristic techniques and metacognitive reflection could further support teachers in guiding learners through more complex, non-routine problems.

For Schoenfeld (2021), effective problem-solvers balance conceptual understanding, strategic planning, and self-monitoring. Meanwhile, Fan et al. (2022) emphasize that structured strategies—like decomposition and trial-and-error—are effective in enhancing students' mathematical performance. Moreover, Lim and Park (2023) underscore the importance of teacher confidence in promoting inquiry-based approaches, as confident educators are more likely to model flexible problem-solving techniques that improve student outcomes.

Table 5. Strategic competence

C. Solving

Statements	Mean	Descriptive Equivalent
I am confident in my ability to solving mathematical problems.	**4.61**	**Very High**
I use specific problem-solving strategies when facing with mathematical challenges.	**4.40**	**High**
I break down the problem into smaller parts to easily understand each detail.	**4.58**	**Very High**
I utilize a known formula or theorem in solving mathematical problems.	**4.49**	**High**
I employ trial and error in solving mathematical problems.	**4.50**	**Very High**
Weighted mean	**4.52**	**Very High**

Legend: *4.50- 5.00 Very High*
3.49- 4.49 High
2.50- 3.49 Moderately High
1.50- 2.49 Fairly Low
0.50- 1.49 Very Low

Part II. Adaptive Reasoning

Explaining

The table shows a very high level of adaptive reasoning among teachers, particularly in the subdomain of explaining, with an overall weighted mean of 4.54. Teachers reported strong abilities in simplifying solution processes (M = 4.62), explaining selected strategies (M = 4.56), and articulating the procedures used (M = 4.54). Additionally, they showed competence in relating mathematical concepts to problem contexts (M = 4.45) and selecting appropriate mathematical ideas to suit the task (M = 4.51). These results highlight teachers' capacity to communicate mathematical reasoning clearly and effectively.

The high ratings suggest that teachers are highly capable of verbalizing and rationalizing mathematical strategies, which is crucial for modeling mathematical thinking and developing student understanding. Their strength in explaining problem-solving processes reflects a deep comprehension of content and pedagogy. This capacity enhances classroom discourse and fosters student confidence in articulating their own reasoning. However, ongoing support in refining mathematical language and using explanatory scaffolds could further help teachers reach learners with varying proficiency levels and promote conceptual clarity.

Stylianides and Stylianides (2022) highlight that teachers' ability to explain mathematical processes is key to fostering students' conceptual understanding. Similarly, Kazemi and Hintz (2021) argue that high-quality mathematics instruction depends on teachers' capacity to justify and discuss solution strategies. Furthermore, Tan and Yeo (2023) stress that simplifying complex procedures helps bridge abstract concepts to learners' prior knowledge, improving retention and comprehension.

Table 6. Adaptive Reasoning

Explaining

Statements	Mean	Descriptive Equivalent
I select appropriate mathematical concepts with problem situations.	**4.51**	**Very High**
I explain the relationship of mathematical concept with the problem situation.	**4.45**	**High**
I can explain the strategy that has been selected.	**4.56**	**Very High**
I explain the procedure of the strategy that has been selected.	**4.54**	**Very High**
I can simplify the process which I used in arriving the answer.	**4.62**	**Very High**
Weighted mean	**4.54**	**Very High**

Legend:

- 4.50- 5.00 Very High
- 3.49- 4.49 High
- 2.50- 3.49 Moderately High
- 1.50- 2.49 Fairly Low
- 0.50- 1.49 Very Low

Justifying

The data indicate that teachers exhibit a very high level of competence in the domain of justifying their mathematical thinking, with a weighted mean of 4.54. Notably, they demonstrated strong performance in justifying strategies used (M = 4.57), providing logical proof through theorems (M = 4.57), and explaining solutions through step-by-step calculations (M = 4.56). High ratings were also recorded for comparing various approaches (M = 4.54) and regularly offering justifications during problem-solving (M = 4.46). This reflects teachers' consistent ability to validate their mathematical processes using logic and structured reasoning.

