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Application of GeoGebra to the Teaching of Vector Algebra

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ABSTRACT

The study was conducted to explore how GeoGebra can enhance the teaching and learning of vector algebra, thereby improving students' conceptual understanding of vector algebra. The study employed a pre-experimental design, which is a one-group pretest-posttest approach, where the researcher had the opportunity to assess the impact of GeoGebra integrated instruction on students' learning outcomes after it has been used as an instructional tool. The statistical comparison between pre-test and post-test scores, indicates a massive increase in the average performance with a mean pre-test score of 29.90 to a post-test mean of 63.30. While the standard deviations for both pre-test (21.28) and post-test (16.9281) scores demonstrated higher variability, the paired t-test results revealed statistically significant difference between the pre-test and post-test mean scores. Further analysis performed indicates a very large effect size suggesting that GeoGebra was valuable to teaching of vector algebra. The descriptive analysis from SPSS revealed more insights into students' perceptions and confidence regarding the use of GeoGebra. A greater proportion of students acknowledged the advantages of the tool, such as improved comprehension and interactive visualization, whereas a minority maintained a neutral stance. The ratings on the instructional materials underscored the importance of diverse resources, with suggestions for improvement focusing on more interactive tutorials and varied exercises. Overall, GeoGebra depicts as promising and potential educational tool for vector algebra instruction and the need to be integrated into our mathematics education at all levels.

1. Introduction

The past decade has seen technology integrating into virtually every aspect of life, including work and education. Even though mathematics has always been one of the key fields of education, it is widely believed to be one of the most challenging subjects (Borovik & Kondratiev, 2023). Moreover, students do often perceive vector algebra to be lacking practical significance for their daily lives and future careers. This issue is further compounded by various mis conceptions, such as equating vector addition to scalar addition and misunderstanding operations such as the dot and cross products (Owusu, Bonyah, & Arthur, 2023). These misconceptions can be addressed through targeted teaching methods which will significantly enhance students' understanding and appreciation of vector concepts (Handhika & Sasono, 2021). Various software titles for vector algebra, such as "Vector Math App," have been developed to facilitate learning. Even though some, like "Vector Magic" and "VectorWorks," have their limitations in educational effectiveness, GeoGebra has proven itself to be a promising solution for vector algebra instruction at all levels because of its comprehensive and user-friendly features (Ziatdinov & Valles, 2022). GeoGebra, a dynamic mathematics software developed by Markus Hohenwarter, offers a comprehensive solution geared towards the requirements of modern education. With functionalities spanning geometry, algebra, and calculus, GeoGebra addresses the deficiencies of existing software and aligns with the demands of the knowledge economy era (Arbain & Shukor, 2015). GeoGebra is versatile and this makes it a very valuable asset for educators who seek to enhance students' understanding of mathematics. GeoGebra was adopted in China since 2010.

Adopting it particularly in the primary and secondary schools, portrays its effectiveness in assisting teachers in lesson delivery and improving students' com prehension of all aspects of mathematics especially geometry. By integrating numbers and shapes, employing dynamic displays, and enabling exploration of mathematical laws, GeoGebra gives empowerment to students to deepen their understanding as well as enhance their geometric thinking (Ziatdinov & Valles, 2022). GeoGebra stands tall as a pivotal tool in mathematics education, especially for elucidating the complexities of vector algebra. These complexities include understanding vector addition and subtraction, interpreting the geometric significance of dot and cross products, comprehending vector magnitudes and directions, and normalizing vectors. GeoGe bra's integration into technology with pedagogy does not only enhance learning outcomes but also fosters a deeper understanding of these mathematical concepts, therefore exemplifying its significance in modern education.

However, the teaching of vector algebra, a fundamental component of mathematics education at secondary and tertiary levels, often faces significant challenges in terms of student engagement and conceptual understanding. Traditional teaching methods primarily rely on static representations and rote memorizations, hindering students' ability to visualize and internalize abstract vector concepts. This disconnects between theoretical instruction and practical comprehension frequently leads to students struggling with spatial reasoning, vector operations, and applications in various scientific and engineering domains. GeoGe bra, an interactive geometry, algebra, and calculus application, offers a promising solution to these pedagogical challenges by providing dynamic visualization tools and interactive learning environments (Aytekin & Kiymaz, 2019). Moreover, there is a lack of comprehensive research examining the efficacy of using GeoGebra in enhancing the teaching and learning of vector algebra specifically (Ziatdinov & Valles, 2022).

Integrating GeoGebra into vector algebra instruction could potentially trans form traditional teaching methodologies, fostering a more intuitive and engaging learning experience. This research aims to investigate the application of GeoGebra in teaching vector algebra, focusing on its impact on students' conceptual understanding, engagement, and overall academic performance. By conducting a systematic study involving a di verse cohort of students, this research will explore the extent to which GeoGebra can bridge the gap between abstract vector concepts and students' cognitive frameworks (Owusu, Bonyah, & Arthur, 2023). The outcomes of this study will provide valuable insights into best practices for incorporating technology into mathematics education and contribute to the development of innovative instructional strategies that enhance learning outcomes in vector algebra. The major purpose of this study is to investigate the efficacy of using GeoGebra as a teaching tool in vector algebra instruction, which is aimed at enhancing the learning experience and improving comprehension of abstract mathematical concepts. By giving adherence to GeoGebra's inter active capabilities, the study seeks to improve students' understanding of concepts of vector algebra when GeoGebra is used as an instructional tool in teaching vector algebra. Using various pedagogical approaches, which include inquiry-based learning, flipped classrooms, and collaborative problem solving, the study aims to foster students' engagement and confidence in vector algebra. Ultimately, the study aims at evaluating the effectiveness of GeoGebra in improving learning outcomes and mathematical visualization. The objectives of the study are: to investigate the efficacy of using GeoGebra in vector algebra instruction; to determine whether GeoGebra's dynamic visualization enhances students' understanding of vector algebra when it is used as a teaching tool for vector algebra instruction; and to explore students' perceptions and experiences through qualitative methods such as the use of questionnaires to evaluate students' attitudes towards GeoGebra, their perceived effectiveness of instructional materials, and the extent to which GeoGebra helps them in the understanding of vector algebra

2. Problem Statement

The teaching of vector algebra, a fundamental component of mathematics education at secondary and tertiary levels, often faces significant challenges in terms of student engagement and conceptual understanding.

