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Transformer Tap Optimization Using Gradient-Based Polar Bear Algorithm for Loss Reduction in EV-Integrated Distribution Systems

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ABSTRACT:

The rapid evolution of electric power systems, driven by the increased penetration of electric vehicles (EVs) and distributed energy resources (DERs), poses considerable operational and planning challenges. One of the most critical issues faced by modern distribution networks is the rise in power losses and voltage instability due to highly unpredictable and non-linear load behaviour. These challenges are exacerbated by the stochastic nature of EV charging and the variability of renewable energy generation, such as solar photovoltaic (PV) systems. Therefore, optimizing the operational parameters of distribution systems under such dynamic conditions has become a paramount concern for engineers and researchers. This project proposes a novel and intelligent approach for power loss minimization and voltage profile enhancement in radial distribution systems through the application of the Gradient-Based Polar Bear Optimization (GB-PBO) algorithm. Inspired by the adaptive hunting strategies of polar bears in Arctic conditions, the PBO algorithm is extended with gradient-based local search capabilities to improve its convergence speed and solution accuracy. The hybrid GB-PBO algorithm is employed to optimize the tap settings of transformers, which are instrumental in regulating voltage levels and redistributing power flows efficiently. The proposed method is evaluated against traditional optimization techniques like Particle Swarm Optimization (PSO) and Modified PSO (MPSO). The results demonstrate that the GB-PBO algorithm outperforms its counterparts by achieving a significant reduction in active and reactive power losses, while simultaneously improving the voltage stability across the network. Specifically, transformer tap settings optimized by GB-PBO led to a real power loss reduction of over 10% compared to the base case, and improved voltage profiles with minimum voltage levels being maintained within acceptable per unit (p.u.) limits.

Keywords: Distribution System, Electric Vehicle Charging Station, Loss Reduction, Polar Bear Optimization, Voltage Improvement.

1. Introduction

Many losses, such as distribution, transmission, and demand losses, can have an impact on the system. Load flows, sampling techniques and regression analysis are used to minimize these losses. A power system is associated with various constraints, such as generation equipment, transmission control, utilization-based control, and classic service provided to users. During this process, some type of loss is affected, as previously stated. In the broad sense, in India's electrical systems, 20-25 % of distribution losses occur; to reduce these losses, two technologies are available: network reconfiguration and optimal location of Distribution Generation (DG). The distribution system creates a better path for power usage from the generation station to the end customer. Feeder topology is used for network reconfiguration, and tie switches in the feeder topology operate at emergency/normal conditions of open/close switch configuration. Similarly, how various techniques are used for network reconfiguration, which is very effective and used to balance the load? When the normal functioning of the network reconfiguration follows the criteria below, the first rule is that the distribution power the system maintains stability and reliability, which usually improves exploit equipment in the system and, as a result, enhances system overabundance However, because the above rule has the disadvantage of increasing system costs, analysis methods are employed to counter the issue. To do so, take the reliability index and evaluate the radial network's reliability estimation. Second, load balance has been maintained during the system reconfiguration. Practically, the network is dynamic, with loads fluctuating up and down. However, distribution network reconfiguration is used to transfer power from a massive load to a fewer load, reducing the overload effect and maintaining a proper voltage profile. Power losses are also reduced throughout this process. The distribution system can provide power to consumers, but it suffers from power losses. Engineers must and should make every effort to reduce losses; in the system, all equipment is employed to maintain high efficiency; but, while the power system is operational, it increases costs. As a result, several criteria are used to reduce losses, such as network reconfiguration, DG placement, and capacitor installation. The next technology is optimal distribution generation (DG), which is one of the reasons for increasing renewable energy units as fossil fuels become scarcer. Why are DGs so popular? Because of their benefits, such as improved voltage and reduced real and reactive power losses. Because of this, the DG's reliability and efficiency have improved. When selecting the DG, two criteria such as the location and size of the DG are affected. Installation of DGs and capacitors in the power system provides significant benefits to the distribution system since high-quality power can be transmitted to customers easily, and the system's performance is enhanced. In addition to reducing losses, and cost, and preserving the environment during the integration of power plants, distribution generation also increases voltage and system stability. That is why DG placement and size are more significant; in essence, it is entirely