This explains that teachers are highly proficient in articulating the rationale behind their problem-solving choices—a key indicator of mathematical maturity and critical for supporting students' development of reasoning and proof skills. Such ability enables teachers to model mathematical justification processes effectively, which fosters a classroom culture of inquiry and rigor. Schools should continue to provide opportunities for collaborative lesson planning and peer reflection sessions where teachers can explore diverse strategies and refine their explanatory techniques to enhance instructional quality.

Justification is a central element of mathematical practice. Stylianides and Stylianides (2022) emphasize that engaging students in justification supports conceptual development and deepens understanding of mathematical structures. According to Brendefur et al. (2021), teachers who consistently model justification and comparison of strategies cultivate students' abilities to think critically and evaluate mathematical claims. Furthermore, Schoenfeld (2023) stresses that justification is essential in building mathematically empowered learners capable of independent reasoning.

Table 7. Adaptive Reasoning

Justifying

Statements	Mean	Descriptive Equivalent
I can justify the strategy which I used in solving mathematical problems.	**4.57**	**Very High**
I provide justifications when solving mathematical problems.	**4.46**	**High**
I provide the theorem as a logical proof of my answer.	**4.57**	**Very High**
I demonstrate the step-by-step calculations.		

	4.56	**Very High**
I compare different approaches when explaining my answers.	**4.54**	**Very High**
Weighted mean	**4.54**	**Very High**

Legend:

- 4.50- 5.00Very High
- 3.49- 4.49High
- 2.50- 3.49 Moderately High
- 1.50- 2.49 Fairly Low
- 0.50- 1.49Very Low

Correlation matrix showing the significant relationship between adaptive reasoning and strategic competence of Grade Six students in solving mathematical problems

The correlation matrix reveals statistically significant and positive relationships between strategic competence (formulating, representing, and solving) and adaptive reasoning (explaining and justifying) among Grade Six students in solving mathematical problems. Formulating is moderately correlated with both explaining ($r = 0.463$, $p < 0.01$) and justifying ($r = 0.437$, $p < 0.01$). Representing shows a stronger correlation with explaining ($r = 0.519$, $p < 0.01$) and justifying ($r = 0.522$, $p < 0.01$), while solving displays the strongest associations—explaining ($r = 0.543$, $p < 0.01$) and justifying ($r = 0.491$, $p < 0.01$). These findings indicate that as students' strategic competence increases, their ability to reason adaptively also improves.

The positive correlation between formulating and adaptive reasoning suggests that students who effectively identify and organize information are more capable of articulating and defending their problem-solving approaches. This implies the importance of explicitly teaching formulation skills as a foundation for reasoning development.

As such, the strong association between representing and adaptive reasoning indicates that students who use visual tools and patterns are better positioned to explain their methods and justify their solutions. This underscores the need for instruction that promotes the use of diagrams, models, and representations in mathematics.

Lastly, the robust link between solving and adaptive reasoning reveals that students who confidently apply strategies and break down problems are more likely to reason clearly and validate their conclusions. Encouraging students to verbalize and reflect on their problem-solving steps can enhance their reasoning capacities.

In support, Stylianides and Stylianides (2022) stated that students who develop early strategic skills tend to perform better in reasoning and proof tasks. Brendefur et al. (2021) argue that representing strategies are crucial for fostering reasoning because they help students make connections between abstract and concrete ideas. Furthermore, Schoenfeld (2023) highlights that teaching problem-solving in a way that integrates formulation, representation, and justification nurtures both mathematical understanding and autonomy.