Traditional teaching methods primarily rely on static representations and rote memorization, hindering students' ability to visualize and internalize abstract vector concepts. This disconnects between theoretical instruction and practical comprehension frequently leads to students struggling with spatial reasoning, vector operations, and applications in various scientific and engineering domains.

GeoGebra, an interactive geometry, algebra, and calculus application, offers a promising solution to these pedagogical challenges by providing dynamic visualization tools and interactive learning environments (Aytekin & Kiymaz, 2019).

However, there is a lack of comprehensive research examining the efficacy of using GeoGebra in enhancing the teaching and learning of vector algebra specifically (Ziatdinov & Valles, 2022). Integrating GeoGebra into vector algebra instruction could potentially transform traditional teaching methodologies, fostering a more intuitive and engaging learning experience.

This research aims to investigate the application of GeoGebra in teaching vector algebra, focusing on its impact on students' conceptual understanding, engagement, and overall academic performance. By conducting a systematic study involving a diverse cohort of students, this research will explore the extent to which GeoGebra can bridge the gap between abstract vector concepts and students' cognitive frameworks (Owusu, Bonyah, & Arthur, 2023).

The outcomes of this study will provide valuable insights into best practices for incorporating technology into mathematics education and contribute to the development of innovative instructional strategies that enhance learning outcomes in vector algebra.

3. Significance of the study

The major purpose of this study is to investigate the efficacy of using GeoGebra as a teaching tool in vector algebra instruction, which is aimed at enhancing the learning experience and improving comprehension of abstract mathematical concepts. By giving leverage to GeoGebra's interactive capabilities, the study seeks to improve students' understanding concepts of vector algebra when GeoGebra is used as an instructional tool in teaching vector algebra. Using various pedagogical approaches, which include inquiry-based learning, flipped classrooms, and collaborative problem-solving, the study aims to foster students' engagement and confidence in vector algebra. Ultimately, the study aims at evaluating the effectiveness of GeoGebra in improving learning outcomes and students' perceptions of using technology for mathematical visualization.

4. Objectives of the Study

1. To Investigate the Efficacy of using GeoGebra in Vector Algebra Instruction.
2. To determine whether GeoGebra dynamic visualization enhances the students understanding of vector algebra when it is used as a teaching tool for vector algebra instruction.
3. To explore students' perceptions and experiences through qualitative methods such as the use of questionnaires, to evaluate students' attitudes towards GeoGebra, their perceived effectiveness of instructional materials, and the extent to which GeoGebra help them in the understanding of vector algebra.

5. Limitations of the study

The study on the application of GeoGebra to teaching and learning of vector algebra has several limitations. Some of these limitations span from lack of a control group, could be influenced by the Hawthorne effect where participants or students may change their behaviour simply because they may be aware they are being studied, thereby influencing the results. Additionally, the research only provides short-term data, which involves a small and possibly homogenous sample. These limitations suggest the need for a more detailed experimental designs in future studies in order to draw definitive conclusions about its effectiveness.

6. Literature Review

Vector algebra is crucial in mathematics and applied across scientific and engineering disciplines. Recent research highlights alternative pedagogical strategies using dynamic mathematics software like GeoGebra to enhance teaching and learning of vector algebra. GeoGebra facilitates understanding of vector concepts such as addition, subtraction, scalar multiplication, dot and cross products, vector projection, and vector fields by allowing dynamic interaction with these concepts. It supports experimentation with vector operations, providing immediate feedback and reinforcing conceptual understanding. The software's interactive platform enables real-world application of vector algebra, aiding in geometric constructions and visualizations across various contexts, including physics and engineering. Studies by (Handhika & Sasono, 2021), (Zulnaidi, Oktavika, & Hidayat, 2020), and (Gono, 2016) emphasize the effectiveness of GeoGebra in fostering intuitive grasp and active learning. Geometric approaches and frameworks like Geometric Algebra (Berlinet, 2014; Turgut, Smith, & Andrews-Larson, 2022; Hestenes, 1990) also highlight the integration of geometry in teaching vector algebra. Advances in technology and virtual environments, as discussed by (Smith et al., 2023), enhance engagement and understanding by providing immersive, hands-on experiences. The constructivist approach supports using tools like GeoGebra for experiential learning, promoting deeper understanding of vector algebra (Berlyne, Piaget, & Piercy, 2003). GeoGebra's built-in tools for vector operations and its dynamic analysis capabilities (Aytekin & Kiymaz, 2019; Saralar-Aras, 2022) demonstrate its effectiveness in making abstract concepts accessible and engaging

7. Methodology

The study investigating the use of GeoGebra in teaching vector algebra employed a mixed-method approach to assess its effectiveness on students' understanding. The research integrated quantitative and qualitative methods to gain a comprehensive view of GeoGebra-enhanced instruction. The intervention involved using GeoGebra as the primary instructional tool, English as the medium of instruction, and administering a pre-test to assess baseline knowledge. The structured intervention spanned two weeks, with two sixty-minute sessions per week, focusing on activity-oriented, student-centered lessons delivered sequentially. The pre-experimental design used a one-group pretest-posttest method to evaluate changes in student understanding, with qualitative data collected through questionnaires to gain insights into students' perceptions. The target population was fifty randomly selected Form 3 science students from Sacred Heart Senior High School in Sunyani West, Ghana. Sampling involved drawing papers marked "YES" or "NO" to ensure an unbiased selection. Data collection included structured interactions, assessments, and questionnaires to evaluate student progress and experiences. Ethical considerations ensured informed consent, voluntary participation, fair se lection, and respect for participants' rights. The study aimed to bridge abstract algebraic formal ism with geometric intuition, enhancing conceptual understanding and engagement through GeoGebra.

