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Markovian Decision Theory in Multi-Purpose/Multi-Objective Anambra-Imo River Basin Planning and Management

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ABSTRACT :

Effective use of water lies in the harness of largest quantities of water flowing in our rivers. Proper and best use of water entails proper planning and management for its utilization, control of quality and quantity. The study aimed to use Markovian Decision Theory in multi-purpose/multi-objective Anambra-Imo River basin project planning and management to help determine levels of development to be apportioned to various purposes for water resources projects. The river basin projects that are critical to the river basin of water supply, irrigation agriculture, hydro-electric power generation, erosion control and flood control were used from the data generated from Bill of Engineering Measurement and Evaluation (BEME) to optimize investment on the courses of action while considering the net benefits of Economic efficiency, regional economic redistribution, environmental quality improvement, social well-being and youth empowerment for the political, ecological and health concern of the people. The methodology used was markovian decision theory analysis based on capital allocation of ₦22.78 billion released from 2015 to 2020 fiscal year. The results show that the expected returns for the five-year period is ₦109.17 billion when the amount of ₦22.78billion released as budgeted for the five (5) year period is deducted from ₦109.17 billion generated, a profit margin of ₦86.39billion would be made from the investment. The work concludes that Markovian decision theory can be applied in optimum policy decision as a basis for inventory theory which is applicable to real life situation's inventory, maintenance, replacement, hydro-electric power generation, cash flow management for the river basin planning and management. It provides a baseline for the future on obtaining superior estimates for institutional use in water resources planning and development.

Keywords: Expected returns, investment, markovian, markov decision theory, optimum policy

1. Introduction

Anambra-Imo River basin is one of the twelve (12) River Basin development Authority that situates at South eastern Nigeria. The River Basin which is referred to as watershed consists of system of rivers which converges towards the same terminus. Such terminus could be a sea, lakes or sometimes an internal water body. The basins and tributaries as watershed are more limited in size which are also referred to as catchment area. River basins and watershed can also be defined as the line separating two river basins. The water management challenges to agriculture are to maximize agricultural production with less water from river basins that are already stressed. The judicious assessment of new water infrastructures in open water basin is necessary to ascertain the possibility of better operations for the benefit of the communities. The nature of the environment has demarcated all land on earth surface into one part of the River basin or another. Although the effort to Control Rivers were initiated many years ago, the late 19th and early 20th centuries gave birth to emergence of River Basins as unit of managing water resources for planning and development.

There is need for a growing interest in institutional processes that would bring together fragmented water users into an integrated planning, allocation and management framework in order to achieve a sustainable River Basin management. Borrow (1998) opined that River basin development planning and management is the process of identifying the best way in which a river and its tributaries may be used to meet competing demands while maintaining river health. These includes the allocation of scarce water resources between different users and purposes, choosing between environmental objectives, competing human needs and choosing between competing food risk management requirements (Molle, 2006). Many serious crises related to floods, degradation of water quality, acute water shortage and degradation of ecological health emerged as a result of the increasing complexity of many of the River Basins occasioned by increase in development with upsurge in population. In many river basin planning, there are various approaches that are ultimately playing significant roles to the adaptation of the local circumstances. The application of markovian decision theory in multi-purpose/multi-objective river basin development project planning and management, helps to determine the levels of development to be apportioned to various purposes for water resources projects. The consideration of the benefits of Economic efficiency, regional economic redistribution, social well-being, Youth empowerment and environmental quality improvement while using multi-purpose projects of water supply, irrigation, hydro-electric power, erosion control and flood control would be relevant to some political, ecological health and sustainability of the environmental infrastructures in the river basin.

The inability of management of river basin to control the whole basin and lack of baseline data with inadequate monitoring are some of the problems identified that hinders River basin development planning and management (Ezenweani, 2017). Klare (2001) also opined that there is politics to determine who is to be employed, what is on the agenda and how river basin development planning and management proceeds which affects them. Eme (2015) also stated that "A nation with economic and great water resources potentials as Nigeria cannot prosper when the benefits of water resources development are not utilized for the welfare of the citizens". The concerned stakeholders in the government and river basin development authorities must ensure that adequate benefits are derived from the resources development and utilization to justify required decision to be made. The selected five multi-purpose river basin projects of water supply, irrigation agriculture, hydro-electric power generation, erosio0n control and flood control are the major challenges at the river basin.