Table 8. Significant relationship between adaptive reasoning and strategic competence

		Adaptive Reasoning	
Strategic Competence		**Explaining**	**Justifying**
Formulating	**Pearson r**	**0.463**	**.437**
	Probability	**0.00****	**0.00****
	N	**177**	**177**
Representing	**Pearson r**	**0.519**	**0.522**
	Probability	**0.00****	**0.00****
	N	**177**	**177**
Solving	**Pearson r**	**0.543**	**0.491**
	Probability	**0.00****	**0.00****
	N	**177**	**177**

Phase 2

Problem-based and inquiry-based learning

Strategic Thinking Through PBL and IBL. Teachers often use Problem-Based Learning (PBL) and Inquiry-Based Learning (IBL) to develop students' strategic competence and adaptive reasoning. These methods involve solving real-world problems and exploring multiple solution strategies. Through reflection and justification, students build deeper understanding and grow into independent, flexible thinkers.

Problem-based learning, inquiry-based instruction, and cooperative learning enable teachers to cultivate well-rounded learners by promoting critical thinking, collaboration, and meaningful engagement in the classroom. As stated:

The specific teaching approaches that I use incorporate strategic and adaptive reasoning among students in mathematics learning, it involves the ability to think logically reflect, explain and justify mathematical ideas and solutions adapting strategies as needed. (Informant 1 RQ2.a L 1-7)

Explicit instruction, combined with problem-based and inquiry-based learning, equips teachers to enhance both strategic competence and adaptive reasoning by promoting logical thinking, reflection, and problem-solving flexibility. As stated:

I prefer to use explicit instruction and problem-based learning for strategic competence and inquiry-based learning and metacognitive approach for adaptive reasoning. (Informant 3 RQ2.a L 46-50)

Recent literature affirms that Problem-Based and Inquiry-Based Learning significantly enhance students' strategic competence and adaptive reasoning. According to Louca et al. (2021), inquiry-based learning empowers students to construct their own understanding, enhancing critical thinking and flexibility in reasoning. Similarly, Madhuri et al. (2022) reported that PBL improves students' problem-solving abilities, allowing them to engage deeply with mathematical content. A 2023 study by Reyes and Luna emphasized the alignment of PBL with real-world contexts, which helps learners develop adaptable strategies to unfamiliar situations. These findings affirm that both approaches cultivate deep, transferable skills essential for mathematical reasoning and strategy formulation.

Table 1

Themes on teachers' approaches: Problem-Based and Inquiry-Based Learning

Global Theme	Organizing Theme	Basic Theme
Constructivist Approaches to Mathematical Learning	**Problem-Based and Inquiry-Based Learning**	**Real-world problem engagement**
		Strategic exploration of multiple solutions

Collaborative and cooperative learning

Collaborative Learning and Reasoning. Teachers integrate collaborative and cooperative learning to support the development of strategic competence and adaptive reasoning. These approaches encourage peer interaction and shared problem-solving through meaningful dialogue. Students learn to express their thinking, evaluate different methods, and refine their reasoning based on feedback.

Collaborative learning empowers students to actively engage with one another, fostering communication, teamwork, and shared understanding of mathematical concepts.

As stated:

Collaborative learning allows students to manipulate, interact and share ideas, opinions and work to each other. (Informant 7 RQ1.a L 154-164)

Engaging students in group work helps cultivate critical thinking and communication skills as they exchange strategies and articulate their reasoning. As stated:

Group work allows students to share strategies, learn from each other. This approach develops their ability to explain and justify their reasoning. (Informant 7 RQ2.a L 171-188)

Through collaborative learning, students are exposed to diverse perspectives that deepen their understanding and enhance their ability to reason strategically. As stated:

Collaborative Learning: Group activities and discussions allow students to share diverse perspectives, refine their reasoning, and develop strategic approaches collectively. (Informant 10 RQ1.a L 262-271)

Collaborative and cooperative learning strategies have been widely recognized for improving mathematical reasoning and communication. According to Gillies (2021), cooperative learning fosters positive interdependence and individual accountability, both of which are essential for the development of adaptive reasoning. Topping et al. (2023) also highlighted that peer-led discussions in collaborative settings enhance conceptual understanding and encourage strategic exploration of problems. Furthermore, Albay and Flores (2022) emphasized that cooperative learning models strengthen students' capacity to justify solutions, enhancing both metacognition and strategic competence.