8. Data Analysis

To evaluate the effectiveness of GeoGebra in teaching and learning of vector algebra within a pre-experimental design, we formulated and tested the following hypothesis:

• H0: There is no statistically significant difference between the pre-test and post-test mean scores of students after using GeoGebra in the teaching and learning of vector algebra.

• HA: There is statistically significant difference between the pre-test and post-test mean scores of students after using GeoGebra in the teaching and learning of vector algebra.

A single group of students were assessed with a pre-test before using GeoGebra and a post-test afterward. Paired (dependent) t-tests was conducted using Python to determine if there was a statistically significant difference between the pre-test and post-test scores. Descriptive statistics provided an overview of the data distribution, and Cohen's d was calculated to measure the effect size, indicating the practical significance of the intervention. SPSS version 25 was also used to analyse the student's questionnaire. The SPSS analysis revealed whether the use of GeoGebra significantly improved students' understanding of vector algebra, providing valuable insights into its instructional impact.

9. Results and Discussion

The instruction of vector algebra, through the utilization of GeoGebra software within a pre-experimental framework where a singular group engages in pre-test and post-test assessments, illustrates the capacity of this technology in improving students' grasp of concepts and procedures. The following figures; 1, 2 and 3 depict the pre-test and post-test results of two sampled student. Previously, the pre-test results highlighted difficulties in problem representation and a limited conceptual understanding of vector algebra. The students' responses indicated gaps in understanding the relationships between vectors and operations such as addition, subtraction, and scalar multiplication.

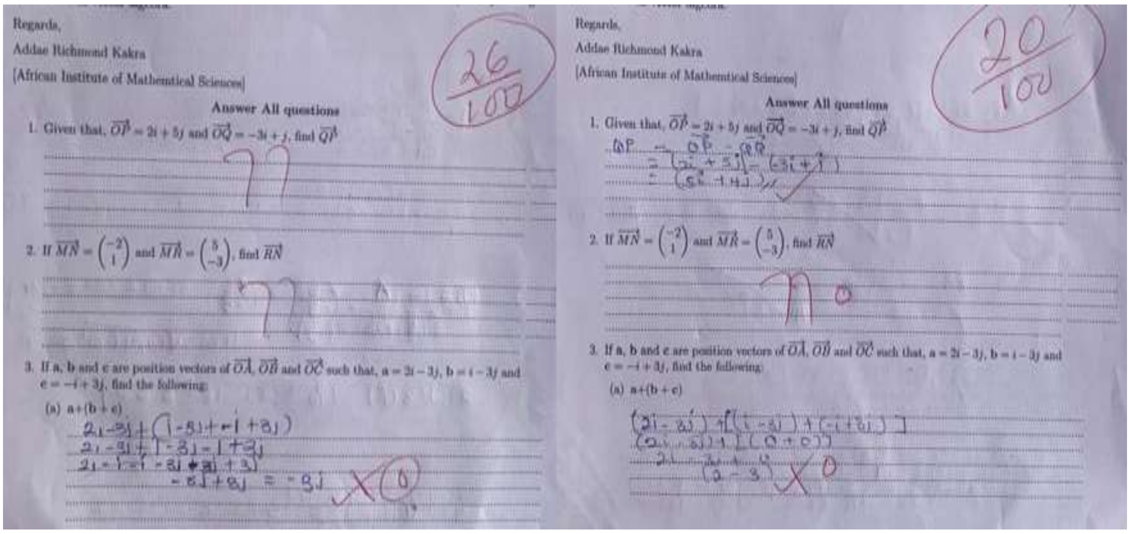

Figure 1: EG student showing initial knowledge during the pre-test.

