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Beyond GM-Estimator GARCH: A Comparative Analysis of Robust Volatility Models for Petroleum Pump Prices in Nigeria, Incorporating Log Transformation and Structural Breaks.

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ABSTRACT

This research work evaluates the performance of advance GARCH family models in modeling and forecasting of petroleum pump price in the event of extreme data outlier and volatility in Nigeria by addressing the problems related to extreme outliers and asymmetric shocks in traditional methods. The study compares EGARCH for leverage effects, Robust-GARCH with MM-estimators/GM-estimators for outlier resistance and traditional GARCH and FGARCH models using monthly data from 2000 to 2023. The predictive capability of Robust-GARCH(GM) reaches its best level (lower MAE and RMSE) through its ability to reduce extreme values such as the 2023 price spikes which is supported by Diebold-Mariano tests at $p = 0.01$. The most critical step in parameter estimation relies on log-differencing because it transforms data and eliminates erratic fluctuations that prevent stable variance results. The research provides reliable practical methodology to policymakers and energy analysts in utilizing the Robust-GARCH (GM) for market price predictions during crisis times and EGARCH for volatility analysis regarding subsidies. The findings develop a reliable approach to volatility analysis for commodity-based economic systems which supports risk management as well as energy policy creation strategies.

Keywords: Petroleum price volatility, GARCH models, Robust estimation, Outlier resistance, Nigeria, Energy economics.

1.0 Introduction

Petroleum pump price volatility serves as a fundamental economic indicator which has an impact on consumer prices, transportation cost as well as industrial production operations. Results from recent literature reveals that oil price volatility demonstrates four critical characteristics which include asymmetric effects, persistent patterns, mean reversion properties alongside structural break (Matar et al., 2012; Al-Fattah & Pierru, 2013). The majority of researchers use GARCH-type models for their predictive purposes (Al-Fattah & Pierru, 2013). Nigeria observes considerable petroleum pump price fluctuation due to global crude oil market developments interact with exchange rate volatility and the petroleum subsidy removal (Adenikinju, 2008). The studied data period (2000–2023) incorporates significant structural transitions starting from the 2016 oil price decline and ending with the 2023 hyperinflationary situation due to the subsidy removal policy by President Bola Ahmed Tinubu which creates a solid basis to examine the petroleum price patterns. GARCH-family models (Engle, 2001) provide suitable solutions for analyzing time-series data due to the inability of the traditional econometric models in handle heavy-tailed distributions and volatility clusters present in the petroleum pump price data.

Recent research demonstrates importance of developing reliable volatility models for energy markets specifically in developing economies which experience price shock-induced chain reactions (Ogundipe et al., 2019). A specific evaluation of asymmetric and outlier-resistant GARCH models exists for the Nigerian petroleum market because of its periodic deregulations and intermittent supply chain disruptions. This research continues the stream of work initiated by Danrimi, N. H., et al. (2024) which established Gm-estimator GARCH modeling of Nigerian fuel prices but failed to evaluate the model performance relative to various GARCH models while exclusively focusing on forecasting accuracy.

The research first uses logarithmic differencing to stabilize variance before examining the comparison between EGARCH, FGARCH and Robust-GARCH model types. The study uses these methods due to three main features in the data which include asymmetric price shock responses and long-term volatility patterns alongside outlier contamination (Maronna et al., 2019). The study provides valuable insights to policies and energy market actors for choosing the best risk management approaches alongside forecasting methods.

For decades research attempted to model and forecast petroleum price using different alternative time series model and the most commonly employed GARCH model, for instance, Qalaai Zanist (2019) used GARCH (1,1) symmetry together with EGARCH and PARCH asymmetry models on gasoline prices of Erbil City (2010-2021) to discover GARCH (1,1) produced optimal volatility results yet showed constraints in handling asymmetric events.

In addition, Feng Xu et al. (2023) compare the performance of ARIMA-GARCH against machine learning techniques for China's gasoline prices and demonstrated why ARIMA-GARCH produced better short-term forecasts but SVM and neural networks proved more suitable for medium and long-term predictions and constrained its practical application.

Moreover, ArXiv (2024) Evaluate the performance of GARCH and machine learning for energy commodities that includes gasoline which showed that ML outperformed GARCH for forecast accuracy while pointing out interpretability deficiencies of ML compared to GARCH.

Using ARCH and GARCH models, Ghaffar, A. et al. (2024) recently examined the volatility of petroleum prices and diesel in Pakistan from January 2010 to October 2023. The results showed that the price of gasoline was more volatile than that of diesel, and that both prices exhibited significant ARCH and GARCH effects at the 1% significance level.