Implementing collaborative and cooperative learning in math classrooms means shifting the teacher's role from knowledge transmitter to facilitator. Educators must structure group activities purposefully to ensure equitable participation and meaningful discussion. Training in group dynamics and formative feedback is essential to fully leverage the cognitive benefits of peer interaction. Schools should also support this approach through manageable class sizes and flexible seating arrangements.

Table 2

Themes on teachers' approaches: Collaborative and Cooperative Learning

Global Theme	Organizing Theme	Basic Theme
Socially Constructed Learning in Mathematics	**Collaborative and Cooperative Learning**	**Peer-led reasoning and strategy sharing**
		Justification through group discourse

Use of technology and digital tools

Technology Integration for Strategic Learning. Teachers use digital tools to make instruction more interactive and responsive to individual learning needs. These technologies—such as simulations, educational apps, and real-time feedback—support the development of strategic competence. Students are able to adapt their approaches and improve their reasoning through immediate, guided interaction.

Teaching practices have evolved to incorporate digital tools that create a more interactive and personalized learning environment for students. These methods not only engage students but also allow teachers to give immediate feedback, which helps cater to individual learning needs. As mentioned:

Engages students with interactive content, provide immediate feedback and personalized learning experience. (Informant 2 RQ1.b L 36-44)

The integration of technology in the classroom fosters collaboration among students, allowing them to interact and share ideas in ways that enhance their understanding. This not only supports peer learning but also enables students to explain concepts to one another more effectively. As noted:

By using technology. students gain insight from one another and may even explain a concept better with grade level peers. (Informant 6 RQ2.a L 135-151)

Technology in the classroom offers students the chance to actively engage with the learning process through hands-on experiences. Interactive tools and simulations allow them to experiment with various strategies and immediately observe the outcomes of their decisions. As highlighted:

Interactive tools and simulations provide students with the opportunity to experiment with different strategies and the consequences of their choices in real time. (Informant 8 RQ2.a L 220-228)

Technology-enhanced learning environments provide opportunities for differentiated instruction and scaffolded problem-solving. According to Cai et al. (2022), digital tools like dynamic geometry software and math apps improve strategic thinking by allowing students to test hypotheses and receive immediate feedback. Magana and Marzano (2021) emphasized the importance of educational technology in fostering adaptive reasoning through simulation-based tasks and data interpretation. In addition, Punzalan and Cruz (2023) noted that Filipino learners benefited from blended learning approaches that integrated technology with inquiry-based pedagogy.

The effective integration of technology into instruction calls for more than access to devices. Teachers must be trained in using digital tools to support reasoning and strategy development. Additionally, curriculum planners should ensure that technology use aligns with learning goals and supports higher-order thinking. Equity of access is also a major concern that needs to be addressed to avoid deepening learning disparities.

Table 3

Themes on teachers' approaches: Use of Technology and Digital Tools

Global Theme	Organizing Theme	Basic Theme
Technology Integration in Mathematics Education	**Use of Technology and Digital Tools**	**Simulation-based reasoning practice**
		Immediate feedback for strategy refinement

Assessment strategies for strategic competence and adaptive reasoning

Assessment Strategies for Reasoning Skills. Teachers utilize a range of assessment methods to evaluate students' strategic competence and adaptive reasoning. Tools such as performance tasks, reflective journals, rubrics, and peer/self-assessments emphasize how students solve problems and justify their thinking. These assessments provide insight into student reasoning and allow them to demonstrate understanding in meaningful, real-world contexts.