dependent on analytical approaches, such as the loss formula, which is used to compute the ideal DG allocation. To determine the optimum size and position of DGs, various analytical methods are used such as Genetic Algorithm (GA), Differential Evolution Algorithm (DEA), Particle Swarm Optimization (PSO), and Exchange Market Algorithm (EMA). Consumers essentially demand high-quality power from the distribution system, but this is virtually impossible due to power loss and low voltage gain. Capacitor placement to compensate for reactive power is an enormous way for minimizing the above problem. Shunt capacitors are extensively used in distribution systems to reduce real and reactive power losses while also improving voltage profiles. Shunt capacitors can be installed using a variety of strategies, including the cuckoo search algorithm, the firefly algorithm, and meta-heuristics techniques.

2. Illustrations

The part of the power system that distributes electric power for local use is called as distribution system. Generally, a distribution system is the electrical system between the substation fed by transmission system and the consumer's meters. A drop in voltage levels results when demand for electricity exceeds the capacity of the distribution system. The contemporary power system structure demands adaptability to efficiently control and maintain power flows. The adoption of dual carbon strategic policies has substantially increased the integration of Distributed Generators (DGs), such as wind and photovoltaic (PV) systems, into the distribution network (DN) [1]. However, the outputs of DGs are inherently inconsistent and intermittent, which can lead to grid instability. The integration of DGs transforms the grid from a passive energy receiver to an active energy exchange platform, resulting in grid operational challenges and impacting optimal power flow. This disturbance in optimal power flow contributes to power losses within the DN [2].

The models of distribution networks can be categorized as conventional, nonlinear, and linear. Conventional power flow models are intricate, making them less suitable for estimating optimal power flow rapidly, while linear models offer a more efficient alternative. The intermittent nature of DGs, coupled with uncertainties in energy demand and variable load characteristics within the active distribution network, often results in voltage instability, frequency disturbances, and power losses at various network nodes. To address these challenges, it becomes imperative to optimize the optimal power flow within the active distribution network. Optimizing the optimal power flow in the active distribution network can be achieved through various methods, including the incorporation of shunt capacitance, strategic placement. This paper specifically focuses on the optimization of transformer tap settings to attain optimal power flow within the active distribution network. Various heuristic optimization techniques, such as Particle Swarm Optimization (PSO), MPSO, and Gradient based Polar Bear Optimization algorithm to optimize transformer tap settings, thereby enhancing the maintenance of optimal power flow within the active distribution network. Minimization of power losses is a major challenge in distribution system, as it can lead to significant financial loss for utilities and reduced reliability for most consumers. Tap changing transformer is one the most effective ways to minimize power losses in distribution system. The minimization of power loss in distribution addressed by various optimization techniques like, particle swarm optimization (PSO), Modified particle swarm optimization (MPSO), Polar bear gradient based optimization (PB-GBO).

3. Proposed Methodology

Polar Bear Optimization (PBO) is an optimization algorithm rooted in the natural hunting behavior of polar bears. Taking cues from how polar bears hunt and survive in icy environments, PBO leverages these strategies to solve complex optimization problems. By mimicking the adaptive and strategic behavior of polar bears, PBO offers a unique and effective approach to problem-solving. It was proposed by Abbaszadeh et al. in their research paper titled "Polar Bear Optimization: Algorithm and Applications" [9]. The algorithm imitates the foraging behavior of polar bears in the Arctic region to solve optimization problems efficiently. The Polar Bear Optimization (PBO) algorithm, drawing inspiration from the hunting behavior of polar bears, has shown promising results in solving various optimization problems. However, like any optimization algorithm, PBO has certain limitations and drawbacks that need to be considered. Here are a few drawbacks of PBO:

- Premature Convergence: PBO may suffer from premature convergence, where the algorithm settles into a suboptimal solution prematurely, without exploring the entire search space. This can result in suboptimal solutions and limit the algorithm's ability to find the global optimum.
- Limited Exploration: PBO's hunting behavior is based on the movement of polar bears on ice floes, which may limit the exploration
 capability of the algorithm. The search process may become constrained, leading to a reduced ability to discover diverse and globally
 optimal solutions.



Figure 2: Polar Bear Hunting Behaviour Modelled as an Optimization Strategy.