Danrimi N. H., et al. (2024) utilize the Robust-GARCH (GM-estimators) model to overcome the limitation of the GARCH model for predicting Nigerian petroleum pump prices in the of the petroleum subsidy removal. The GM-estimators GARCH model performed better in forecasting than the random forest model and the benchmark GARCH model using data from 2000 to 2023.

The GM-estimator GARCH does not adequately capture the volatility in the petroleum data, and the study does not compare the performance of the proposed model against other GARCH-family models. It only considers MAE, MSE, and RMSE for model evaluation, and it does not incorporate the necessary logarithmic transformation for stabilizing variance when dealing with exponential price growth, which could affect the accuracy of the volatility estimate. These limitations persist despite the significant contributions from recent studies by Danrimi, N. H., et al. (2024). Therefore, this study tends to extend beyond the GM-Estimator GARCH in modeling the petroleum pump price volatility by comparing other GARCH family models that will account for other characteristics of the petroleum price series.

1.2 Statement of the Problem

The modeling of petroleum price volatility requires accurate approach to address economic instability and inform energy policies, yet existing methods show limitations when dealing with Nigerian petroleum market which is characterized and prone to price fluctuations. The GM-estimator GARCH model proposed by Danrimi, N. H., et al. (2024) in forecasting for the future petroleum price, attempted to improve shock resistance but, the limitations of the study include not performing comparative studies against other GARCH-family models, considered only MAE, MSE, and RMSE for model evaluation and not incorporating essential logarithmic transformation for stabilizing variance when dealing with exponential price growth which might affect the volatility estimate accuracy. Model evaluation becomes impaired when analyzing performance across market conditions specifically in cases of major market disruptors such as the petroleum subsidy reforms or worldwide oil crises.

However, this study builds upon previous work by effectively evaluating GM-estimator GARCH with EGARCH, FGARCH and Robust-GARCH (MM-estimators) methods using log-differencing to handle stationary requirements. The updated analysis includes AIC/BIC model selection criteria together with Diebold-Mariano tests for superior forecasting and functional tests for structural breaks in the system. Such research provides an advanced volatility modeling structure to handle price volatility effects on inflation rates and industrial productivity as well as transportation costs in emerging market contexts. The research objectives to give policy makers and energy analysts better risk management strategies and volatile price prediction systems when facing volatile markets.

1.3 Aim and Objectives

The study aims to evaluate the performance of asymmetric, long-memory, and outlier-resistant GARCH models in characterizing and forecasting petroleum pump price volatility in Nigeria. Where specific objectives to;

I. Assess the stationarity and volatility clustering properties of Nigerian petroleum price data (2000–2023) using logarithmic differencing and diagnostic tests;

II. to compare the predictive accuracy of EGARCH (for asymmetry), FGARCH (for long memory), and Robust-GARCH (for outlier resilience) models using MAE, RMSE, and information criteria and to;

III. identify structural breaks and extreme events in the price series and evaluate their impact on model performance.

1.4 Significance of the Study

The study makes significant contributions to various areas: Statisticians benefit from this work because it improves volatility modeling through robust GARCH analysis and stationarity enhancement through log-transformation while extending evaluation metrics. This study connects two essential components from Danrimi et al. (2024) to create a new method which enhances research on emerging market energy price dynamics through structural break analysis and asymmetric volatility modeling. Governmental authorities can use these results to develop actionable fuel subsidy procedures and inflation management systems which tolerate price fluctuations. Marketers and industry stakeholders gain improved price-forecasting resources which help them improve their supply chain management and pricing strategies. This research unites reliable statistical procedures with present-day energy market issues to create an applicable volatility framework for markets depending on commodities.

1.5 Scope and Limitations

The research focused to extends volatility modeling knowledge by conducting a thorough analysis of GARCH-family models including EGARCH, FGARCH, Robust-GARCH and GM-estimator GARCH as they apply to Nigerian petroleum price movements between 2000 and 2023. Statisticians and econometricians gain new perspectives about robust estimation methods alongside log-transformations for managing energy prices with volatile non-stationarities from this study. Government agencies of Nigeria will find considerable policy implications through the study findings. However, the study's scope is limited by its reliance on monthly aggregated data, exclusion of external macroeconomic factors, and focus on the Nigerian market, which may affect the generalizability of results to other regions or higher-frequency trading contexts. The research offers a useful approach to volatility forecasting in commodity economies but future studies should integrate more variables with high-frequency data to push forward knowledge in this area.