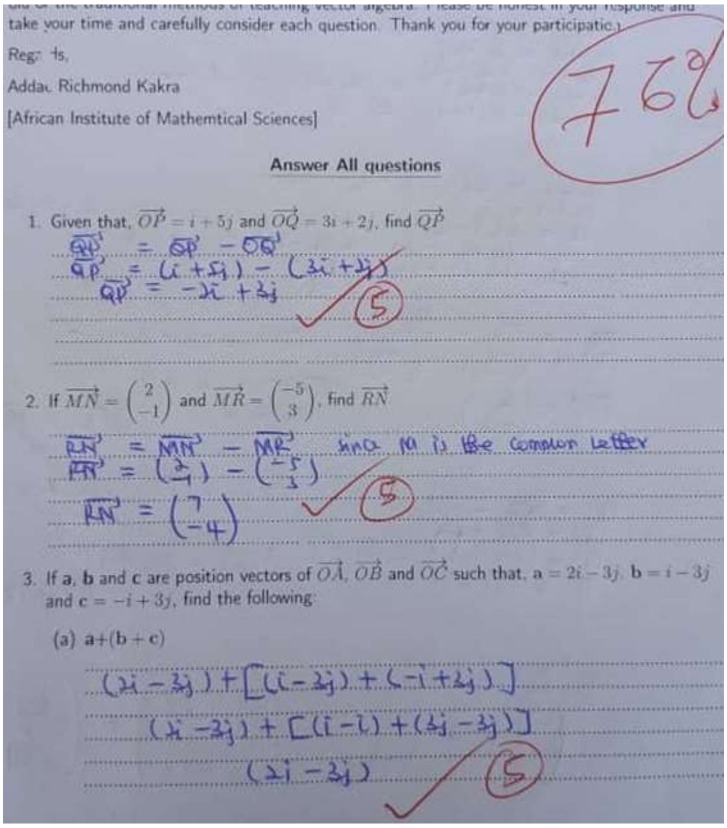

Figure 2: Student's post-test script showing post-intervention knowledge

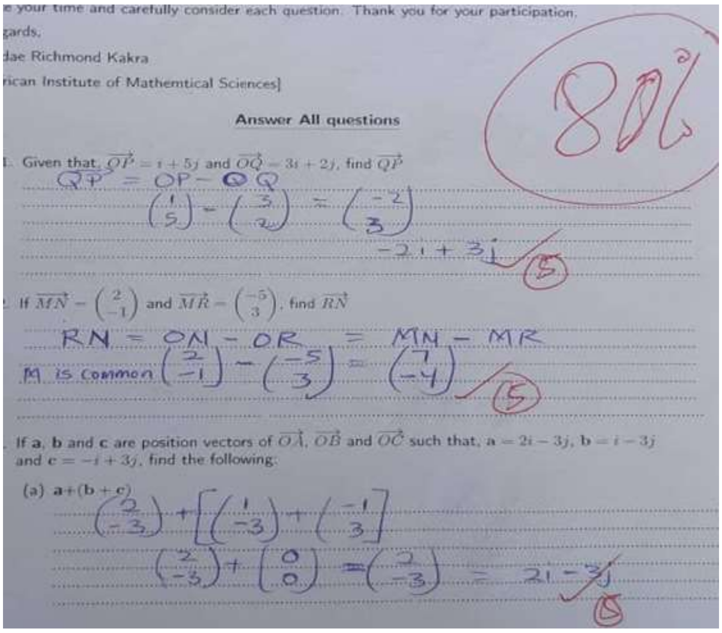

Figure3: Student’s post-test script showing post-intervention knowledge

From the GeoGebra facilitated intervention course, the post-test results show a massive improvement when GeoGebra was introduced as an instructional tool. The student demonstrated excellent problem-solving skills and a better conceptual understanding of vector algebra. The dynamic manipulation and immediate feedback provided by GeoGebra allowed the student to visualize vector operations more clearly and understand their properties more concretely. It is evident in Table 1 below that the students’ marks have improved, reflecting a great level of learning gains when the GeoGebra software was used in vector algebra instruction.

Table1: Shows the Pre-test and Post-test Scores for the students

Marks (%)	**Pre-test Results**	**Post-test Results**
0–9	8	0
10–19	13	0
20–29	8	1
30–39	6	4
40–49	5	6
50–59	4	8
60–69	3	13
70–79	2	9
80–89	1	6
90–99	0	3
Total	**50**	**50**

Table 2: Pre-test and Post-test Statistics

Statistic	Value
Pre-test Mean	29.9000
Pre-test Standard Deviation	21.2800
Post-test Mean	63.3000
Post-test Standard Deviation	16.9281

To determine whether there has been an improvement in the students' understanding in vector algebra when GeoGebra is used for vector algebra instruction, the researcher calculated the means and standard deviation for both tests. Table 2 above provides a comprehensive comparison of pre-test and post-test mean scores of the performance outcomes of the students before and after the GeoGebra-based instructional course.

The mean pre-test score was recorded as 29.90, which is lower than the post-test mean score of 63.30, demonstrating a major increase in the performance after using GeoGebra for teaching students vector algebra. In terms of the standard deviation, the pre-test scores recorded 21.28 and post-test 16.93, both indicating higher variability and diverse levels of students' understanding of vector algebra.

Overall, the data suggest a massive increase in the average scores with considerable high variability in student performance both before and after the GeoGebra-based instructional course in vector algebra.

Comparing students' performance in vector algebra with GeoGebra as an instructional tool, the study employed the paired t-test statistic to analyze the pre-test and post-test data. The relevance of the paired t-test is to help determine whether the intervention with GeoGebra-based instructional course has a statistically significant effect on the students' conceptual and procedural understanding in vector algebra under the following research hypothetical assumptions.

Hypothesis

H_0: There is no statistically significant difference between the pre-test and post-test mean scores of students after using GeoGebra as an instructional tool in teaching vector algebra.

H_a: There is statistically significant difference between the pre-test and post-test mean scores of students after using GeoGebra as an instructional tool in teaching vector algebra.

Table 3: Paired t-test results for Pre-test and Post-test Scores

Paired t-test Results	Value
Mean Difference	33.40
Standard Deviation of Differences	8.48
t-statistic	27.86
Degrees of Freedom	49
p-value	0.0000

According to Table 3 above, the paired t-test results show that, there exist statistically significant difference between the pre-test and post-test mean scores when GeoGebra was used as instructional tool for the teaching vector algebra. The t-statistic recorded 27.86, and the p-value is 0.0000, which is less than the conventional significance level of 0.05, indicating strong evidence the null hypothesis.

In conclusion, the results of the paired t-test suggest that the null hypothesis **H_0** should be rejected at an alpha level of 0.05 and **H_a** should be accepted, suggesting a 95% confidence level that the use of GeoGebra has a positive impact on the performance of the students.

Furthermore, the **Cohen's d analysis** was also performed to find out the effect size in the Pre-test and Post-test means score. **Table 4** below shows the analysis of the Cohen's d.

Table 4: Cohen's d Analysis for Pre-test and Post-test Scores

Statistic	Value
Mean diff between Post-test and Pre-test	33.4
Pooled Standard Deviation	19.2276
Cohen's d	1.7371

From the analysis of the Cohen's d in Table 4 above, the statistical interpretation indicates that the effect size of implementing GeoGebra in the teaching and learning of vector algebra is **very large**. With a Cohen's d value of **1.7371**, it suggests that there exists statistical difference between the pre-test and post-test mean scores of students after using GeoGebra for vector algebra instruction.

Moreover, the mean difference between the Pre-test and Post-test scores was **33.4**. This was accompanied by a relatively large standard deviation of **19.2276**, which supports the conclusion of the previous analysis that the implementation of GeoGebra as an instructional tool for vector algebra indeed has influence on the performance of students in vector algebra.

Analyzing students' perception on GeoGebra as instructional tool

Table 5: Gender and Age Distribution of the Respondents

Category	Frequency	Percent (%)
Gender		
Male	24	48.0
Female	26	52.0
Total (Gender)	50	100.0
Age		
16–19	50	100.0
Total (Age)	50	100.0

Table 5 above illustrates the gender and age distribution among a cohort of 50 individuals who participated in the research. Of the total sample, 24 were male students, constituting 48% of the cohort, while the remaining 26 were female students, representing 52%.

In terms of age, all 50 individuals are situated within the 16 to 19 age brackets, encompassing 100% of the cohort. This dataset underscores an almost equal division between males and females, suggesting that the entire cohort is composed of individuals in their late teenage years.