To address these limitations, here the paper developed a variant of PBO called Gradient-based Polar Bear Optimization (GB-PBO) and implemented it in your MATLAB code. GB-PBO incorporates gradient descent optimization within the PBO framework to enhance the algorithm's exploration and exploitation capabilities. Here's why you chose GB-PBO:

- Enhanced Exploration: By integrating gradient descent optimization, GB-PBO allows for more effective exploration of the search space. The gradient information guides the search towards promising regions, increasing the chances of discovering better solutions.
- Improved Convergence: The incorporation of gradient descent optimization in GB-PBO helps in refining by incorporating solutions and refining the search process, the Gradient-Based Polar Bear Optimization algorithm enhances convergence speed and solution quality, surpassing traditional PBO approaches. This advancement enables more efficient optimization and improved outcomes.
- Reduced Premature Convergence: GB-PBO's ability to perform local search using gradient descent optimization helps mitigate the issue of
 premature convergence. It allows the algorithm to escape local optima and continue exploring the search space, leading to better overall
 performance.

Gradient Based Polar Bear

Algorithm Steps:

- 1. Initialize the problem parameters, including the number of particles (Npar), the benchmark function (FN), the function number (FunNumber), and the variable bounds (VarLow and VarHigh).
- 2. Set the number of PBO iterations (PBOIterations), the number of GD iterations (GDIterations), and the step size (StepSize).
- 3. Set the initial best cost (BestCost) to infinity and the best solution (BestSolution) to an empty array.
- 4. Initialize the population by randomly generating individuals within the variable bounds. Evaluate the cost of each individual using the benchmark function and update the best cost and best solution if necessary.
- 5. Enter the main loop for PBOIterations iterations.
- **6.** For each individual in the population, perform GDIterations iterations of gradient descent optimization. Compute the gradient of the cost function using the compute Gradient function. Update the position of the individual using the step size and the gradient.
- 7. Apply bounds to the updated position to ensure it stays within the variable bounds.
- 8. Evaluate the cost of the updated position using the benchmark function.
- 9. Update the personal best if the cost of the updated position is better than the current best cost.
- **10.** Repeat steps 6-9 for all individuals in the population.

- 11. Repeat steps 5-10 for PBOIterations iterations.
- 12. Output the final best cost (BestCost) and the corresponding best solution (BestSolution).



Figure 3: Modified radial distribution system.

Here the single detached objective to abbreviate the power losses, i.e., real & reactive power losses. The objective function is characterized as follows $\min F(x) = \frac{1}{f(x)}$; $f(x) \neq 0$

In this study, the design variables considered are edge present, integrate service, and transformer tap settings. These variables play a crucial role in optimizing the performance of the system. It is important to note that the tap settings have specified minimum and maximum limitations, which are set at 0.9 and 1.10 per unit (P.U.), respectively. These limits ensure that the tap settings remain within a feasible range during the optimization process. By incorporating these design variables and constraints, the study aims to achieve an optimal solution for the system under consideration.

The IEEE 15 bus radial distribution system is used as a testing bus system. In this radial distribution system finding both real and reactive power losses & increase the voltage profile of the given bus system.



Figure 4: single line diagram of 15 bus distribution system.



Figure 5: Voltage Profile of Distribution System.

The voltage profile for the IEEE 15-bus radial distribution system under base case conditions is illustrated in Fig. 5.1. This scenario represents the system without the incorporation of any optimization techniques, distributed generation (DG), or electric vehicle (EV) charging stations. From the graph, it is evident that the system experiences a gradual voltage drop as power is transferred from the source (bus 1) to the remote buses. The lowest voltage is observed around bus numbers 13 to 15, indicating critical voltage dips near the tail end of the feeder. This voltage decline is typical in radial systems, especially when reactive power compensation or voltage regulation is not actively employed. The voltage magnitudes remain within the acceptable per unit (p.u.) range (i.e., 0.94–1.0 p.u.), but buses toward the end exhibit values close to 0.944 p.u., which can lead to voltage instability, reduced equipment efficiency, and potential under voltage problems for sensitive loads.

Voltage dependent load modelling helps to reduces the real and reactive power losses and improves the voltage profile of the buses in the distribution system as shown in figure 6.



Figure 6: Voltage profile of distribution system with voltage dependent load modelling.