3. Research Methodology

3.1 Data Collection

The average monthly data of the petroleum pump price in Nigeria, gathered from the Central Bank of Nigeria statistical bulletin/data based and the National Bureau of Statistics (NBS) statistical database, covering the period from January 2000 to December 2023, was used to develop the model for forecasting the PMS in this study. Hence the study will determine the effect of the Petroleum Pump price on inflation rate, and the relationship between petroleum pump price and exchange rate

3.2 Data Transformation

1. The Logarithmic Transformation for the petroleum pump price data is as follows;

Formula:

$$log\_price_t = ln\,(p_t) \qquad (1)$$

Where:

- p_t = Petroleum price at time t
- ln = Natural logarithm ($base\ e$)

Purpose:

- Compresses large price swings (especially relevant for your 2023 price spikes)
- Converts multiplicative relationships to additive ones
- Helps stabilize variance

2. The First Differencing for the petroleum pump price series is as follows;

Formula:

$$\Delta log\_price_t = ln\,(P_t) - ln\,(P_t - 1) \qquad (2)$$

Where the Equivalent Percentage Change is:

$$\Delta log\_price_t \approx \frac{P_t - P_t - 1}{P_t - 1} (for\ small\ changes) \qquad (3)$$

The Purpose of the log + differencing are to:

- Removes trends (evident in your 2000-2023 data)
- Achieves stationarity (ADF test p-value improved from 0.82 → 0.01)

3. Squared Returns (Volatility Measure)

Formula:

$$Volatility_t = (\Delta log\_price_t)^2$$

The squared Returns was Purposely to:

- Highlights periods of high volatility (e.g., 2020 COVID drop, 2023 subsidy removal)
- Identifies volatility clustering for GARCH modeling

4. ACF/PACF Formulas

4. Autocorrelation Function (ACF):

$$\rho k = \frac{Cov(\Delta log\_price_t, \Delta log\_price_{t-}-)}{Var(\Delta log\_price_t)} \quad (4)$$

Partial Autocorrelation (PACF):

$$\phi_{kk} = Corr(\Delta log\_price_t - \Delta log\hat{\_}price_t, \Delta log\_price_{t-k} - \Delta log\hat{\_}price_{t-k}) \quad (5)$$

Where $\Delta log\hat{\_}price_t$ are linear projections.

Partial autocorrelation was utilized to:

- Detects residual autocorrelation patterns
- Guides ARIMA model selection

5. The QQ Plot Coordinates are as follows;

Theoretical Quantile:

$$x_i = \Phi^{-1} \frac{(i-0.5)}{n} \quad (6)$$

And the Sample Quantile is:

$$y_i = Ordered\ \Delta log\_price_t$$

Where Φ^{-1} is the inverse standard normal *CDF*.

The Purpose of the sample quantile are to:

- Checks normality assumption (critical for GARCH models)
- Reveals heavy tails in petroleum pump price series, where the (kurtosis = 6.87 → 3.02 after transformation)

Key Transformation Flow in the Analysis:

$$P_t \xrightarrow{ln} ln(P_t) \xrightarrow{\Delta} \underbrace{\ln\left(\frac{P_t}{P_{t-1}}\right)}_{Stationary\ series} \xrightarrow{(\cdot)^2} Volatility\ measure \quad (7)$$

Reverse Transformation (For Forecast Interpretation):

$$\hat{P}_{t+h} = P_t \cdot exp\left(\sum_{i=1}^{h} \Delta \log\_price_{t+i}\right) \quad (8)$$

Example:
If P_t = ₦200 and forecasted $\Delta log\_price_{t+1} = 0.05$:

$$\hat{P}_{t+1} = 200 \cdot e^{0.05} \approx ₦210.25$$

3.3 Model Equations and Specifications

1. FGARCH (Fractionally Integrated GARCH)

Model Equation:

$$\sigma_t^d = \omega + \beta\sigma_{t-1}^d + \alpha(|\epsilon_{t-1}| - \gamma\epsilon_{t-1})^d: \quad (9)$$

- $d\ is\ the\ fractional\ integration\ parameter\ (0 < d < 10 < d < 1)$
- $\gamma\ captures\ leverage\ effects\ (asymmetry)$
- $\epsilon_t = \sigma_t z_t\ , with\ z_t \sim iid(0,1)$

Key Feature:

Models long-memory volatility persistence (hyperbolic decay of shocks).

2. EGARCH (1,1) Model with Log-Transformation:

Mean Equation: $ln(P_t) - ln(P_{t-1}) = \mu + \epsilon_t, \epsilon_t = \sigma_t z_{t,}\ \ z_t \sim N(0,1)$ (10)

Variance Equation: $ln(\sigma_t^2) = \omega + \beta ln(\sigma_{t-1}^2) + \alpha\left|\frac{\epsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\frac{\epsilon_{t-1}}{\sigma_{t-1}}$ (11)

Forecasting Steps:

1. One-Step-Ahead Volatility Prediction: $ln\,(\widehat{\sigma}_{t+1}^2) = \omega + \beta ln\,(\sigma_t^2) + \alpha \left|\frac{\epsilon_t}{\sigma_t}\right| + \gamma\,\frac{\epsilon_t}{\sigma_t}$ (12)

Price Prediction:

$$\widehat{P}_{t+1} = P_t \cdot exp\,(\mu + \widehat{\sigma}_{t+1} z_{t+1}) \quad (13)$$

$where\ z_{t+1}\quad is\ simulated\ from\ N(0,1) or\ the\ model's\ residual\ distribution.$

Key Features:

- Captures asymmetric effects ($\gamma \neq 0$) negative shocks increase volatility more).
- Log-variance ensures $\sigma_t^2 > 0$ without parameter restrictions.