2. Aim and Objectives of the Study

The aim of the study is to utilize the markovian decision theory in multi-purpose/multi-objective Anambra-Imo river basin projects planning and management in order to help determine levels of development to be apportioned to various purposes for water resources projects as identified with the following objectives:

1. To ascertain the benefits under various objectives of economic efficiency, regional economic Redistribution, Social well-being, youth empowerment and environmental quality improvement in Multi-Objective River basin planning and management development projects.
2. To apply the Markovian Decision theory in multipurpose/multi-objective River Basin Planning and Management project optimization of water supply, irrigation agriculture hydro-electric power generation, Erosion Control and Flood control for the optimal benefits of inhabitants of the river basin.

3. Literature Review:

The application Markovian decision theory as used in multi-purpose/multi-objective Anambra-Imo River basin development projects is a decision problem. It is necessary to explain the following terms:

3.1. Definitions on the Markovian Decision Theory According to Taha (2002)

Markovian - for a process to be Markovian the future must depend only on the present and past.
Markovian chain - is a special case of Markovian processes (i) used to study the short -and long-run behavior of certain stochastic systems
Stochastic process - is the family of random variables which indicate the states at a time that actually represent the (exhaustive and mutually exclusive) outcomes of the system at that time or are referred to as the coin tossing game with a number of trials. Each trial may be viewed as a point in time. The resulting sequence of trials forms a stochastic process. The state of the system at any trial is either a head or tail.
Transition probability - is the conditional probability of the system being given which is referred to as the one-step transition.
Stochastic matrix – This is the matrix P which is called a homogeneous transition because all the transition probabilities are fixed and independent of time.
The Transition matrix P - together with the initial probability associated with the states completely defines a Markov chain.
Imbedded Markov chains - are referred to a situation where the length of the interval of a Markov chain will depends on the characteristics of the system and hence may not be equal.
Two step or second order transition probabilities shows the probability of going from one state to another state in exactly two transitions.
Absolute probabilities is the behaviour of the system in a Markov chain over a short period of time.
Irreducible Markov chain - A Markov chain is said to be irreducible if every state can be reached from every other state after a finite .number of transitions. In this case all the states of the chain communicate.
Transient - A state is transient if it is less than one.
Recurrent (Persistent) - A state is recurrent if it is equal to one-
Periodic- This is referred to as a state if a return is possible.
Ergodic – describes a recurrent state if it is non-null and aperiodic (not periodic)
Steady-state probabilities: these are result of the long-run properties of Markov chains, which implies that the long-run absolute probabilities are independent.
Optimal policy- The property for an optimal policy is that whatever the initial state and the initial decision are the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision. This principle implies that a wrong decision taken at any stage does not prevent the taking of optimal decisions for the- remaining stages.
Stationary policies refer to a situation when the gardener is interested in evaluating the expected revenue resulting from pre-specified causes of action for a given state of the system. For example, application of fertilizer or not, whenever the soil condition is poor or good respectively.
E is the expected yearly revenue or expected revenue per stage-
State 1: is when the condition of the system is good.
State 2: is when the condition of the system is fair
State 3: is when the condition of the system is poor
A decision problem when the gardener considers whether the gardening activity will continue for a limited number of years or indefinitely, a decision problem is the problem which arises. These situations are referred to as finite stage and infinite stage decision problems. In both cases, the gardener would determine the best course of action (fertilize or not fertilize) given the outcome of the chemical tests (state of the system).
Return function refers to as the expression of the gain or loss during a one-year period, depending on the states between which the transition is made.
Strategy is any course of action or policy available to the decision maker.

Stage - Stage indicates a portion of the decision problem for which a separate decision can be made. At each stage, there are a number of alternatives. The decision-making process involves the selection of one feasible alternative which may be called the stage decision. "Stage decision" may not be optimal for the considered stage, but contributes to making an overall optimal decision for the entire problem.

A State – This is normally defined to reflect the status of the constraints that bind all the stages together. State variables are the variables which specify the condition of the decision process and summarize the current status of the system. The decision making process at each stage is a decision made to change the state of the problem, with the aim at maximizing the return.

Correlation coefficient- (r) represents the linear interdependence of two variables or sets of data.

Contingent- is the truth by virtue of the way things in fact are not by logical necessity.

Contingency Coefficient- (C) represents a future event or circumstance which is possible but cannot be predicted with certainty.

Regression equation - test the level of interaction of components of a system such that it is found how fitted it is by least of square.

3.2. Overview with Gardner Example

The use of the gardener example in explaining this model shows the underlying philosophy that the example paraphrases several important applications in the areas of real life inventory, maintenance replacement, cash flow management, regulation of electric hydro power and water resources. This work presents an application of dynamic programming to the solution of a stochastic decision process that can be described by a finite or infinite number of states. The transition probabilities between the states are described by a Markov chain. The reward structure of the, process is also described by a matrix whose individual elements represents the revenue (or cost) resulting from moving from one state to another. The transition and revenue matrices depend on the decision alternatives available to the decision maker. And in the multipurpose/and multi-objective nature of this work, the purposes and the objectives are in conflicting situation to be satisfied with available limited resources. The objective of the problem is to determine the optimal policy or strategy or action that maximizes the expected revenue over a finite or infinite number of stages. Every year at the beginning of a season a gardener applies chemical tests to check soils condition. Depending on the outcome of the tests the gardener productivity for the new season falls in one of three states: (l) good (E) fair and (3) poor.