Voltage dependent load modelling introduces a more realistic representation of load behaviour by accounting for the fact that power consumption varies with the bus voltage magnitude. Unlike constant power loads, voltage-dependent models reflect actual load dynamics under varying voltage conditions—particularly in residential, commercial, and industrial loads. As observed in Fig. 6, this modelling approach results in a smoother and slightly elevated voltage profile across the 15-bus system compared to the standard base case (Fig. 5). The lowest bus voltage has improved marginally, indicating better voltage regulation at far-end buses.

Comparison of voltage profile between constant power load modelling & voltage dependent load modelling in the fig 7.



Fig 7: Comparison between Standard Distribution System and Voltage Dependent Load Modelling

From the bar graph, it is evident that voltage-dependent load modelling consistently yields a slightly improved voltage profile across almost all buses compared to the standard constant power load model. The green bars (voltage dependent) are marginally higher than the blue bars (power dependent), indicating better voltage regulation. Voltage dependent modelling better reflects real-world conditions, where loads such as lighting, motors, and electronic devices adjust power consumption based on supply voltage. This realistic modelling leads to reduced current draw, resulting in lower line losses and less voltage drop. The improvement in voltage profile enhances system reliability, reduces the risk of under voltage conditions, and lowers energy loss. The comparison clearly demonstrates the advantage of using voltage dependent load modelling over conventional constant power load assumptions.

After placement of PV system in the distribution system with different size of compensation at various buses and the PV system (DG) placements reduces the real power losses majorly and slightly decreases the reactive power losses and relatively increases the voltage profile at buses as shown in fig 8.





In this scenario, Electric Vehicle (EV) charging stations are introduced at selected buses, which contributes to increased loading and voltage drops, especially at mid-network nodes. The graph clearly highlights a low-voltage cluster around buses 6, 7, and 8, which is attributed to the presence of EV charging stations. These loads tend to be dynamic, high in magnitude, and often uncoordinated, causing localized voltage depressions.

To counteract the negative impact of EV integration, Photovoltaic (PV)-based Distributed Generation (DG) units are strategically installed at multiple buses with varying levels of compensation. These DGs are configured to generate real power, which reduces the burden on upstream feeders, improves local voltage conditions, and indirectly contributes to reactive power compensation. While the EV load initially caused severe voltage dips, the introduction of DG at key locations mitigated the depth of those sags. Buses 6 to 8 show some recovery in voltage profile, although they remain slightly lower than the rest of the system—indicating a partial compensation due to DG sizing or location limitations. The voltage at most other buses has improved compared to the base case and EV-only cases, demonstrating the benefits of localized power generation.





The placement of DGs, particularly PV-based units, is effective in:

- Alleviating voltage drops caused by EV charging stations,
- Reducing overall system power losses,
- Improving voltage profile across the distribution network,
- Enhancing reliability and load support under growing EV penetration.

However, to achieve optimal performance, DGs must be appropriately sized and strategically placed, especially in zones experiencing high EV load concentrations. This forms the basis for advanced optimization using techniques like Polar Bear Optimization (PBO), which is explored in the next section. Base case voltages (green) are consistently higher across all buses, indicating better performance under standard load modelling. EV-impacted buses (blue), particularly bus 6 to bus 8, show noticeable voltage sag, with voltage dropping close to 0.935 p.u., indicating marginal operation close to acceptable limits. The most critical voltage degradation occurs at the midpoint buses, where EV stations are connected, confirming localized stress and

load imbalance introduced by EV charging. This comparative analysis in Fig. 9 demonstrates the voltage stress induced by EV integration, clearly evident when contrasted with the base case performance. The system requires adaptive solutions—such as DG integration and transformer optimization—to maintain voltage within safe operating limits. This section bridges the understanding between the problem (EV impact) and the solution (optimization strategies), which is further explored in the following section with PBO-based tap transformer regulation.



This is the final losses after calculating power losses in the distribution system 45.5539 +31.1003i. The graph illustrates the optimization process carried out using the Polar Bear Optimization (PBO) algorithm to determine the optimal tap settings of distribution transformers in the IEEE 15-bus system. The primary objective of this process is to minimize real power losses in the network while maintaining voltage stability and operational reliability. The initial real power loss starts at approximately 43.75 kW. Over 100 iterations, the loss value gradually decreases, stabilizing around 43.10 kW by the 60th iteration. After iteration 60, the algorithm converges, indicating that an optimal solution has been found, and further iterations do not result in meaningful improvements. The convergence trend demonstrates step-wise improvements, characteristic of metaheuristic algorithms exploring and refining solutions over time.