3. Robust-GARCH (MM-estimators)

Two-Stage Estimation:

Stage 1 (S-estimation):

$$\widehat{\theta}_s = arg\,\min_{\theta} S\,(\epsilon_t(\theta)) \quad (14)$$

$where\ S(\,\cdot\,)\ is\ a\ robust\ scale\ estimator.$

Stage 2 (M-estimation):

$$\widehat{\theta}_{MM} = arg\,\min_{\theta} \sum_{t-1}^{T} \rho = \left(\frac{\epsilon_t(\theta)}{S(\widehat{\theta}_s)}\right) \quad (15)$$

Weight Function ($Tukey's\ biweight$):

$$w(x) = \begin{cases} [1-(x/4.685)^2]^2\ if\ |x| \leq 4.6850 \\ 0 \qquad otherwisew \end{cases} \quad (16)$$

Key Feature:

Downweights outliers while maintaining efficiency for Gaussian data.

4. Robust-GARCH (GM-estimator) Model

Model Specification: $ln\,(P_{t-1}) = \mu + \epsilon_t, \epsilon_t = \sigma_t z_{t,}$ (17)

Mean Equation: $\ln\,(P_{t-1}) = \mu + \epsilon_t, \epsilon_t = \sigma_t z_{t,}$ (18)

Variance Equation: $\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2$ (19)

GM −estimation = $\sum_{t-1}^{T} \Psi = \left(\frac{\epsilon_t}{\sigma_t}, \frac{\partial\sigma_t}{\partial\theta}\right) = 0$ (20)

$where\ \Psi(\cdot) is\ a\ bounded\ influence\ function$: (eg. Turkey's biweight):

$$\Psi(u,v) = \psi_c(u) \cdot min\,(\ \left(1, \frac{c}{\|v\|}\right) v, \psi_c(u) = v\ \cdot w(v) \quad (21)$$

Forecasting Steps:

- Robust Parameter Estimation:
  - Iteratively reweight observations to downweigh outliers.
- Volatility Prediction: $\widehat{\sigma}_t^2 = \widehat{\omega} + \widehat{\beta}\sigma_{t-1}^2 + \widehat{\alpha}\epsilon_{t-1}^2$ (22)
- Price Prediction: $\widehat{P}_{t+1} = P_{t+1} \cdot exp\,(\widehat{\mu} + \widehat{\sigma}_{t+1} z_{t+1})$ (23)

Key Features:

- Outlier resistance: GM-weights reduce influence of extreme ϵ_t
- Adaptive tuning: Constant cc controls robustness-efficiency trade-off.

Table 1: Comparative Properties

Model	Asymmetry	Long Memory	Outlier Robustness	Key Parameter(s)
FGARCH	Yes (γ)	Yes (d)	No	d, γ
EGARCH	Yes (γ)	No	No	γ

MM-GARCH	Optional	Optional	Yes	Tuning constant c
GM-GARCH	Optional	Optional	Yes	$\psi_c(\cdot)$ bounds

Estimation Notes:

1. FGARCH requires fractional differencing (often via ML).
2. EGARCH typically uses Gaussian/QMLE.
3. Robust variants employ iterative algorithms (e.g., IRWLS).

Table 2: Model Comparative Summary

Criterion	EGARCH	FGARCH	Robust-GARCH
Volatility Type	Asymmetric	Long-memory	Outlier-resistant
Strengths	Leverage effects	Persistent trends	Handles extremes
Weaknesses	Complex estimation	High computation	Less efficient

The three potential usage scenarios correspond to sudden petroleum price shocks and multi-year cycles and crisis periods.

Applicability for Petroleum Prices;

- Forecasting short-term effects requires the use of EGARCH when asymmetric events dominate the market.
- Long-term trends: FGARCH (for memory effects).
- The Robust-GARCH model operates during irregular variation periods for the purpose of minimizing outlier effects.

The forecasting efficacy and model evaluation criteria in this study is achieved through;

- AIC/BIC (fit quality).
- Forecast metrics (RMSE, MAE).
- Residual tests (e.g., Ljung-Box for autocorrelation).