3.2.1. The Infinite - Stage Model

In the steady-state behavior of a Markovian process is independent of the initial state of the system. We are interested in evaluating polices for which the associated Markov chains allow the existence of a steady- state solution.

The essence of Exhaustive Enumeration method is to solve the infinite -stage problem calls for evaluating all possible stationary polices of the decision problem- This is equivalent to an exhaustive enumeration process and can be used only if the number of stationary policies is seasonably small- Suppose that the decision problem has total of S_i stationary polices and assume that P^s and. R^s are the (one-step) transition and revenue matrices associated with the policy, S = 1, 2, …. s. the following are the steps of exhaustive enumeration method

Step 1

Compute V_i^s the expected one- step (one-period) revenue of policy’s given state = ij, for i = 1, 2, …m

Step 2

Compute $\pi^s j$ the long-run stationary probabilities of the transition matrix P^s associated with policy S- These probabilities when they exist are computed from the equations-

$$\pi^s p^s = \pi^s$$

$$\pi^s{}_1 + \pi^s{}_2 + \ldots + \pi^s{}_n = 1$$

$$\text{where } \pi^s = (\pi^s{}_1 + \pi^s{}_2 + \ldots + \pi^s{}_n)$$

Step 3

Determine E^s the expected revenue of policy s per transition step (period) by using the formular

$$E^s = \sum_{i=1}^{n} \pi_i^s v_i^s$$

Step 4

The optimal policy S is determine such that

$$E^s = \max_{s} (E^s)$$

4. Methodology

The methodology used was the application of Markovian Decision Theory analysis in determining the multi-purpose/multi-objective Anambra-Imo River Basin planning and Management. These include the acquisition of relevant data, processing of the data to obtain net benefits values, determine optimal benefits using markovian decision model to the allocation of resources to various planning and objectives of the River Basin.

5. Data Analysis and Results

5.1 Calculation of Observed/Expected Contingency Values for Coefficient of Correlation

Table 1: Observed/Expected Contingency Table using Net Benefits

S/ N	Purpose	Economic Efficiency (B_1)	Regional Economic Redistribution (B_2)	Social Wellbeing (B_3)	Youth Empowerment (B_4)	Environmental Quality Improvement (B_5)	Total
1.	Water supply	4.52 **(4.79)**	3.50 **(3.32)**	4.97 **(3.51)**	2.54 **(3.39)**	3.03 **(3.54)**	**18.56**
2.	Irrigation	4.66 **(4.78)**	3.46 **(3.31)**	1.57 **(3.50)**	3.69 **(3.38)**	5.13 **(3.53)**	**18.51**
3.	Hydroelectric power	4.95 **(4.90)**	2.29 **(3.40)**	3.34 **(3.59)**	4.44 **(3.47)**	3.95 **(3.62)**	**18.97**
4.	Erosion control	6.61 **(4.81)**	2.95 **(3.40)**	4.93 **(3.53)**	2.61 **(3.41)**	1.54 **(3.56)**	**18.64**
5.	Flood control	3.02 **(4.48)**	4.28 **(3.11)**	2.60 **(3.28)**	3.55 **(3.17)**	3.91 **(3.31)**	**17.36**
	Total	**23.76**	**16.48**	**17.41**	**16.83**	**17.56**	**92.04**

The calculation of the expected contingency values are carried out using the formula,

For Row 1 of the Table 1 we have;

1st Row

$$\text{Row } (a) = \frac{18.56 \times 23.76}{92.04} = 4.79$$
$$(b) = \frac{18.56 \times 16.48}{92.04} = 3.32$$
$$(c) = \frac{18.56 \times 17.41}{92.04} = 3.51$$
$$(d) = \frac{18.56 \times 16.83}{92.04} = 3.39$$
$$(e) = \frac{18.56 \times 17.56}{92.04} = 3.54$$

2nd Row

$$(a) = \frac{18.51 \times 23.76}{92.04} = 4.78$$
$$(b) = \frac{18.51 \times 16.48}{92.04} = 3.31$$
$$(c) = \frac{18.51 \times 17.41}{92.04} = 3.50$$
$$(d) = \frac{18.51 \times 16.83}{92.04} = 3.38$$
$$(e) = \frac{18.51 \times 17.56}{92.04} = 3.53$$