The efficacy of using GeoGebra as an instructional tool was assessed based on students' confidence in the understanding of vector algebra before using GeoGebra.

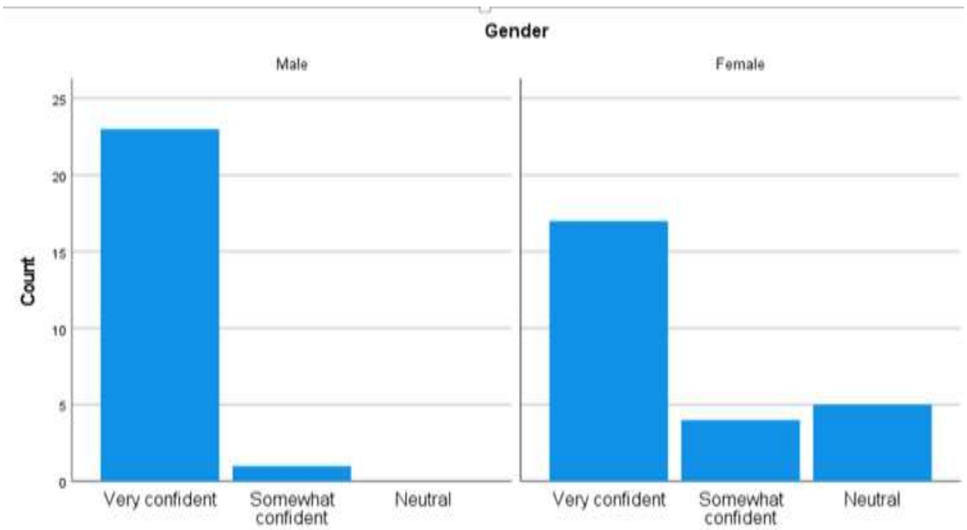

Figure 4: A bar graph illustrating the distribution of confidence levels in vector algebra concepts prior to the utilization of GeoGebra, categorized by gender.

Figure 4 depicts the level of confidence male and female students had regarding vector algebra concepts prior to utilizing GeoGebra. The chart shows that a majority of male students (22), representing 44%, and female students (18), also representing 36%, demonstrated high levels of confidence in their comprehension of concepts of vector algebra. An equivalent number of male and female students, 2 (4%) each, expressed some level of confidence, while a minority of students felt neutral, including 1 (2%) male and 3 (6%) females. On the whole, male students appeared marginally more assured than their female counterparts. The utilization of GeoGebra has the potential to bridge these confidence disparities and enhance overall student involvement and grasp of vector algebra principles.

After introducing GeoGebra as an instructional tool for the teaching of vector algebra, the researcher further assessed students' understanding of vector algebra concepts. This was when the researcher was interested in knowing whether the introduction of GeoGebra helped in vector algebra delivery effectively.

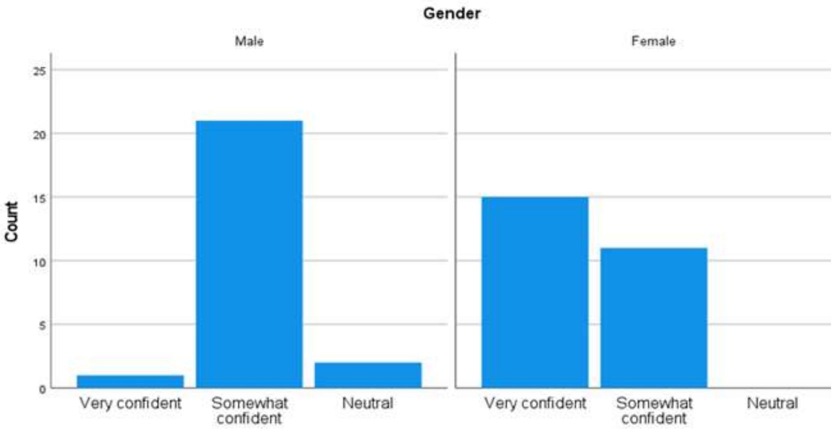

Figure 5: Bar Graph showing the distribution of males and females’ students who were confident in their understanding of vector algebra concepts after using GeoGebra

Figure 5 above delineates the levels of assurance in comprehending vector algebra principles amongst male and female students subsequent to utilizing GeoGebra. 21 male students, representing 87.5%, expressed a certain degree of confidence; 1, representing 4.2%, expressed high confidence; and 3, also depicting 12.5%, remained neutral.

Conversely, 16 female students, representing 59.3%, felt highly confident, 11, representing 40.7%, felt moderately confident, and none were neutral. A noticeable shift is observed in comparison to their initial confidence levels: male students mainly transitioned from "Highly Confident" to "Moderately Confident," suggesting a more accurate self-evaluation following in-depth interaction with the subject matter. Female students exhibited a substantial rise in the "Highly Confident" category, implying that GeoGebra might have been especially effective in enhancing their confidence. The neutral category experienced a decrease among male students and was absent among female students, indicating a trend towards more distinct levels of confidence.

In general, GeoGebra had a positive impact on students’ confidence in grasping vector algebra, with males embracing a more restrained self-assessment and females displaying heightened confidence.

Furthermore, the researcher was also interested in knowing whether or not GeoGebra dynamic visualization enhanced their understanding in vector algebra. The chart below depicts the answer for the research question.