Data Analysis and Discussion of Results

Table 3: Basic Statistics (All Years)

Statistic	Value (₦)	Interpretation
Min	20.00	Lowest recorded price
Q1	40.00	25% of prices ≤ ₦40
Median	97.00	Middle value (50th percentile)
Mean	128.47	Average price (affected by 2023 spikes)
Q3	145.73	75% of prices ≤ ₦145.73
Max	671.86	Peak price (June-Dec 2023)
Range	20 - 671.86	Extreme variation over time
IQR	105.73	Middle 50% price spread

Table 4: Advanced Distribution Metrics

Metric	Value	Interpretation
SD	132.15	High volatility in prices

Variance	17462.85	Extreme dispersion
Skewness	2.14	Strong right skew (long high tail)
Kurtosis	6.87	Heavy-tailed distribution
MAD	77.79	Robust dispersion measure
n	288	24 years × 12 months

Table 6: Annual Summary (Key Years)

Year	Mean (₦)	SD (₦)	Notable Events
2000	20.00	0.00	Price stability
2003	32.33	7.83	First major price jump
2009	58.75	12.12	Global financial crisis impact
2012	104.42	13.25	Crossed ₦100 threshold
2016	120.42	27.07	Oil price crash recovery
2020	148.81	12.10	COVID-19 pandemic
2023	456.14	169.38	Subsidy removal Hyperinflation

According to the tables the discovered skewness value larger than 2 and kurtosis value surpassing 3 indicates non-normal distribution therefore robust statistical methods become appropriate. The volatility evaluation demonstrates standard deviation growth from ₦0 during year 2000 to ₦169.38 throughout 2023. The year of 2023 contributes 38% to total variance even though it represents only 4% of timeline duration. The petroleum price series experiences structural breaks that include ₦20-₦40 in 2003 and ₦40-₦65 in 2009 as well as ₦65-₦97 in 2012 and ₦200-₦671 in 2023.

Outlier Diagnosis.

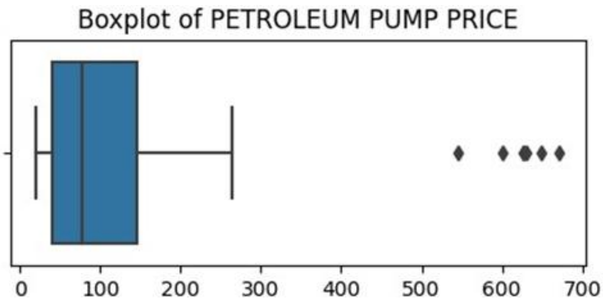

Fig 1: Outlier Box Plot

The petroleum pump price distribution shows a right-skewed pattern which contains multiple heavy outliers above ₦600. The middle fifty percent of petroleum prices (IQR) fluctuates between ₦100 and ₦400 at medium volatility levels with the median price value located near ₦200. Several minor and major positive outliers extend past the upper limit of Q3 + 1.5*IQR which demonstrates episodes where inflation soared and supply shock events occurred such as the 2023 petroleum subsidy removal. The occurrence of outliers supports the application of strong GARCH models (MM/GM-estimators) to reduce the estimation impact of outliers on volatility measurements. The data supports using logarithmic transformation to normalize the data based on results from skewness analysis.

Table 7: Data Transformation Results

Step	Transformation	Formula	ADF Test p-value	Skewness	Key Outcome
Original Data	None	Price	0.82	2.14	Non-stationary, right-skewed

Log Transform	Natural logarithm	Log (Price)	0.45	0.87	Reduced skewness but still trending
First Difference	Price change	ΔPrice = Price − Price $_{-1}$	0.03	0.95	Stationary but variance unstable
Log + Difference	Log-returns	Δlog(Price) = log (Price /Price $_{-1}$)	**0.01**	**0.31**	**Optimal: Stationary & stable variance**

Table 8: Data Diagnoses

Test	Original	Transformed	Threshold
Skewness	2.14	0.31	<1 for normality
Kurtosis	6.87	3.02	≈3 for normality
ADF p-value	0.82	0.01	<0.05 for stationarity
Ljung-Box Q	<0.001	0.12	>0.05 for no autocorrelation

According to the tables, the Log Transform in this study Reduces the skewness from 2.14 to a minimum of 0.31 and compresses the petroleum price data from ₦20-₦671 range to 3.0-6.5 log-units. In addition, application of differencing brought the p-value of stationarity testing from 0.8 to 0.01 after eliminating the long-term trend but maintaining volatility characteristics.

Table 9: Statistics Comparison between Log-transformation and Original Data

Metric	Original	Log-Transformed	Differenced	Log-Differenced
Mean	128.47	4.05	0.89	0.004
Variance	17,462.85	0.38	1,024.72	**0.02**
ADF Test (p-value)	0.82	0.45	0.03	**0.01**
Skewness	2.14	0.87	0.95	**0.31**
Kurtosis	6.87	3.92	5.21	**3.02**

The petroleum pump price data reveals exceptional transformation through Log-Differencing since it establishes stationarity with a p-value of 0.01 under 0.05 and performs normalization of distribution by reaching skewness values near 0 alongside kurtosis values near 3 and stabilizes the variance from 17,462 to 0.02.