3rd Row

$$(a) = \frac{18.97 \times 23.76}{92.04} = 4.90$$
$$(b) = \frac{18.97 \times 16.48}{92.04} = 3.40$$
$$(c) = \frac{18.97 \times 17.41}{92.04} = 3.59$$
$$(d) = \frac{18.97 \times 16.83}{92.04} = 3.47$$
$$(e) = \frac{18.97 \times 17.56}{92.04} = 3.62$$

4th Row

$$(a) = \frac{18.64 \times 23.76}{92.04} = 4.81$$
$$(b) = \frac{18.64 \times 16.48}{92.04} = 3.34$$
$$(c) = \frac{18.64 \times 17.41}{92.04} = 3.53$$
$$(d) = \frac{18.64 \times 16.83}{92.04} = 3.41$$
$$(e) = \frac{18.64 \times 17.56}{92.04} = 3.56$$

5th Row

$$(a) = \frac{17.36 \times 23.76}{92.04} = 4.48$$
$$(b) = \frac{17.36 \times 16.48}{92.04} = 3.11$$
$$(c) = \frac{17.36 \times 17.41}{92.04} = 3.28$$
$$(d) = \frac{17.36 \times 16.83}{92.04} = 3.17$$
$$(e) = \frac{17.36 \times 17.56}{92.04} = 3.31$$

5.2. Calculation of values for Pearson's Product Moment correlation coefficient

Table 2: Table of values for Pearson's Correlation Calculation

S/N	Observed X	Expected Y	XY	X²	Y²
1	4.52	4.79	21.6508	20.4304	22.9441
2	3.5	3.32	11.62	12.25	11.0224
3	4.97	3.51	17.4447	24.7009	12.3201
4	2.54	3.39	8.6106	6.4516	11.4921
5	303	3.54	1072.62	91809	12.5316
6	4.66	4.78	22.2748	21.7156	22.8484
7	3.46	3.31	11.4526	11.9716	10.9561
8	1.57	3.5	5.495	2.4649	12.25
9	3.69	3.38	12.4722	13.6161	11.4244
10	5.13	3.53	18.1089	26.3169	12.4609
11	4.95	4.9	24.255	24.5025	24.01
12	2.29	3.4	7.786	5.2441	11.56
13	3.34	3.59	11.9906	11.1556	12.8881
14	4.44	3.47	15.4068	19.7136	12.0409
15	3.95	3.62	14.299	15.6025	13.1044
16	6.61	4.81	31.7941	43.6921	23.1361
17	2.95	3.34	9.853	8.7025	11.1556
18	4.93	3.35	16.5155	24.3049	11.2225
19	2.61	3.41	8.9001	6.8121	11.6281
20	1.54	3.56	5.4824	2.3716	12.6736
21	3.02	4.48	13.5296	9.1204	20.0704
22	4.28	3.11	13.3108	18.3184	9.6721
23	2.6	3.28	8.528	6.76	10.7584
24	3.55	3.17	11.2535	12.6025	10.0489
25	3.91	3.31	12.9421	15.2881	10.9561
Total	**92.04**	**92.04**	**346.5895**	**373.2898**	**346.4137**

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][nY^2 - (\sum Y)^2]}}$$

$$r = \frac{25(346.5895) - (92.04)(92.04)}{\sqrt{[25(373.289)^2 - (92.04)^2]\,[25(346.4137)^2 - (92.04)^2]}}$$

$$\frac{8664.7375 - 8471.3616}{\sqrt{(860.854)(188.9809)}} = \frac{193.3759}{403.3491} = 0.4794$$

Degree of freedom (D_f) = $P_1 + P_2 - 2 = 25 + 25 - 2 = 48$

The result at 0.05 level of significance and 48 degree of freedom (d_f) critical r = 0.2787. Since the calculated r value of 0.4794 is greater than the critical value of r = 0.2787, the null hypothesis is rejected and the alternative hypothesis is accepted. The conclusion is that there is a significant relationship between the observed and expected values in the correlation analysis. This shows that there is a relationship between the river basin purposes and the objectives/benefits.

