



## Mathematics Model of COVID-19 Spreads Using SEIR Models

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### ABSTRACT:

The COVID-19 outbreak, which began in 2019, prompted a significant amount of research into understanding its spread. Dr. Lin initially developed a simple model of COVID-19 transmission using the SIR (Susceptible-Infected-Recovered) framework [1]. Subsequently, numerous studies expanded on this approach. One notable contribution was by Mr. McGhee, who, with Dr. Lin's support, explored the spread of COVID-19 through an SEIR (Susceptible-Exposed-Infected-Recovered) model that incorporated the impact of vaccinations. This paper provides a brief overview of the SEIR model, its application to coronavirus transmission, and the specific modeling efforts related to COVID-19.

### Introduction

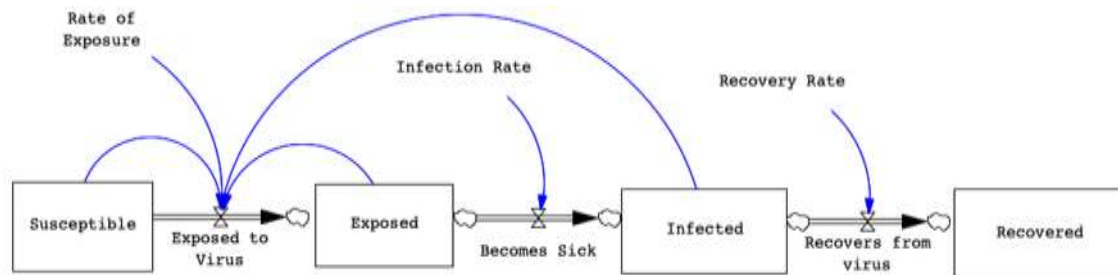
SIR models are the foundational frameworks used to model the transmission of diseases and viruses within populations. Building upon the SIR model, more advanced models have been developed to better capture the complexities of disease spread. One such model is the SEIR model, an extension of the SIR framework, which incorporates an additional compartment for individuals who are exposed to the disease but not yet infectious. The SEIR model seeks to improve the accuracy of disease transmission predictions by considering not only susceptible, infected, and recovered populations, but also the exposed individuals, who are in the incubation phase before becoming infectious.

The SEIR model is a compartmental model used in epidemiology to describe the spread of infectious diseases. It divides the population into four distinct groups:

1. **Susceptible (S):** Individuals who are not yet infected but are at risk of contracting the disease.
2. **Exposed (E):** Individuals who have been exposed to the virus but are not yet infectious. This represents the incubation period of the disease.
3. **Infected (I):** Individuals who are infected and can spread the disease to others.
4. **Recovered (R):** Individuals who have recovered from the disease and are assumed to have immunity (either temporary or permanent, depending on the disease).

The SEIR model helps to simulate the dynamics of disease spread, accounting for the latency period (Exposed phase) before individuals become infectious. It can be extended to include other factors, such as vaccination or interventions, making it a useful tool for predicting disease trends and evaluating control strategies.

The diagram of SEIR model is as following:

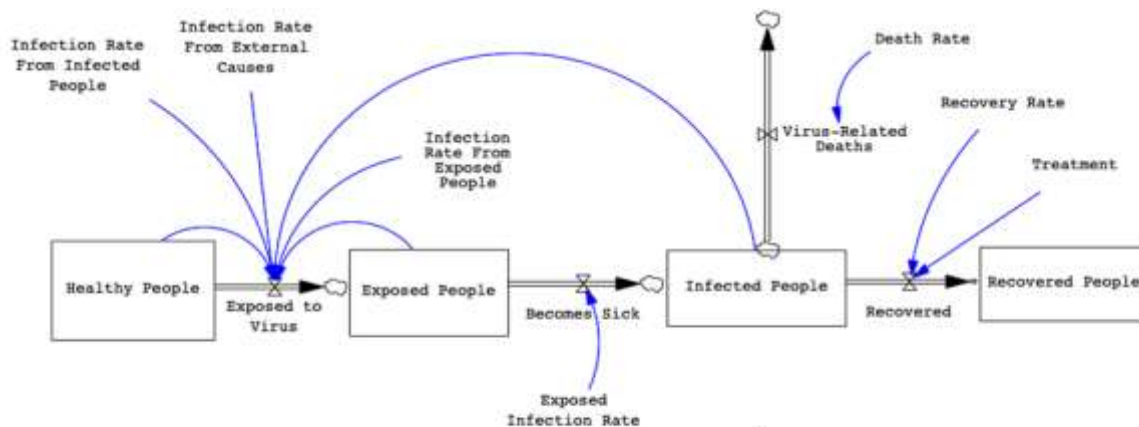


## SEIR Models: Coronaviruses

Coronaviruses are a family of viruses known for causing diseases in animals and humans. Named for their crown-like spikes, there are 4 main types of coronaviruses: alpha, beta, gamma, and delta. Human coronaviruses were identified sometime in the 1960s and have been known to cause mild respiratory tract illnesses such as coughs and colds in humans. There have been three specific coronaviruses that have been known to cause more serious disease and even death.

Severe Acute Respiratory Syndrome (SARS) is a respiratory illness caused by a coronavirus (SARS-CoV). SARS is an airborne virus transmitted virus contact or exposure to the virus. There are several ways that you can directly be exposed to the virus from person-to-person contact, airborne spread, or droplet spread. SARS can also be indirectly transmitted via contaminated surfaces. SARS was first reported in Asia in early 2003 before spreading to a dozen countries. Symptoms of SARS ranges from high fever, chills, shortness of breath, headache, body aches and mild respiratory issues. There is no cure or vaccine for the disease. Treatment for SARS is usually based on patients' symptoms; however, for severe cases intubation and ventilation for patients may be the only option. There have not been any known cases of SARS since 2004 as the virus was contained after its global outbreak in 2003. In 2012, the SARS coronavirus was deemed a select agent meaning it has a potential to pose a severe threat to public health and safety.

Middle Eastern Respiratory Syndrome (MERS) is a respiratory illness caused by a beta coronavirus (MERS-CoV). MERS is transmitted through contact with the virus or infected animals. MERS was first reported in Saudi Arabi in 2012 and since has been reported in a total of 27 countries mainly in the Middle East, Africa, and Asia. Infections range from no symptoms to respiratory symptoms such as fever, shortness of breath, cough, pneumonia or even death. Some people have also reported diarrhea, nausea and vomiting as symptoms. People with certain conditions such as diabetes and, lung disease and other immunocompromised people are considered high risk for the disease. There are several means of transmission from human-to-human contact, to contact with infected animals (specifically camels) however transmission is not fully understood. Like other coronaviruses, there are several ways that you can directly be exposed to the virus from person-to-person contact, airborne spread, or droplet spread through respiratory secretions. Prevention can include but is not limited to avoiding contact with camels, drinking raw camel milk or urine, eating improperly cooked meats, and infected humans. All cases to date have been linked to the Arabian Peninsula. We can use what we know from these coronaviruses and SEIR models to help us create a more advanced model for a covid-19 transmission model.



## SEIR Models: Covid-19

Covid-19 is a disease caused by a novel coronavirus (SARS-CoV-2). Covid-19 is an infectious *airborne respiratory disease transmitted through either direct or indirect contact*. A person can be exposed via person-to-person contact, droplet spread or indirectly through contaminated surfaces. Due to its very contagious nature, Covid-19 was first identified in Wuhan, China in December of 2019 before quickly spreading throughout the world and becoming a pandemic. Common symptoms include fever, chills, shortness of breath, loss of taste/smell, headache, and other flu-like symptoms. Symptoms can be expected between 2-14 days of exposure and range from mild to severe illness and in some cases even death. Treatment for Covid-19 varies from patient to patient as symptoms could require medical intubation. A vaccine and other forms of treatment have also been developed throughout the pandemic.

The true issues with Covid-19 stem from its ability to replicate and spread before carriers become symptomatic. Therefore, some carriers show symptoms while others do not. To effectively model Covid-19 transmission we must account for all possibilities of virus transmission. This includes infection from external causes, exposed people, and infected people. To begin we will first compare initial covid conditions to present conditions where we have more effective treatments and a vaccine to help fight viral transmission. The original diagram is an example of initial Covid-19 conditions prior to social distancing, masking, and vaccines.