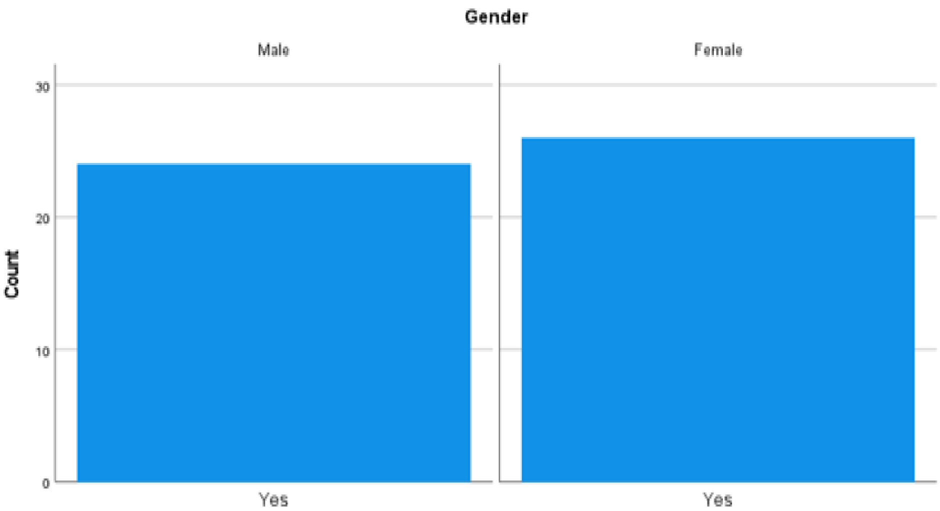

Figure 6: Bar Graph showing the males and females who agreed that GeoGebra dynamic visualization enhanced their understanding

Figure 6 below shows that, indeed, GeoGebra dynamic visualization enhanced students' understanding of vector algebra. This was when 22 (91.7%) of the respondents representing males responded positively to the fact that GeoGebra dynamic visualization really helped them understand vector algebra, with 27 (100%) respondents representing females supporting the same fact.

Finally, the research analyzed the students' experiences when GeoGebra was used as an instructional tool for vector algebra as compared to the old or traditional methods of teaching vector algebra. It was amazing to record a fascinating response. The chart below shows the response rate from the participants.

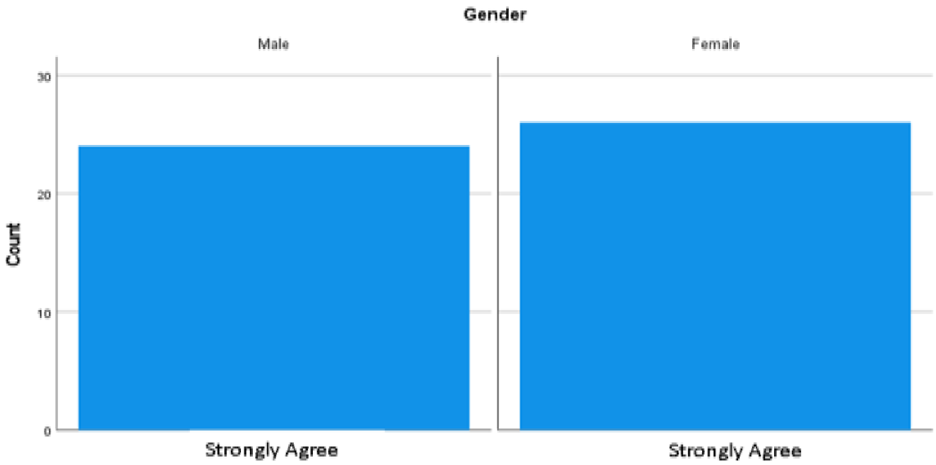

Figure 7: Bar Graph showing males and females who strongly agreed that GeoGebra helped them understand vector concepts better and described their experience with GeoGebra instruction as excellent

From Figure 7 above, it is evident that participants strongly agreed to the fact that their experiences with GeoGebra for vector algebra instruction were excellent compared to the traditional methods of vector algebra instruction. 91.7% representing 22 males and 100% representing 27 females support the assertion that GeoGebra actually enhanced their understanding of vector algebra and there is a need for it to be implemented in our curriculum and mathematics education.

10. Conclusion

The findings from the analysis and the survey responses shed more light on the effectiveness and perception of using GeoGebra as an instructional tool in teaching vector algebra. From the massive increase in average scores in the pre-test to the post-test, the paired t-test results showed a statistically significant difference, indicating that GeoGebra has a significant impact on student performance after it has been used to teach vector algebra.

The effect size from Cohen's d analysis further supports this conclusion. The survey responses revealed very strong perceptions among students regarding GeoGebra's utility. A greater number of respondents responded positively and acknowledged the benefits of dynamic visualization, while only a few others remained neutral.

In general, it is evidently clear that GeoGebra is indeed an educational tool for teaching vector algebra, which needs to be implemented in our mathematics education at all levels.

11. Recommendations

From the findings, the following recommendations were made with regard to the application of GeoGebra to the teaching and learning of vector algebra:

1. The researcher advocates that comprehensive training and support for educators is key to effectively integrating GeoGebra into all teaching practices.
2. The researcher proposes that a wider range of instructional materials, including interactive tutorials, step-by-step problem-solving guides, and more varied exercises, should be developed in order to cater to all diverse learning styles and preferences.
3. The research also pleads with all stakeholders involved in the design and implementation of the mathematics curriculum to consider the integration of GeoGebra as an instructional tool and as an ICT tool to enhance teaching—not only in mathematics but also in all disciplines that may require the use of this remarkable software to practically explain concepts more effectively.

By adhering to these recommendations, educators, stakeholders, and institutions can harness the full potential of GeoGebra as a powerful tool for teaching vector algebra, which will eventually enhance student learning outcomes and experiences.

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13. Appendices

Appendix A: Proposed Intervention

A two-week GeoGebra-based intervention course was taught after the pre-test with two visits each week. Each visit lasted for sixty minutes. A variety of activities, discussions, and strategies were employed during the intervention. Lessons were activity-oriented and focused on a teacher-to-student-centered approach.

WEEK ONE Activity 1A

Topic	Vector Algebra
Sub-Topic	
Objective	By the end of the visual lesson, students will be able to identify, familiarize, and use the tools on GeoGebra menu bar effectively for their intended works on Vector Algebra and more.
T.L.M.	GeoGebra Classic Software
Duration	Sixty minutes
Procedure	Click on the link below and perform the following activity https://www.geogebra.org/m/fveugaet.