5.3. Calculation of Chi-square Contingency Values

Table 3: Chi-square table of values

S/N	The Observed Values (O)	The Expected values (E)	O – E	$(O – E)^2$	$(O – E)^2/E$
1	4.52	4.79	-0.27	0.0729	0.0161
2	3.5	3.32	0.18	0.0324	0.0093
3	4.97	3.51	1.46	2.1316	0.4289
4	2.54	3.39	-0.85	0.7225	0.2844
5	3.03	3.54	299.46	89676.29	0.0858
6	4.66	4.78	-0.12	0.0144	0.0031
7	3.46	3.31	0.15	0.0225	0.0065
8	1.57	3.5	-1.93	3.7249	2.3725
9	3.69	3.38	0.31	0.0961	0.0260
10	5.13	3.53	1.6	2.56	0.4990
11	4.95	4.9	0.05	0.0025	0.0005
12	2.29	3.4	-1.11	1.2321	0.5380
13	3.34	3.59	-0.25	0.0625	0.0187
14	4.44	3.47	0.97	0.9409	0.2119
15	3.95	3.62	0.33	0.1089	0.0276
16	6.61	4.81	1.8	3.24	0.4902
17	2.95	3.34	-0.39	0.1521	0.0516
18	4.99	3.35	1.58	2.4964	0.3976
19	2.61	3.41	-0.8	0.64	0.2452
20	1.54	3.56	-2.02	4.0804	2.6496
21	3.02	4.48	-1.46	2.1316	0.7058
22	4.28	3.11	1.17	1.3689	0.3198
23	2.6	3.28	-0.68	0.4624	0.1778
24	3.55	3.17	0.38	0.144	0.0407
25	3.91	3.31	0.6	0.36	0.0021
Total	**92.04**	**92.04**			**9.6987**

Chi-square (χ^2) = **9.6987**

The contingency coefficient, C = $\sqrt{\frac{\chi 2}{N+\chi 2}} = \sqrt{\frac{9.6987}{92.04+9.6987}} = 0.3088$

$$\frac{0.3088}{0.8} = 0.3859$$

The correlation of attributes (r) is given as:

r = $\sqrt{\frac{\chi 2}{N(K-1)}} = \sqrt{\frac{9.6987}{92.04(5-1)}} = \sqrt{\frac{9.6987}{92.04(4)}} = 0.1623.$

$\therefore$ r = 0.1623

The χ^2 values of 9.6987 as interpreted from the Chi-square (χ^2) table of probability values at 0.05 level of significance. The degree of freedom was determined from the frequency table by the number of rows minus one time the number of columns minus one = (r – 1) (c – 1) d_f = (5 – 1) (5 – 1) = 16; The critical Chi-square (χ^2) value obtained at 0.05 level of significance and degree of freedom d_f =16 is 26.30 since the calculated χ^2 value (9.6887) < $\chi^2_{0.05}$ (26.30), the null hypothesis was rejected and the alternative hypothesis accepted. This shows that there is a relationship between the state of nature (the river basin purposes) and the objectives/the net benefits in Anambra-Imo River Basin Development for their multi-purpose/multi-objectives projects.

5.4. The Application of Markovian Decision Theory in multi-purpose/multi-objective project optimization at Anambra-Imo River Basin

$$P^1 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 0.30 & 0.25 & 0.20 & 0.15 & 0.1 \\ 0 & 035 & 0.3 & 0.2 & 0.15 \\ 0 & 0 & 0.45 & 0.35 & 0.2 \\ 0 & 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad R^1 = \begin{bmatrix} 5.2 & 4.03 & 5.72 & 2.92 & 3.48 \\ 5.36 & 3.98 & 1.81 & 4.24 & 5.90 \\ 5.69 & 2.63 & 3.84 & 5.11 & 4.54 \\ 7.60 & 3.39 & 5.67 & 3.00 & 1.77 \\ 3.47 & 4.92 & 2.99 & 4.08 & 4.50 \end{bmatrix} \begin{array}{c} Water\ supply \\ Irrigation \\ Hydro-power \\ Erosion\ control \\ Flood\ control \end{array}$$

$$P^2 = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{bmatrix} 0.40 & 0.3 & 0.15 & 0.1 & 0.05 \\ 0.05 & 0.45 & 0.35 & 0.1 & 0.05 \\ 0.05 & 0.15 & 0.55 & 0.15 & 0.1 \\ 0.06 & 0.10 & 0.11 & 0.6 & 0.13 \\ 0.08 & 0.12 & 0.15 & 0.20 & 0.45 \end{bmatrix} \qquad R^2 = \begin{bmatrix} 4.52 & 3.50 & 4.97 & 2.54 & 3.03 \\ 4.66 & 3.46 & 1.57 & 3.69 & 5.13 \\ 4.95 & 2.29 & 3.34 & 4.44 & 3.95 \\ 6.61 & 2.95 & 4.93 & 2.61 & 1.54 \\ 3.02 & 4.28 & 2.60 & 3.55 & 3.91 \end{bmatrix} \begin{array}{c} Water\ supply \\ Irrigation \\ Hydro-power \\ Erosion\ control \\ Flood\ control \end{array}$$