#### Notation

Healthy People: People susceptible to the virus

Exposed People: Exposed to virus (symptomatic or asymptomatic)

Infected People: Infected with virus symptoms

Recovered People: People who have recovered from Covid-19

Vaccinated People: People who have been vaccinated for Covid-19

ecr: infection rate due to external causes

epr: infection rate from exposed people

ipr: infection rate from infected people

eir: exposed people infection rate

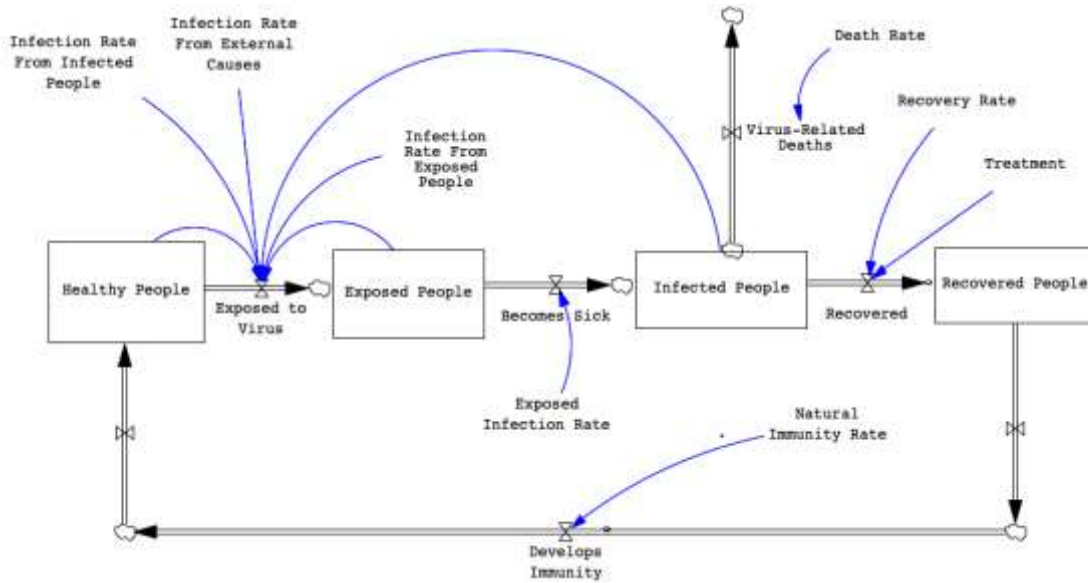
dr: death rate due to covid-19

rr: recovery rate of infected people

vir: infection rate for vaccinated people

vr: vaccination rate for healthy people from covid-19

With parameters and variables, we can model covid-19 transmission based on our initial conditions (assumptions).



This diagram can be used to model initial conditions of Covid-19. We learned that Recovered people develop some type of waning immunity that will allow them to be reinfected again. This SEIR model can be expressed in equations by

The change in Healthy People can be expressed as

$$\frac{dH}{dt} = -ecr * H - epr * H * E - ipr * H * I$$

The change in Exposed People can be expressed as

$$\frac{dE}{dt} = ecr * H + epr * H * E + ipr * H * I - eir * E$$

The change in Infected People can be expressed as

$$\frac{dI}{dt} = eir * E - rr * I - dr * I$$

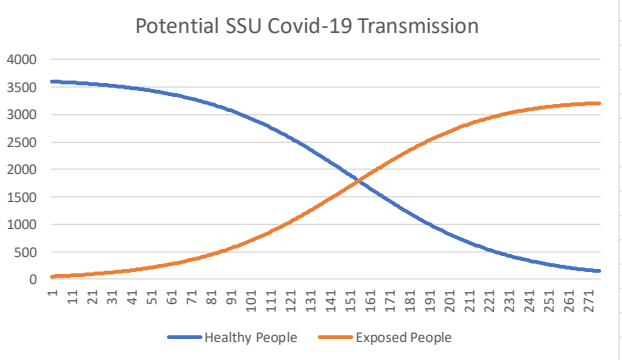
The change in Recovered People can be expressed as