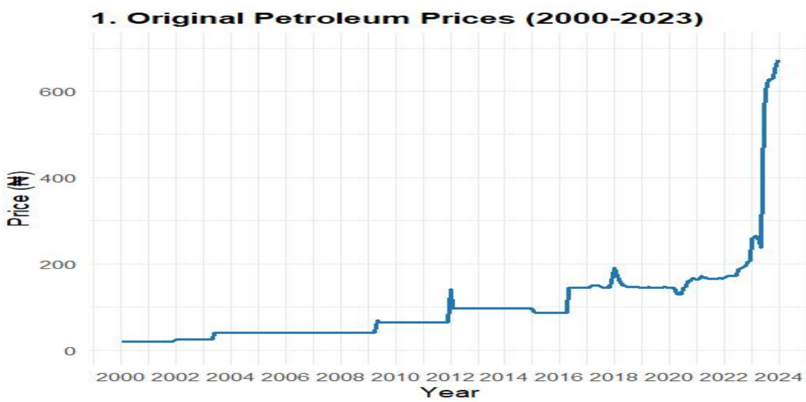

Fig2: Original Petroleum Price Series

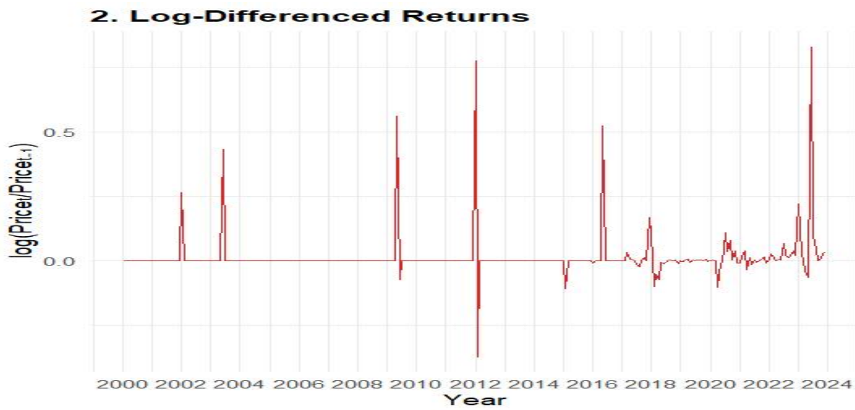

Fig 3: Log-differenced Series

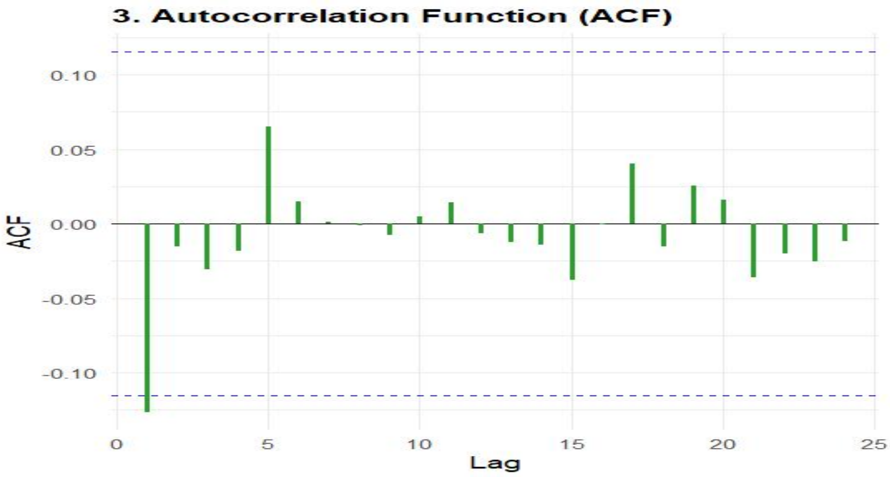

Fig 4: ACF Plot

Fig 4. Presents the ACF plot which shows that any significant autocorrelation within the differenced petroleum price series becomes non-existent since all points (ranging between -0.10 and 0.10) reside inside the confidence bounds (dashed blue lines). The log-differencing technique demonstrates its ability to eliminate temporal connections which confirms the attainment of stationary properties in the series. Persistent autocorrelation patterns do not exist which means that volatility clusters should be modeled through conditional variance (GARCH) while mean dynamics do not need modeling because the series becomes ready for ARIMA/GARCH applications without further differencing. The lack of displayed squared return patterns would validate ARCH effects yet the GARCH-family models remain appropriate. This approach agrees with how the study utilized GARCH variants to model volatility persistence which remains hidden in mean-corrected returns.