Soil condition at 1 = very good, 2 = good, 3 = fair, 4 = poor, 5 = very poor

Consider the case in which no fertilizer is used (k = 1)

$$V_i^k = \sum_{i=1}^{m} P_{ij}^k \ r_{ij}^k$$

$Water\ Supply\ \ V_1^1 = .3 \times 5.2 + .25 \times 4.03 + .2 \times 5.72 + .15 \times 2.92 + 0.1 \times 3.48 = 4.4975 \approx 4.50$

$Irrigation\ \ V_2^1 = 0 \times 5.36 + .35 \times 3.98 + .3 \times 1.81 + .2 \times 4.24 + .15 \times 5.90 = 6.606 \approx 6.6$

$Hydro-power\ \ V_3^1 = 0 \times 5.69 + 0 \times 2.63 + .45 \times 3.84 + .35 \times 5.11 + .2 \times 4.54 = 4.4245 \approx 4.4$

$Erosion\ control\ \ V_4^1 = 0 \times 7.60 + 0 \times 3.39 + 0 \times 5.67 + .5 \times 3.00 + .5 \times 1.77 = 2.385 = 2.4$

$Flood\ control\ \ V_5^1 = 0 \times 3.47 + 0 \times 4.92 + 0 \times 2.99 + 0 \times 4.08 + 1 \times 4.50 = 4.5$

$V_1^2 = 0.4 \times 4.52 + .3 \times 3.50 + .15 \times 4.97 + .1 \times 2.54 + .05 \times 3.03 = 4.312 \approx 4.3$

$V_2^2 = 0.05 \times 4.66 + .45 \times 3.46 + .35 \times 1.57 + .1 \times 3.69 + .05 \times 5.13 = 2.965 \approx 3.0$

$V_3^2 = 0.05 \times 4.95 + .15 \times 2.29 + .55 \times 3.34 + .15 \times 4.44 + .1 \times 3.95 = 3.489 \approx 3.5$

$V_4^2 = 0.06 \times 6.61 + .1 \times 2.95 + .11 \times 4.93 + .6 \times 2.61 + .13 \times 1.54 = 3.0001 \approx 3.0$

$V_5^2 = 0.08 \times 3.02 + .12 \times 4.28 + .15 \times 2.60 + .20 \times 3.55 + .45 \times 3.91 = 3.6147 \approx 3.6$

Stage 5

	V_i^k		Optimal solution	
(i)	**K=1**	**K=2**	$f_5(i)$	k^*
1	4.5	4.3	4.5	1
2	6.6	3.0	6.6	1
3	4.4	3.5	4.4	1
4	2.4	3.0	3.0	2
5	4.5	3.6	4.5	1

States		*Courses of action*
1	-	very good
2	-	good
3	-	fair
4	-	Poor
5	-	Very poor

At this stage the optimal solution for water supply, irrigation agriculture, hydro-electric power generation and flood control can be achieved without maintenance. It is only on erosion control that maintenance be required to optimize result.

Stage 4

$V_i^k = P_{11}^k f_5^{(1)} + P_{12}^k f_5^{(2)} + P_{13}^k f_5^{(3)} + P_{14}^k f_5^{(4)} + P_{15}^k f_5^{(5)}$			**Optimal solution**	
i.	$k = 1$	$k = 2$	$f_4^{(i)}$	K^*
1.	$4.5 + .3 \times 4.5 + .25 \times 6.6 + .20 \times 4.4 + .15 \times 3.0 + .1 \times 4.5 = 9.28$	$4.3 + .4 \times 4.5 + .3 \times 6.6 + .15 \times 4.4 + .1 \text{ x } 3.0 + .05 \times 4.5 = 9.26$	9.28	1
2.	$6.6 + 0 \times 4.5 + .35 \times 6.6 + .3 \times 4.4 + .2 \times 3.0 + .15 \times 4.5 = 11.51$	$3.0 + .05 \times 4.5 + .45 \times 6.6 + .35 \times 4.4 + .1 \text{ x } 3.0 + .05 \times 4.5 = 8.26$	11.51	1

3.	$4.4 + 0 \times 4.5 + 0 \times 6.6 + .45 \times 4.4 + .35 \times 3.0 + .2 \times 4.5 = 8.33$	$3.5 +.05\times 4.5 + .15 \times 6.6 + .55 \times 4.4 + .15 \text{ x } 3.0 + .1 \times 4.5 = 8.04$	8.33	1
4	$2.4 + 0 \times 4.5 + 0 \times 6.6 + 0 \times 4.4 + +.5 \times 3.0 + .5 \times 4.5 = 6.15$	$3.0 +.06 \times 4.5 + .1 \times 6.6 + .11 \times 4.4 + .6 \text{ x } 3.0 + .13 \times 4.5 = 6.80$	6.80	2
5	$4.5 + 0 \times 4.5 + 0 \times 6.6 + 0 \times 4.4 + 0 \times 3.0 + 1 \times 4.5 = 9.0$	$3.6+.08 \times 4.5 + .12 \times 6.6 + .15 \times 4.4 + .20 \text{ x } 3.0 + .45 \times 4.5 = 8.04$	9.0	1

At this stage all the other projects except the erosion control will not require maintenance to achieve optimum result.