$$\frac{dR}{dt} = rr * I - R$$

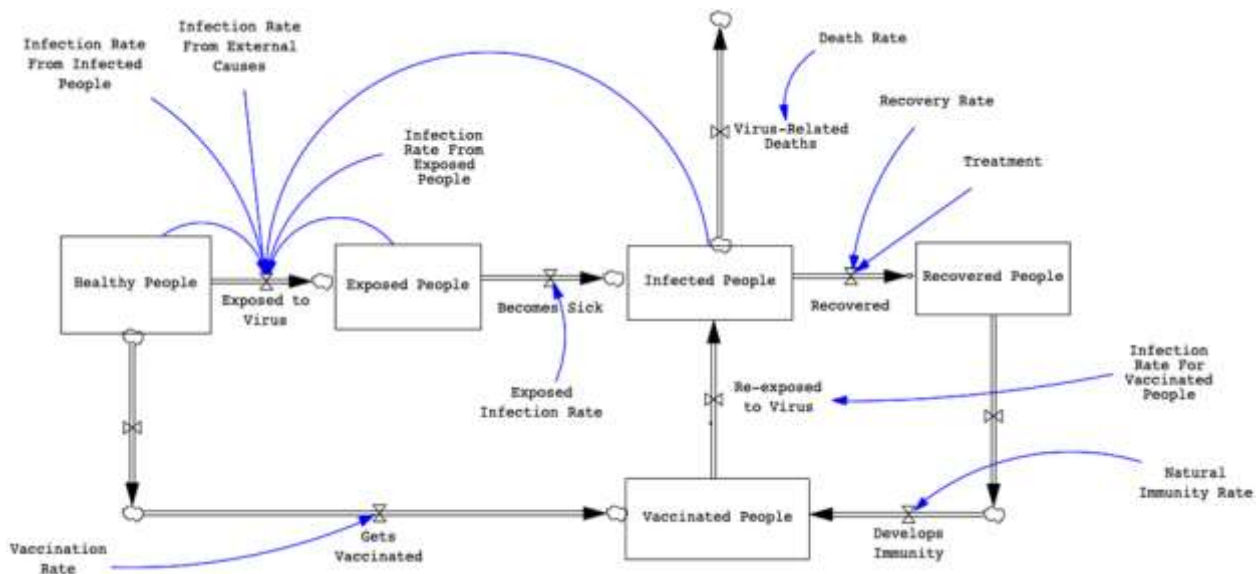
Rewriting the equations using the difference equation for approximating differential equations we receive 4 equations that we will use in excel to receive our solutions.

$$\begin{aligned} \frac{H(t + \Delta t)}{\Delta t} &= H(t) + \Delta t * (-ecr * H - epr * H * E - ipr * H * I) \\ \frac{E(t + \Delta t)}{\Delta t} &= E(t) + \Delta t * (ecr * H + epr * H * E + ipr * H * I - eir * E) \\ \frac{I(t + \Delta t)}{\Delta t} &= I(t) + \Delta t * (eir * E - rr * I - dr * I) \\ \frac{R(t + \Delta t)}{\Delta t} &= R(t) + \Delta t * (rr * I) \end{aligned}$$

Time	Healthy People	Exposed People	Infected People	Recovered People	Parameters	Initial Conditions
0	3600	50	15	1	delta t	0.01 H(0)=3600
0.01	3598.2531	51.7119	14.92005	1.09	InfectedPerson	0.000745 E(0)=50
0.02	3596.4633	53.46550165	14.84052613	1.1795203	External Cause R <sub>i</sub>	0.0001 I(0)=15
0.03	3594.629536	55.26184017	14.76142613	1.268563457	ExposedPerson	0.000745 R(0)=0
0.04	3592.750719	57.10197388	14.68274773	1.357132014	Death Rate	0.003
0.05	3590.825737	58.9869846	14.60448868	1.4452285	Recovery Rate	0.6
0.06	3588.853452	60.917978	14.52664676	1.532855432		
0.07	3586.832704	62.89608405	14.44921973	1.620015313	Exposed Infection	0.07
0.08	3584.762303	64.92245738	14.37220539	1.706710631		
0.09	3582.641037	66.99827768	14.29560153	1.792943863		
0.1	3580.467666	69.12475009	14.21940598	1.878717473		
0.11	3578.240923	71.3031056	14.14361654	1.964033908		
0.12	3575.959515	73.53460141	14.06823107	2.048895608		
0.13	3573.622121	75.8205213	13.9932474	2.133304994		
0.14	3571.227392	78.16217607	13.91866339	2.217264478		
0.15	3568.77395	80.56090381	13.84447691	2.300776459		
0.16	3566.260391	83.01807035	13.77068585	2.38384332		
0.17	3563.685279	85.53506958	13.69728809	2.466467435		
0.18	3561.047151	88.11332376	13.62428155	2.548651164		
0.19	3558.344511	90.75428393	13.55166413	2.630396853		
0.2	3555.575837	93.45943019	13.47943376	2.711706838		
0.21	3552.739574	96.230272	13.40758838	2.792583441		
0.22	3549.834136	99.06834855	13.33612593	2.873028971		