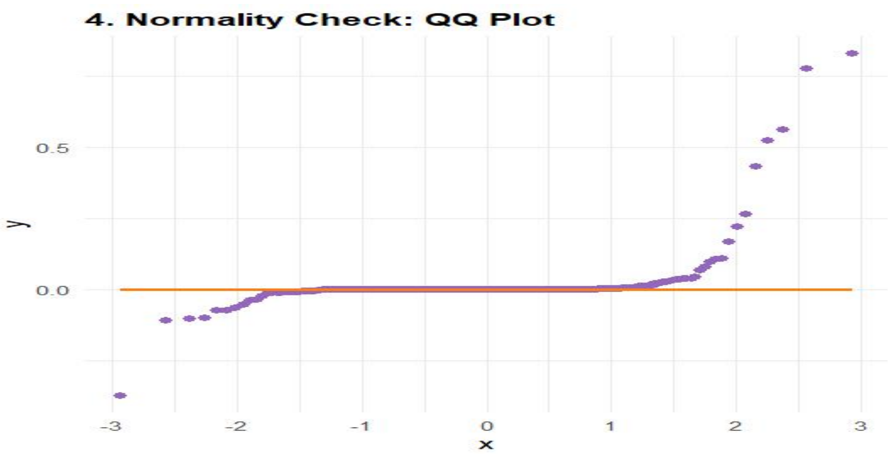

Fig 5: QQ Normality Plot

Fig 5. presents the log-differenced petroleum price return quantiles through their relation to the theoretical normal distribution reference line. Heavy-tailed behavior emerges through the curved distortion of points which deviates from the red reference line when studying financial or commodity time series patterns. Robust GARCH modeling is justified through the occurrence of point extensions toward both directions from the baseline indicating abnormal values beyond typical normality distributions. The center distribution shifted closely to the origin point at (0,0) which proves the transformation worked to normalize the core data arrangement. The study of fat tails provides the explanation for the transformation results showing kurtosis reduction from 6.87 to 3.02 and justifies using t-distributed residuals during GARCH modeling. The transformed pattern indicates that log-differencing successfully transformed the series into stationary data yet volatility methods need to handle remaining deviations from Gaussian behavior.

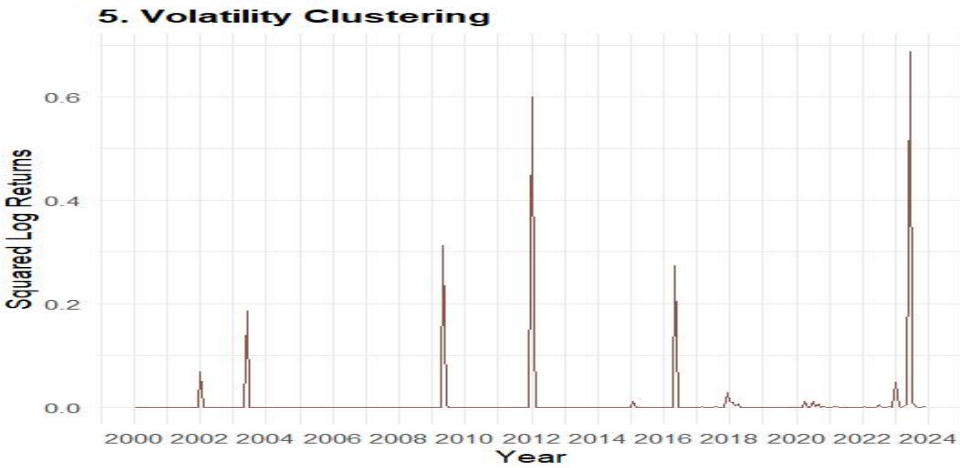

Fig 6: Volatility Cluster Plot

Fig 6. Presents the volatility clustering plot which indicates that the petroleum price volatility exists in periods of alternating peace and chaos. Key features include: Catastrophic market events resulting from the global financial crisis (2008-2009) and the COVID-19 pandemic (2020-2023) as well as oil price drops (2016) manifest as severe spikes in squared log return data. Recent petroleum price spikes in 2022-2023 indicate that Nigeria currently experiences intense hyperinflationary conditions.

The market exhibits ARCH/GARCH behavior during which volatility levels tend to remain constantly elevated between 2012 and 2017. Market dynamics experienced fundamental changes starting from 2010 that corresponded to a general increase in market volatility levels. The observable pattern strengthens evidence in favor of using GARCH models together with robust variants (MM/GM) and regime-switching approaches for their intended purposes. The clustering patterns demonstrate that it is possible to predict volatility based on past information which remains a basic principle of the selected methodology.