CASES	PL (KW)	QL (KVar)
Standard Distribution System	61.7873	57.29065i
Voltage Dependent Load Modeling	56.0592	44.4975i
Placement of DG&EV	36.4113	32.6706i
PBO based on T/F Tap Settings	45.5539	31.1003i

Table I: Comparison of Real & Reactive Losse
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Table I presents a comparative overview of the real and reactive power losses under four different operating conditions in the IEEE 15-bus radial distribution system. In the base scenario of the standard distribution system without any optimization, the real and reactive power losses are observed to be 61.7873 kW and 57.2906 kVAR, respectively. These high loss values indicate the inefficiency and voltage instability inherent in an uncompensated system. By implementing voltage-dependent load modelling, which accounts for the sensitivity of loads to voltage variations, a notable reduction in losses is achieved. The real power loss is reduced to 56.0592 kW and the reactive power loss to 44.4975 kVAR, signifying an improvement in both energy efficiency and voltage regulation compared to the base case.

S. No.	Algorithms	Active Power Losses	Reactive Power Losses
1	PSO [16]	46.4196	28.4731
2	MPSO [17]	43.1205	32.2073
3	Proposed PBO	45.5539	31.1003

Table II presents a comparative analysis of different optimization algorithms applied to reduce power losses in the distribution system, namely Particle Swarm Optimization (PSO), Modified Particle Swarm Optimization (MPSO), and the Proposed Polar Bear Optimization (PBO). The PSO algorithm achieves real power losses of 46.4196 kW and reactive power losses of 28.4731 kVAR. Although it provides a reasonably optimized solution, the conventional PSO may suffer from premature convergence and local optima entrapment in complex search spaces.

Improving upon PSO, the Modified PSO (MPSO) shows the best performance in minimizing active power losses, reducing them to 43.1205 kW. However, it results in slightly higher reactive power losses (32.2073 kVAR) compared to PSO. This indicates that while MPSO enhances the active loss performance, it may trade off some efficiency in reactive power compensation.

The proposed Gradient-based Polar Bear Optimization (PBO) algorithm strikes a balanced optimization, achieving active power losses of 45.5539 kW and reactive power losses of 31.1003 kVAR. While its real power loss is marginally higher than that of MPSO, it offers superior reactive power loss reduction compared to MPSO and shows competitive performance overall. The PBO's nature-inspired design with enhanced gradient search capabilities helps in exploring and exploiting the solution space efficiently, making it a robust choice for real-time voltage and power loss optimization tasks in distribution networks.

4. Conclusion

This paper has systematically addressed the problem of power losses and voltage instability in radial distribution systems under evolving load conditions, such as electric vehicle (EV) integration and variable demand. Starting with the standard IEEE 15-bus distribution system, the analysis revealed significant real and reactive power losses, along with poor voltage profiles, particularly at distant buses. To improve system performance, voltage-dependent load modelling was introduced, offering a more realistic representation of consumer load behaviour. This alone resulted in noticeable reductions in power losses and marginal improvement in voltage levels. The impact of EV charging stations was then analysed, and it was found to cause further voltage depression and increased losses due to the unpredictable and high-magnitude nature of EV loads. To mitigate this issue, photovoltaic-based distributed generation (DG) units were strategically placed at selected buses. This not only improved the voltage profile but also led to the lowest real and reactive power losses among all configurations, proving the effectiveness of localized generation and renewable energy integration. Finally, to optimize the operational efficiency of the system, Polar Bear Optimization (PBO) was applied to regulate transformer tap settings. The proposed PBO algorithm effectively minimized power losses while maintaining a high-quality voltage profile. Compared to conventional methods like PSO and MPSO, PBO demonstrated competitive performance, especially in terms of reducing reactive losses, and showcased its robustness and adaptability in handling complex optimization problems.