We can use the same concept to model covid in the more current conditions. Transmission of Covid-19 in most places has not actually decreased; however, there are other factors and variables that have allowed a decrease on death and infection in populations. Although most people have stopped wearing masks and social distancing, the large-scale immunization movement against Covid-19 can be contributed to one of the main reasons we have been able to control the spread and transmission within communities.



The change in Healthy People can be expressed by the equation

$$\frac{dH}{dt} = -ecr * H - epr * H * E - ipr * H * I - vr * H$$

The change in Exposed People can be expressed as

$$\frac{dE}{dt} = ecr * H + epr * H * E + ipr * H * I - eir * E$$

The change in Infected People can be expressed as

$$\frac{dI}{dt} = eir * E - rr * I - dr * I + vir * V * I$$

The change in Recovered People can be expressed as

$$\frac{dR}{dt} = rr * I$$

The change in Vaccinated People can be expressed as

$$\frac{dV}{dt} = vr * H - vir * V * I$$

Rewriting the equations using the difference equation for approximating differential equations for use in excel produces

$$\frac{H(t + \Delta t)}{\Delta t} = H(t) + \Delta t * (-ecr * H - epr * H * E - ipr * H * I - vr * H)$$

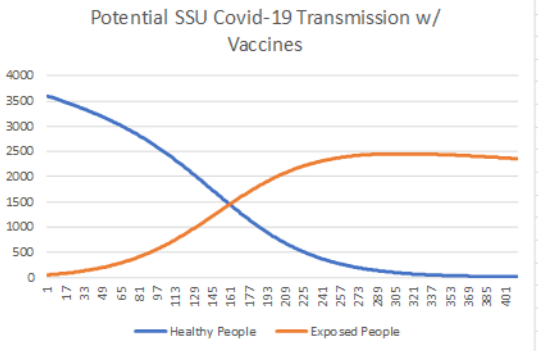
$$\frac{E(t + \Delta t)}{\Delta t} = E(t) + \Delta t * (ecr * H + epr * H * E + ipr * H * I - eir * E)$$

$$\frac{I(t + \Delta t)}{\Delta t} = I(t) + \Delta t * (eir * E - rr * I - dr * I)$$