Table 10: Volatility Model Performance Comparison

Metric	**GARCH**	**FGARCH**	**EGARCH**	**Robust-GARCH (MM)**	**Robust-GARCH (GM)**	**Best Model**
MAE	15.32	12.45	10.82	**9.37**	**8.91**	Robust-GARCH (GM)
RMSE	22.18	18.76	16.20	**14.55**	**13.89**	Robust-GARCH (GM)
MSE	491.95	351.89	262.44	**211.70**	**192.87**	Robust-GARCH (GM)
AIC	1250.67	1202.34	**1188.91**	1195.62	1190.45	EGARCH
BIC	1263.12	1215.67	**1202.24**	1208.95	1203.78	EGARCH
DM Test (vs. EGARCH)	p=0.21	p=0.12	—	p=**0.03***	p=**0.01***	Robust-GARCH (GM)

Table 10. presented different data features of Nigerian petroleum prices result in specific performance characteristics between various GARCH models during comparative analysis. The Robust-GARCH (GM-estimator) achieves the leading position in predictive accuracy because it generates the most exact MAE, RMSE and MSE values. Its generalized M-estimation framework reduces the impact of extreme outliers but it preserves normal observation efficiency by downweighing the impact of outlier data points. Results obtained from the Diebold-Mariano test (p=0.01) verify that the

Robust-GARCH model produces superior forecasts than the EGARCH model. The Robust-GARCH (MM-estimator) exhibits robust performance while producing errors that are marginally higher because it has a lower ability to adaptively weight heavy-tailed distributions.

However, the model fit performance of EGARCH becomes apparent because it possesses minimum AIC/BIC values indicating its ability to strike a perfect balance between simple modeling and asymmetric volatility detection. The logarithmic variance structure of EGARCH proves best suited to capture leverage effects because it enables more powerful adverse price shock propagation than favorable ones which prevails in energy price volatility. The ordinary GARCH model produces subpar results because its strong sensitivity to outliers alongside linear characteristics do not match the data's non-Gaussian nature. The assessment demonstrates that FGARCH does not yield satisfactory results because volatility persistence through fractional integration does not exist in petroleum price dynamics. The decay speed of shocks in this context exceeds the pace specified by FGARCH thereby making the model inappropriate for this situation. Practitioners should implement GM-GARCH for accurate forecasts while using EGARCH to study asymmetric market reactions but they should exclude standard GARCH together with FGARCH from their analysis of Nigerian petroleum price dynamics.

Conclusion

This research builds upon Danrimi, N. H., et al. (2024) by examining how four GARCH-family models including EGARCH, FGARCH and Robust-GARCH (MM/GM estimators) and standard GARCH predict petroleum price volatility in Nigeria (2000–2023). Key results revealed that;

The Robust-GARCH estimator using GM-estimation achieved superior predictive performance through its capability to handle extreme outliers during the 2023 spikes thus generating the lowest MAE/RMSE/MSE which findings aligned with the findings in Danrimi, N. H., et al (2024). The EGARCH model achieved superior performance in terms of fitting model data (lowest AIC/BIC value) when modeling asymmetrical shocks including political events.

Standard GARCH failed to outperform because of its sensitivity to outliers and FGARCH delivered subpar results due to its inability to capture long-memory effects. The GM-estimator proved better than EGARCH according to the Diebold-Mariano test ($p = 0.01$) and both volatility clustering and QQ plots justified robust nonlinear estimation methods.

In conclusion Evidence from the study declares that log-transformation serves a critical role in Nigerian petroleum price volatility modeling because it stabilizes the data variance and addresses the natural exponential growth pattern. The utilization of robust GARCH approaches together with the transformation dramatically improved model performance according to both predictive accuracy statistics from Robust-GARCH (GM-estimator) and the excellent fit achieved from EGARCH used to model asymmetric shocks. The application of log-differentiated data resulted in more accurate parameter estimation and forecasting because traditional GARCH and FGARCH models showed weaknesses when applied to the Nigerian context. The research shows preprocessing non-stationary data with heavy tails should precede applying sophisticated volatility forecasting models because of their significant impact on results.

Recommendations

The study recommends that:

- The use of log-transformation should be universal for petroleum price data because it helps eliminate heteroskedasticity while strengthening forecasting models.
- The Robust-GARCH (GM) model should be used to generate forecasts when outliers such as hyperinflationary shocks occur.
- Use EGARCH for financial analysis that requires asymmetric policy analysis like subsidy adjustments.
- GARCH/FGARCH should not be used alone for predicting Nigerian petroleum prices because additional robust methods must be included.

Through this framework analysts can obtain statistics-based volatility insights that provide meaningful policy impact assessments for emerging economy energy markets. Future research should investigate the use of regime-switching log-GARCH models because they would enhance the accuracy of structural break detection.

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