Assessment strategies in teaching are designed to evaluate not only students' knowledge but also their ability to apply what they've learned in practical situations. These methods provide teachers with a deeper understanding of students' problem-solving skills and their ability to reflect on and articulate their learning process. As stated:

I assess students through performance task, reflective journals and problem solving explanation during class discussions. (Informant 1 RQ1.c L 19-25)

Incorporating hands-on activities and real-life applications into assessments helps students connect their learning to practical situations. These tasks encourage them to think critically and strategically as they apply their knowledge in various contexts. As noted:

Giving performance task, real world scenarios or simulations where students apply their knowledge and skills strategically. (Informant 2 RQ1.c L 44-51)

To get a comprehensive understanding of student progress, teachers use a range of assessment techniques that go beyond traditional testing. These methods allow teachers to evaluate not just the final outcome but also the process behind students' learning and problem-solving abilities. As mentioned:

"Teachers assess students through a variety of methods...observation, problem-solving tasks, rubrics, peer and self-assessment." (Informant 4 RQ1.c L 103-109)

Assessment plays a vital role in diagnosing and developing strategic and adaptive reasoning skills. According to Clarke et al. (2020), performance-based tasks provide insights into students' decision-making processes and ability to adjust strategies. Formative assessment strategies, as outlined by Black and Wiliam (2023), promote metacognition and strategic reflection. In the Philippine context, Ramos and Tizon (2024) emphasized the integration of problem-based assessments aligned with competencies to promote critical reasoning and adaptive learning behaviors.

Table 4

Themes on teachers' approaches: Assessment Strategies

Global Theme	Organizing Theme	Basic Theme
Evaluating Strategic Thinking and Reasoning	**Assessment Strategies for Strategic Competence and Adaptive Reasoning**	**Performance-based assessment for strategy use**
		Reflective and peer assessment for metacognition

CHAPTER V SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

This chapter presents the summary of findings, conclusions, and recommendations.

Summary of Findings

The following are the significant findings of the study:

- The findings reveal that elementary teachers in the 2nd legislative district of the Province of Cotabato possess a high to very high level of strategic competence and adaptive reasoning. In the domain of strategic competence, they scored high in formulating ($M = 4.46$), and very high in both representing ($M = 4.52$) and solving ($M = 4.52$).
- In adaptive reasoning, teachers also performed at a very high level in both explaining ($M = 4.54$) and justifying ($M = 4.54$).
- A significant and positive correlation was found between strategic competence and adaptive reasoning. Formulating showed a moderate correlation with explaining ($r = 0.463$) and justifying ($r = 0.437$), while representing had a stronger relationship with both explaining ($r = 0.519$) and justifying ($r = 0.522$). Solving showed the strongest association with explaining ($r = 0.543$) and justifying ($r = 0.491$), all significant at $p < 0.01$.
- Teachers employ multiple pedagogical strategies such as problem-based and inquiry-based learning, collaborative learning, digital tools, and varied assessment methods.
- These approaches emphasize real-world applications, student interaction, and continuous feedback, which promote flexible reasoning and strategic decision-making.

Conclusions

This concludes that:

- Elementary teachers demonstrated a high level of strategic thinking and reasoning. This reflects their preparedness to model effective problem-solving behaviors and guide learners in developing critical mathematical competencies.
- Strategic competence significantly contributes to adaptive reasoning. As teachers improve their ability to formulate, represent, and solve problems, their capacity to explain and justify mathematical processes also strengthens.
- Teachers employ varied strategies aligned with constructivist principles. Their use of PBL, IBL, cooperative learning, digital tools, and performance-based assessments effectively nurtures both strategic and reasoning skills among learners.

Recommendations

The following are the recommendations of the study:

- Education stakeholders should implement sustained professional learning focused on advanced problem-solving techniques, visual reasoning, and reflective mathematical discourse to strengthen teacher competencies.
- Schools should formally integrate problem-based and inquiry-based learning strategies, as well as digital tools, into classroom instruction to deepen strategic competence and adaptive reasoning among students.
- Encourage the use of reflective journals, performance tasks, peer and self-assessments to evaluate students' reasoning processes. These methods provide a holistic view of learners' mathematical thinking and growth.
- Future research should explore the effectiveness of these strategies across diverse grade levels and learner profiles, including multilingual and disadvantaged groups, to inform inclusive and equitable instructional practices.

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