$$\frac{R(t + \Delta t)}{\Delta t} = R(t) + \Delta t * (rr * I)$$

$$\frac{V(t + \Delta t)}{\Delta t} = V(t) + \Delta t * (vr * H - vir * V * I)$$

Time(days)	Healthy People	Exposed People	Infected People	Recovered People	Vaccinated People	Parameters	Initial Conditions
0	3600	50	15	1	0	Delta t	0.01 H(0)=3600
0.01	3592.4931	51.7119	14.94455	1.09	5.76	InfectedPersonRa	0.000745 E(0)=50
0.02	3584.95752	53.4632923	14.89080485	1.1796673	11.5078168	ExternalCausesRe	0.0001 I(0)=15
0.03	3577.392407	55.25504963	14.83878033	1.269012129	17.24340611	Exposed Persons	0.000745 R(0)=1
0.04	3569.796891	57.08805935	14.78849276	1.358044811	22.96672222	DeathRate	0.003 V(0)=0
0.05	3562.17009	58.96322346	14.73995908	1.446775768	28.67771796	Recovery Rate	0.6
0.06	3554.511108	60.88145875	14.6931968	1.535215522	34.37634468		
0.07	3546.819035	62.84369684	14.64822404	1.623374703	40.06255226	VaccinationRate	0.16
0.08	3539.092947	64.85088423	14.60505952	1.711264047	45.73628903	VaccInfected Rate	0.0002
0.09	3531.331904	66.90398238	14.5637226	1.798894404	51.39750178		
0.1	3523.534955	69.00396774	14.52423321	1.88627674	57.04613575	Exposed Infection	0.07
0.11	3515.701132	71.15183179	14.48661197	1.973422139	62.68213457		
0.12	3507.829455	73.34858105	14.45088008	2.060341811	68.30544028		
0.13	3499.918928	75.59523712	14.41705943	2.147047091	73.91599826		
0.14	3491.968541	77.89283664	14.38517253	2.233549448	79.51373224		
0.15	3483.977272	80.24243132	14.3524256	2.319860483	85.09859427		
0.16	3475.944082	82.64508788	14.32729338	2.405991939	90.67051469		
0.17	3467.86792	85.10188804	14.30134948	2.491955699	96.22942709		
0.18	3459.747719	87.61392846	14.27743609	2.577763796	101.7752633		
0.19	3451.582401	90.18232066	14.25557908	2.663428412	107.3079535		
0.2	3443.370871	92.80819094	14.23580504	2.748961887	112.8274259		
0.21	3435.112023	95.4926803	14.21814124	2.834376717	118.3336069		
0.22	3426.804735	98.23694431	14.2026157	2.919685564	123.8264212		
0.23	3418.447873	101.042153	14.1892571	3.004901259	129.3057914		
0.24	3410.040289	103.9094905	14.1780949	3.090036801	134.7716385		
0.25	3401.580823	106.8401554	14.16915924	3.175105371	140.2238814		
0.26	3393.068301	109.83536	14.16248103	3.260120326	145.662437		