Stage 3

$V_i^k + P_{11}^k f_4^{(1)} + P_{12}^k f_4^{(2)} + P_{13}^k f_4^{(3)} + P_{14}^k f_4^{(4)} + P_{15}^k f_4^{(5)}$			**Optimal solution**	
i.	**k = 1**	**k = 2**	$\mathbf{f_3^{(i)}}$	K^*
1.	$4.5 + .3 \times 9.28 + .25 \times 11.51 + .20 \times 8.33 + .15 \times 6.80 + .1 \times 9.0 = 3.75$	$4.3+.4\times 9.28 + .3 \times 11.51 + .15 \times 8.33 + .1 \text{ x } 6.80 + .05 \times 9.0 = 13.84$	13.75	1
2.	$6.6 + 0 \times 9.28 + .35 \times 11.51 + .3 \times 8.33 + .2 \times 6.80 + .15 \times 9.0 = 15.84$	$3.0+.05\times 9.28 + .45 \times 11.51 + .35 \times 8.33 + .1 \text{ x } 6.80 + .05 \times 9.0 = 12.69$	15.84	1
3.	$4.4 + 0 \times 9.28 + 0 \times 11.51 + .45 \times 8.33 + .35 \times 6.80 + .2 \times 9.0 = 12.33$	$3.5 +.05\times 9.28 + .15 \times 11.51 + .55 \times 8.33 + .15 \text{ x } 6.80 + .1 \times 9.0 = 12.19$	12.33	1
4	$2.4 + 0 \times 9.28 + 0 \times 11.51 + 0 \times 8.33 + .5 \times 6.80 + .5 \times 9.0 = 10.3$	$3.0 +.06 \times 9.28 + .1 \times 11.51 + .11 \times 8.33 + .6 \text{ x } 6.80 + .13 \times 9.0 = 10.87$	10.87	2
5	$4.5 + 0 \times 9.28 + 0 \times 11.51 + 8.33 + 0 \times 6.80 + 1 \times 9.0 = 13.5$	$3.6+.08 \times 9.28 + .12 \times 11.51 + .15 \times 8.33 + .2 \text{ x } 6.80 + .45 \times 9.0 = 12.38$	13.5	1

At this third stage also all the multi-purpose projects except erosion control will not require any maintenance to achieve optimal solution.

Stage 2

$V_i^k + P_{11}^k f_3^{(1)} + P_{12}^k f_3^{(2)} + P_{13}^k f_3^{(3)} + P_{14}^k f_3^{(4)} + P_{15}^k f_3^{(5)}$			**Optimal solution**	
i.	$k = 1$	$k = 2$	$f_2^{(i)}$	K^*
1.	$4.5 + .3 \times 13.75 + .25 \times 15.84 + .2 \times 12.33 + .10.87 + .1 \times 13.50 = 18.03$	$4.3+.4\times 13.75 + .3 \times 15.84 + .15 \times 12.33 + .1\, x\, 10.87 + .05 \times 13.50 = 18.16$	18.16	2
2.	$6.6 + 0 \times 13.75 + .35 \times 15.84 + .3 \times 12.33 + .2 \times 10.87 + .15 \times 13.50 = 20.04$	$3.0+.05\times 13.75 + .45 \times 15.84 + .35 \times 12.33 + .1\, x\, 10.87 + .05 \times 13.50 = 16.89$	20.04	1
3.	$4.4 + 0 \times 13.75 + 0 \times 15.84 + .45 \times 12.33 + .35 \times 10.87 + .2 \times 13.50 = 16.45$	$3.5 +.05\times 13.75 + .15 \times 15.84 + .55 \times 12.33 + .15\, x\, 10.87 + .1 \times 13.50 = 16.33$	16.45	1
4	$2.4 + 0 \times 13.75 + 0 \times 15.84 + 0 \times 12.33 + .5 \times 10.87 + .5 \times 13.50 = 14.59$	$3.0 +.06 \times 13.75 + .1 \times 15.84 + .11 \times 12.33 + .6\, x\, 10.87 + .13 \times 13.50 = 15.04$	15.04	2
5	$4.5 + 0 \times 9.28 + 0 \times 15.84 + 0 \times 12.33 + 0 \times 10.87 + 1 \times 13.5 = 18.0$	$3.6+.08 \times 13.75 + .12 \times 15.84 + .15 \times 12.33 + .2\, x\, 10.87 + .45 \times 13.50 = 16.70$	18.0	1

The result at this stage also shows that all multipurpose projects of water supply, irrigation agriculture, hydro-electric power and flood control will achieve optimal result except erosion control that will require maintenance during the period.