REFERENCES

- G. Gangil, S. K. Goyal and M. Srivastava, "Optimal Placement of DG for Power Losses Minimization in Radial Distribution System using Backward Forward Sweep Algorithm," 2020 IEEE International Conference on Advances and Developments in Electrical and Electronics Engineering (ICADEE), 2020, pp. 1-6, doi: 10.1109/ICADEE51157.2020.9368941.
- U. Jamil, A. Amin and A. Mahmood, "A comparative study of control techniques for power loss minimization in a distribution network," 2018 1st International Conference on Power, Energy and Smart Grid (ICPESG), 2018, pp. 1-5, doi: 10.1109/ICPESG.2018.8384524.
- A. A. Victoire T, C. Chelladurrai, R. Selladurai, A. N. Kanimozhi P, S. A. Gobu B Jaikumar S and D. SN, "Multi Objective Optimization for Sizing and Placement of Distributed Generators Using a Modified Ant Lion Optimizer Algorithm," 2019 9th International Conference on Power and Energy Systems (ICPES), 2019, pp. 1-6, doi: 10.1109/ICPES47639.2019.9105513.
- 4. K. Mahmoud and N. Yorino, "Optimal combination of DG technologies in distribution systems," 2015 IEEE PES Asia-Pacific Power and Energy Engineering Conference (APPEEC), 2015, pp. 1-5, doi: 10.1109/APPEEC.2015.7381009.
- Nawaz, Sarfaraz, Manish Sharma, and Ankush Tandon. "A new approach for power loss minimization in radial distribution networks." In Applications of artificial intelligence techniques in engineering, pp. 1-7. Springer, Singapore, 2019.
- M. Sedghi, A. Ahmadian and M. Aliakbar-Golkar, "Optimal Storage Planning in Active Distribution Network Considering Uncertainty of Wind Power Distributed Generation," in IEEE Transactions on Power Systems, vol. 31, no. 1, pp. 304-316, Jan. 2016, doi: 10.1109/TPWRS.2015.2404533.
- D. S. Rani, N. Subrahmanyam and M. Sydulu, "Self adaptive harmony search algorithm for optimal capacitor placement on radial distribution systems," 2013 International Conference on Energy Efficient Technologies for Sustainability, 2013, pp. 1330-1335, doi: 10.1109/ICEETS.2013.6533580.
- Sultana, Sneha, and Provas Kumar Roy. "Optimal capacitor placement in radial distribution systems using teaching learning based optimization." International Journal of Electrical Power & Energy Systems 54, pp. 387-398, 2014.
- 9. Vuletić, Jovica, and Mirko Todorovski. "Optimal capacitor placement in radial distribution systems using clustering based optimization." International Journal of Electrical Power & Energy Systems 62, pp. 229-236, 2014.
- Shuaib, Y. Mohamed, M. Surya Kalavathi, and C. ChristoberAsirRajan. "Optimal capacitor placement in radial distribution system using gravitational search algorithm." International Journal of Electrical Power & Energy Systems 64, pp. 384-397, 2015.
- 11. Tamilselvan, V., T. Jayabarathi, T. Raghunathan, and Xin-She Yang. "Optimal capacitor placement in radial distribution systems using flower pollination algorithm." Alexandria engineering journal 57, no. 4 (2018): 2775-2786.
- Al-Ammar, Essam A., Ghazi A. Ghazi, and WonsukKo. "Optimal capacitor placement in radial distribution systems using a fuzzy-dragonfly method." Int J Smart Grid Clean Energy 8 (2018): 116-1124.

- 13. Mujezinović, Adnan, NedimTurković, NedisDautbašić, Maja MuftićDedović, and Irfan Turković. "Use of integer genetic algorithm for optimal allocation and sizing of the shunt capacitor banks in the radial distribution networks." In 2019 18th International Symposium INFOTEH-JAHORINA (INFOTEH), pp. 1-6. IEEE, 2019.
- 14. Rajeshkumar, G. "Hybrid particle swarm optimization and firefly algorithm for distributed generators placements in radial distribution system." Journal of Computational Mechanics, Power System and Control 2, no. 1 (2019): 41-48.
- Aldik, Abdelrahman, and Bala Venkatesh. "Reactive power planning using convex line-wise power balance equations for radial distribution systems." IET Generation, Transmission & Distribution 14, no. 12 (2020): 2399-2406
- 16. Farrag, Mahmoud Ali, Ahmed Hamdy Khalil, and ShaimaaOmran. "Optimal conductor selection and capacitor placement in radial distribution system using nonlinear AC load flow equations and dynamic load model." International Transactions on Electrical Energy Systems 30, no. 5 (2020): e12316.