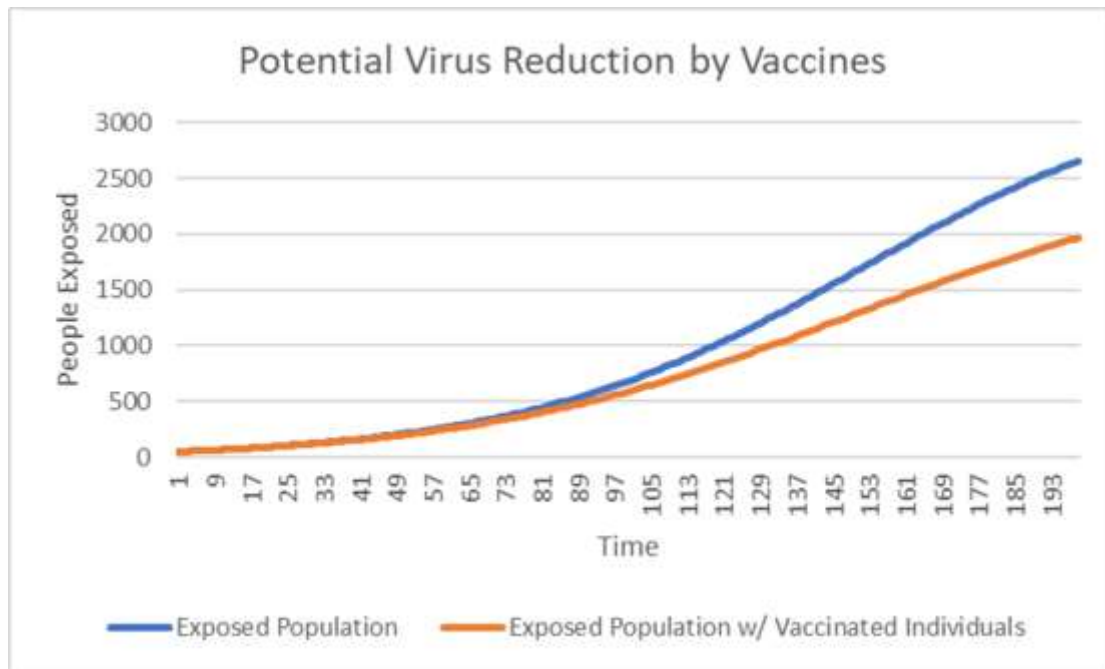


**Conclusion**

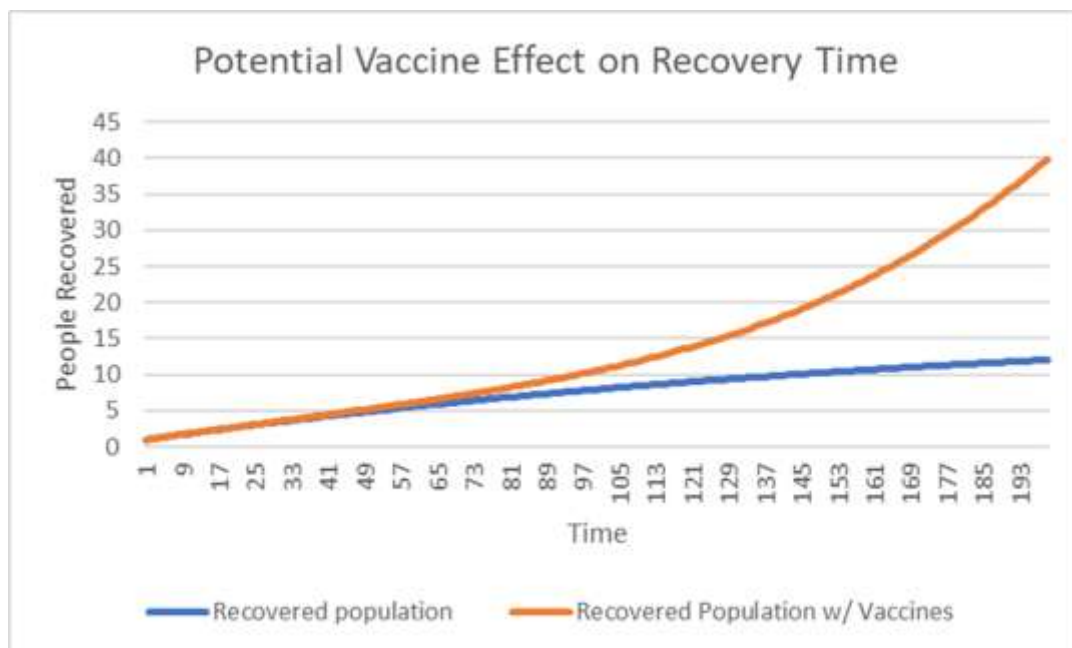
During our research, we encountered various obstacles and unforeseen variables. Our study focused on comparing COVID-19 conditions before and after vaccine distribution, incorporating multiple factors into our models. To our surprise, COVID-19 transmission rates fluctuated significantly throughout the pandemic, with some regions experiencing higher transmission than ever due to emerging variants.

Given these variations, we examined how immunization has shaped the pandemic. Vaccines played a crucial role in protecting high-risk and highly exposed populations while reducing the number of people susceptible to initial infection. They also significantly decreased mortality rates and lessened symptoms for individuals with major health risks. While early measures such as masking and social distancing helped control the virus’s spread, vaccination has been the primary factor in reducing deaths, hospitalizations, and community transmission.

To illustrate this, I developed a model using parameters based on the population of Savannah State University. By keeping assumed rates constant, we isolated the direct impact of vaccination. The results demonstrated that introducing vaccines into the environment led to a lower number of exposures over time. Although vaccines do not completely prevent infections, they reduce transmission within vaccinated populations, ultimately limiting overall exposure within the community.



The Covid-19 pandemic drastically changed the education system in this country. Students across the country were forced to leave their campuses and attend school virtually. Covid-19 transmission has not slowed as variants and waning vaccine immunity will continue to allow reinfection to happen in communities. But the introduction of vaccines and boosters will continue to be the reason why we can not only control Covid-19 spread but also drastically reduce the number of hospitalizations and deaths from Covid-19. In effectively immunizing as many people as possible, we reduce covid-19 exposure to a complete population of people until their immunity wanes. Our model also showed us that vaccines could potentially decrease recovery time in a community. Within our vaccinated population, more people recovered in the same time. This could be contributed to the vaccine's ability to reduce symptoms, hospitalizations, and deaths.



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