WEEK ONE Activity 1B

Topic:	Vector Algebra

Sub-Topic:	Using GeoGebra to define a vector, zero or null vector, unit vector, addition and subtraction of vectors, parallel vectors, and scalar multiplication of vectors.
Objective:	1. Students will use GeoGebra to appreciate that vectors could be represented as directed line segments. 2. Students will use GeoGebra to recognize and accept that the zero vector has a direction even though it is indeterminate. 3. Students will visualize and calculate unit vectors, grasping their role in indicating direction in vector spaces. 4. Students will understand vector addition and subtraction, recognizing the geometric interpretation of these operations. https://www.geogebra.org/m/htgyxdap for Addition and https://www.geogebra.org/m/vhdvwwxr for Subtraction 5. Students will explore parallel vectors using GeoGebra, identifying vectors that have the same or opposite direction with similar or different magnitudes. 6. Students will practice scalar multiplication of vectors using GeoGebra, understanding how multiplying a vector by a scalar affects its magnitude and direction. https://www.geogebra.org/m/k2huzabx
T.L.M.:	GeoGebra Classic Software
Duration:	Sixty minutes.
Procedure:	Click on the link and perform the following activity**:** **https://www.geogebra.org/m/rudat6xp**

WEEK TWO Activity 1A

Field	Details
Topic	Vector Algebra
Sub-Topic	The position vector of a point that divides a given straight line in a given ratio; uniqueness of the centroid.
Objective	1. To demonstrate how to divide a straight-line segment in a given ratio with GeoGebra. Students are allowed to experiment with different ratios and observe how the division point changes, using the formula: $P(x, y) = (mx_2 + nx_1) / (m+n), (my_2 + ny_1) / (m+n)$ https://www.geogebra.org/m/gudssxdf 2. Show how the position vector of the point dividing the line segment is related to the centroid of the segment, using GeoGebra to illustrate with examples. https://ggbm.at/vmdudgbv 3. Present students with geometric problems involving straight lines, ratios, and centroids. Encourage them to use GeoGebra to visualize the problems and explore solutions. https://ggbm.at/tqzcvzgc 4. Provide students with practice problems to solve using GeoGebra, reinforcing understanding of position vectors, line division, and centroids.
T.L.M.	GeoGebra Classic Software
Duration	Sixty minutes

Procedure	1. Discuss the Ratio Theorem using GeoGebra: $\vec{P} = (n\vec{A} + m\vec{B}) / (m+n)$ This formula calculates the position vector $\vec{P}$ dividing segment AB in a ratio m : n. https://www.geogebra.org/m/t98cvstq 2. Explore the Uniqueness of the Centroid using GeoGebra: For a convex polygon with n vertices $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, the centroid (x_c, y_c) is calculated as: $x\_c = (x_1+x_2+\ldots+x_n) / n$ $y\_c = (y_1+y_2+\ldots+y_n) / n$ These ensure a unique centroid for the figure. https://www.geogebra.org/m/entvwjr4

Appendix B: Student Questionnaire

Introduction and Instructions

Thank you for taking the time to complete this questionnaire. The purpose of this survey is to assess the efficacy of using GeoGebra in vector algebra instruction. Your feedback is crucial in helping the researcher improve the use of GeoGebra in educational settings. Please read each question carefully and provide your responses based on your experiences. For questions with multiple-choice options, please select the option that best reflects your opinion or experience by ✓. For open-ended questions, write your responses in the space provided.

Your responses will remain confidential and will only be used for research purposes. Thank you for your participation!

Demographic Information:

1. Gender:
(a) □ Male
(b) □ Female

2. Age:
(a) _____

3. Previous Experience with GeoGebra:
(a) □ None
(b) □ Beginner
(c) □ Intermediate
(d) □ Advanced

Objective 1: Investigate the Efficacy of using GeoGebra as an instructional tool

4. Please indicate your level of agreement with the following statement: "The use of GeoGebra in vector algebra instruction helped me understand vector concepts better."
(a) □ Strongly Agree
(b) □ Agree
(c) □ Neutral
(d) □ Disagree
(e) □ Strongly Disagree

5. Before using GeoGebra, how confident were you in your understanding of vector algebra concepts?
(a) □ Very confident
(b) □ Somewhat confident
(c) □ Neutral
(d) □ Not very confident
(e) □ Not confident at all

6. After using GeoGebra, how confident are you in your understanding of vector algebra concepts?
(a) □ Very confident
(b) □ Somewhat confident
(c) □ Neutral
(d) □ Not very confident
(e) □ Not confident at all

7. Did the dynamic visualizations provided by GeoGebra enhance your understanding of vector algebra concepts?
(a) □ Yes
(b) □ No

Objective 2: Develop GeoGebra-Based Instructional Materials

8. Which of the following GeoGebra-based instructional materials did you find most helpful in learning vector algebra concepts? (Select all that apply)
(a) □ Interactive diagrams
(b) □ Simulations
(c) □ Activities/exercises

9. How would you rate the usability and accessibility of the GeoGebra-based instructional materials provided in your vector algebra instruction?
(a) □ Excellent
(b) □ Good
(c) □ Fair
(d) □ Poor

10. Do you feel that the GeoGebra-based instructional materials effectively facilitated your understanding of vector algebra concepts?
(a) □ Yes
(b) □ No

Objective 3: Explore Student Perceptions and Experiences

11. Overall, what are your thoughts on using GeoGebra for learning vector algebra concepts?
(a) □ Very positive
(b) □ Somewhat positive
(c) □ Neutral
(d) □ Somewhat negative
(e) □ Very negative

12. How would you describe your experience with GeoGebra-based vector algebra instruction compared to traditional methods (e.g., lectures, textbooks)?
(a) □ Excellent
(b) □ Good
(c) □ Fair
(d) □ Poor

13. What suggestions do you have for improving the use of GeoGebra in vector algebra instruction?
(a) □ Provide more interactive tutorials
(b) □ Include step-by-step problem-solving guides
(c) □ Offer more varied types of exercises and activities
(d) □ Improve the user interface for easier navigation
(e) □ Increase the availability of online resources and support
(f) □ Incorporate more real-world applications and examples

Appendix C: Pre-Test Questions

Thank you for participating in this survey! Your feedback is invaluable in helping us understand the effectiveness of GeoGebra in vector algebra instruction.