Stage 1

$V_i^k + P_{11}^k f_2^{(1)} + P_{12}^k f_2^{(2)} + P_{13}^k f_2^{(3)} + P_{14}^k f_2^{(4)} + P_{15}^k f_2^{(5)}$			**Optimal solution**	
i.	$k = 1$	$k = 2$	$f_1^{(i)}$	K^*
1.	$4.5 + .3 \times 18.16 + .25 \times 20.04 + .2 \times 16.45 + .15 \times 15.04 + .1 \times 18.0 = 22.30$	$4.3+.4\times 18.16 + .3 \times 20.04 + .15 \times 16.45 + .1 \text{ x } 15.04 + .05 \times 18.0 = 22.45$	22.45	2
2.	$6.6 + 0 \times 18.16 + .35 \times 20.04 + .3 \times 16.45 + .2 \times 15.04 + .15 \times 18.0 = 24.26$	$3.0+.05\times 18.16 + .45 \times 20.04 + .35 \times 16.45 + .1 \text{ x } 15.04 + .05 \times 18.0 = 21.09$	24.26	1
3.	$4.4 + 0 \times 18.16 + 0 \times 20.04 + .45 \times 16.45 + .35 \times 15.04 + .2 \times 18.0 = 20.67$	$3.5 +.05\times 18.16 + .15 \times 20.04 + .55 \times 16.45 + .15 \text{ x } 15.04 + .1 \times 18.0 = 20.52$	20.67	1

4	$2.4 + 0 \times 18.16 + 0 \times 20.04 + 0 \times 16.45 + .5 \times 15.04 + .5 \times 18.0 = 18.92$	$3.0 + .06 \times 18.16 + .1 \times 20.04 + .11 \times 16.45 + .6 \times 15.04 + .13 \times 18.0 = 19.27$	19.27	2
5	$4.5 + 0 \times 18.16 + 0 \times 20.04 + 0 \times 16.45 + 0 \times 15.04 + 1 \times 18.0 = 22.50$	$3.6 + .08 \times 18.16 + .12 \times 20.04 + .15 \times 16.45 + .2 \times 15.04 + .45 \times 18.0 = 21.03$	22.50	1

At this stage on the fifth year, water supply and erosion control would require maintenance while irrigation, hydro-electric power and flood control projects will not require maintenance to achieve optimal solution.

Using the Gardner example for the five-year transition period, the optimum values for the solution shows that for years 1 and 2. There should be maintenance of the facilities when the state is very good and poor. In year 3, 4 and 5 maintenance is done only when the state is poor.

The result of the performance experiment shows that from 2015 to 2020 the total expected revenue for optimal utilization of Anambra-Imo River Basin assets are:

F(1) = ₦22.45billion for the state of the project in year 1 very good.

F(2) = ₦24.26billion for the state of the project in year 2 is good.

F(3) = ₦20.67billion for the state of the project in year 3 is fair.

F(4) = ₦19.29billion for the state of the project in year 4 is poor.

F(5) = ₦22.50billion for the state of the project in year 5 is very poor.

Assuming the capital allocation to the Anambra-Imo River basin of ₦22.78billion was released the expected returns for the five-year period is ₦109.17billion when this figure of ₦22.786billion is deducted from ₦109.17billion generated a profit margin of ₦86.39billion from the investment.

The investment would result to net benefits from economic efficiency, regional economic redistribution, social well-being, youth empowerment and environmental quality improvement used on the data obtained from bill of Engineering Measurement and Evaluation.

6. Conclusion and Recommendation

I. The analysis shows that the Markovian decision theory was effectively applied in optimum policy decision making in multi-purpose/multi-objective water resources planning and management.

II. The use of Markovian decision theory provides a new basis for inventory theory and it is applicable in real life situations, inventory, maintenance, replacement, hydroelectric power generation, cash flow management, water resources planning and management etc.

III. In any of the multi-purpose/multi-objective water resources planning and management project, the optimization was achieved by application of Markovian decision theory which would prevent the disagreement between the planning Engineer and other interest groups during project authorization.

IV. This study provides a baseline for future on the issue of obtaining superior estimates for institutional use in water resource planning and conjunctive uses of water resources development.

V. The result of the experiments serves as a vital input into the demand management process for long term sustainable multi-purpose/multi-objective water resources projects in Anambra-Imo River Basin Development Authority.

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