Dear Students,

This is a pre-test designed to assess your current understanding and difficulties in vector algebra. Please be honest in your responses because this will help the researcher address your learning needs. Answer the following questions to the best of your ability. Your responses will remain anonymous. Please take your time and carefully consider each question.

Thank you for your participation!

Regards,

Addae Richmond Kakra

1. Given that $\vec{OP}$= 2i + 5j and $\vec{OQ}$= −3i + j, find $\vec{QP}$.
2. If $\vec{MN}$= (−2 1) and $\vec{MR}$= (5 −3) find $\vec{RN}$
3. If **a, b**, and **c** are position vectors of $\vec{OA}$, $\vec{OB}$ and $\vec{OC}$ such that, a = 2i−3j, b = i−3j,

and c = −i+3j, find the following:

(a) a + (b + c)

(b) (a + b) + c

(c) a + b

(d) b - (a + c)

4. The coordinates of PP and QQ are (5, −2) and (3, −6) respectively. Find |$\vec{PQ}$|.
5. Given that $\vec{AB}$=3i−3j and $\vec{PA}$=−i+2j, find |$\vec{PB}$|.
6. Given that q = (2 5) and r = (4 1), find 2q+3r2q + 3r.
7. If r =3i−4jr and s =2i−5js, find |2s - 3r|.
8. △ XYZ is a scalene triangle with vertices X (1, 3), Y (4,2), and Z (6,5). Determine its centroid.
9. Suppose we have a line segment with endpoints A (2, 4) and B (6, 8) in 2D space. Find the position vector P of a point (x, y) that divides the line AB in the ratios 2:1 and -2:1.
10. Given a vector **v** = 3i−4j in a two-dimensional Cartesian coordinate system:

(a) Find the magnitude of v.

(b) Determine the unit vector in the direction of v.

(c) Express v in terms of its unit vector.

11. Construct parallelogram ABCD that A is at the origin. Express the position vectors of the other vertices in terms of vectors u and v, where B = u, D = v, and C = u + v.
    a. Find the midpoint M of the diagonal AC. What is the position vector of M in terms of u and v?
    b. Find the midpoint N of the diagonal BD. What is the position vector of N in terms of u and v?
    c. Show that the midpoints M and N found in (a) and (b) are the same. Use the position vectors to verify that M=N.
    d. Explain why the result from (c) implies that the diagonals of a parallelogram bisect each other.

12. Can vectors with different dimensions be added?
    a. Yes
    b. No
    c. Sometimes
    d. Only if they have the same magnitude

Answer: ....

13. What is the effect on a vector if it is multiplied by a positive scalar?

(a) The direction changes

(b) The magnitude decreases

(c) The magnitude increases

(d) The vector becomes zero

Answer: ....

14. What is the effect on a vector if it is multiplied by a negative scalar?

(a) The direction reverses

(b) The magnitude decreases

(c) The magnitude increases

(d) The vector becomes zero

Answer: ....

15. What happens to a vector if it is multiplied by zero?

(a) The direction changes

(b) The magnitude remains the same

(c) The vector becomes zero

(d) The vector becomes undefined

Answer: ....

16. When are two vectors considered equal?

(a) When they have the same direction

(b) When they have the same magnitude

(c) When they have the same magnitude and direction

(d) When they are in the same plane

Answer: ....

17. What are parallel vectors?

(a) Vectors with the same magnitude

(b) Vectors that have the same or opposite direction

(c) Vectors that lie in the same plane

(d) Vectors that intersect at right angles

Answer: ....

18. According to the triangle law of vector addition, if two vectors aa and bb are represented as two sides of a triangle taken in order, what does the third side represent?

(a) The difference of aa and bb

(b) The sum of aa and bb

(c) The cross product of aa and bb

(d) The scalar product of aa and bb

Answer: ....

19. What is a zero vector?

(a) A vector with a magnitude of one

(b) A vector with an undefined direction

(c) A vector with zero magnitude and undefined direction

(d) A vector with zero magnitude but defined direction

Answer: ....

20. The zero vector is:

(a) A vector with zero magnitude and an undefined direction.

(b) A vector with zero magnitude and a fixed direction.

(c) A vector with unit magnitude and an undefined direction.

Answer: ....

Appendix D: Post-Test Questions

Dear Students,

This is a post-test question designed to assess your understanding after GeoGebra has been used to teach vector algebra. The results of this post-test will inform the researcher whether or not the application of GeoGebra to the teaching of vector algebra has really improved on the old or the traditional methods of teaching vector algebra.

Please be honest in your response and take your time and carefully consider each question.
Thank you for your participation.

Regards,
Addae Richmond Kakra

Answer All Questions

1. Given that OP=i+5j and OQ =3i+2j , find QP.

2. If vector MN=(2 −1) and MR=(−5 3), find vector RN.

3. If a, b, and c are position vectors of OA, OB and OC respectively, such that:
a = 2i−3j, b = i−3j, and c = −I + 3j, find the following:

(a) a+(b + c)a

(b) (a + b) + c(a+ b)

(c) a+ b − (b+ a)

(d) b - 2ab−(a +c)+b−2a

4. The coordinates of P and Q are (-5, -3) and (−3, −6) respectively. Find |PQ|.
5. Given that vector AB =i−3j and PA=−5i+2j, find |PB|.
6. Given that q = (2 −9) and r= (8 −10), find −23q+3r-2/3q

8. △XYZ is a scalene triangle with vertices X(-1, 6), Y(8, -3), and Z(−6,−5). What is its centroid?
9. Suppose we have a line segment with endpoints A(2, 4) and B(6, 8) in 2D space. Find the position vector P(x, y) that divides the line AB in the ratio:

(a) 2 : 1

(b) -2 : 1

10. Construct a parallelogram ABCD such that A is at the origin. Express the position vectors of the other vertices in terms of vectors u and v, where B= u, D = v, and C = u + v.

(a) Find the midpoint M of the diagonal AC. What is the position vector of M in terms of u and v?
(b) Find the midpoint N of the diagonal BD. What is the position vector of N in terms of u and v?