International Journal of Research Publication and Reviews, Vol 6, Issue 3, pp 1485-1491 March 2025

WWW.IJRPR.COM

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Prime Cordial Distance Labeling for Some Graphs

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DO : https://doi.org/10.55248/gengpi.6.0325.1146

A B S T R A C T

In this paper, we define and discuss prime cordial distance labeling, a mathematical graph labeling technique, where vertex labels ensure that the absolute difference between certain distance vertices is a prime number. This unique labeling method has distinct properties, that set it apart from other labeling techniques in Graph Theory.

Keywords: Graph Labeling, Vertex Labeling, Edge Labeling, Prime Labeling, Prime Cordial Labeling, and Prime Cordial Distance Labeling.

1.INTRODUCTION

In 1967, the Graph Labeling was introduced by Alexander Rosa [1]. The notion of prime labeling was introduced by Roger Entringer [9] and in 1982 Tout et al [13] explored prime labeling in Graphs where distinct integers are less than or equal to the number of vertices.

The idea of prime cordial labeling was introduced by Sundaram et al. [12] and in the same paper they have investigated several results on prime cordial labeling. Vaidya and Vihol [15] as well as Vaidya and Shah [16] have discussed prime cordial labeling in the context of some graph operations. Then, the concept of Prime Distance graphs was introduced in the year 2013 by Laison et al [4]. In this paper, we explore prime cordial distance labeling of various graphs.

2.PRELIMINARIES:

Definition 2.1 [3]

A graph G = (V, E) is a mathematical structure consisting of two sets V and E The elements of V are called vertices, and the elements of E are called edges. Each edge has a set of one or two vertices associated to it, which are called its endpoints.

Definition 2.2 [14]

A labeling f: v(G)⟶ $\{0,1\}$ is called binary vertex labeling of G and f(v) is called the label of the vertex v of G under f.

Definition 2.3[14]

If for an edge $e = u$v, the induced edge labelling f^*: $E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(\text{v})|$. Then

$v_f(i)$ = number of vertices of G having label i under

$e_f(i)$ = number of edges of G having label i under f^*, where i = 0 or 1.

Definition 2.4 [3]

The Graph labeling is an assignment of numbers to the vertices or edges or both subject to certain conditions. If the domain of the mapping is the set of vertices (edges), then the labeling is called a vertex labeling (edge labeling).

Definition 2.5 [10]

A bijection f: V→ {1,2,3……. n} induces an edge labeling f': E→ {0,1} such that for an edge uv in G f'(uv)=1 if gcd(f(u), f(v)) =1, and f'(uv)=0 otherwise. Such a labeling is called a prime labeling if f' (uv) ϵ E. We say G is a prime graph if it admits a prime labeling.

Definition 2.6 [14]

A binary vertex labeling f of a graph G is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph G is cordial if it admits cordial labeling.

Definition 2.7 [10]

A prime cordial labeling *of* a graph G with vertex set $V(G)$ is a bijection,

$f: V(G) \to \{1, 2, \ldots \ldots, |V(G)|\}$ and if the induced function $f*: E(G) \to \{0, 1\}$ is defined by

$f^*(e=uv) = \begin{cases} 1 & \text{if gcd } [f(u).f(v)] = 1 \\ 0 & \text{otherwise} \end{cases}$, then the number of edges labeled with 0 and the number of edges labelled with 1 differ by at most 1. A graph which admits prime cordial labeling is called a prime cordial graph.

Definition 2.8 [3]

A path graph P is a simple graph with $|V_p| = |E_p| + 1$ that can be drawn so that all of its vertices and edges lie on a single straight line. A path graph with n vertices and n-1 edges is denoted P_n.

Definition: 2.9 [8]

A connected graph that is regular of degree two is a cycle graph. We denote the cycle graph on n vertices by C_n.

Definition: 2.10 [17]

The Star Graph of order denoted S_n is a simple graph with n vertices with the following properties;

i)one distinguished vertex is of degree n-1,

ii)The remaining vertices are all of degree 1 and are adjacent only to the distinguished vertex.

Definition:2.11 [3]

A graph is connected if for every pair of vertices u and v, there is a walk from u to v.

Definition: 2.12 [18]

If G is a connected graph without any cycles, then G is called a Tree.

Definition: 2.13 [5]

The fish Graph is a special type of graphs. It has the six vertices and seven edges.

Definition: 2.14 [8]

A planar graph is a graph that can be drawn in the plane without crossings that is, so that no two edges intersect geometrically except at a vertex to which both are incident.

Definition: 2.15 [6]

A graph $B_{n,\, m}$ (where n and m are any arbitrary positive integers) is said to be a butterfly graph if "Two cycles of the same order sharing a common vertex with an arbitrary number of pendant edges attached at the common vertex".

Definition:2.16 [7]

The Helm graph H_n is the graph with 2n+1 vertices obtained from an n-wheel graph by adjoining a pendant edge at each node of the cycle.

Definiton.2.17 [2]

Let $\mathbb{L}n$ with $n \geq 2$ be a triangular ladder graph with $2n$ vertices. We denote the vertices and edges of this graph as follows.

$V(_n) = \{v_i: I = 1, 2 \ldots . 2n\}$,

$E(L_n) = \{ V_iV_{i+1}, V_iV_{i+2} : i = 1, 2 \ldots . 2n\text{-}2\} \cup \{v_{2n-1}, v_{2n}\}$.

Definition: 2.18 [11]

A Diamond network for $n \geq 3$ is gained by connection of a single vertex x to all other vertices V_i, (i= 1,2,3……n) of the triangular ladder graph TL_n.

Definition: 2.19 [6]

A Lotus inside a circle denoted LC_n is obtained from C_n: $v_1, v_2, v_3 \ldots \ldots v_n, v_1$ and $K_{1,\, n}$ (with the central vertex v_0 and the pendant vertices $u_1, u_2, u_3 \ldots \ldots u_n$) by joining each u_i to v_i and v_{i+1} (mod n).

3.MAIN RESULTS:

Definition 3.1

A bijection f: V(G)→ $\{1,2\ldots.\,|V(G)|\}$ induces an edge labeling $f^{*:}$ E(G)→ {0,1} such that

$f^*(e=uv) = \begin{cases} 1 & \text{if gcd } [f(u).f(v)] = 1 \\ 0 & \text{otherwise} \end{cases}$ and |f(u)-f(v)| is prime number,

such a labeling is called a **prime cordial distance labeling.**

Theorem 3.2

The path graph, P_n, admits a prime cordial distance labeling.

Proof:

Consider the path graph P with vertices $v_1, v_2 \ldots\ldots v_n$ edges $e_1, e_2, e_3 \ldots\ldots e_{n-1}$.

Define, f: V(G)→ $\{1,2\ldots.\,|V(G)|\}$ induces an edge labeling f^*: $E(G) \rightarrow \{0, 1\}$ such that

$f^*(e=uv) = \begin{cases} 1 & \text{if gcd } [f(u).f(v)] = 1 \\ 0 & \text{otherwise} \end{cases}$ and, |f(u)-f(v)| is prime number.

Therefore, the conditions of prime cordial distance labeling are satisfied. So, it follows that P_n, admits a prime cordial distance labeling.

Illustration 3.3

The Prime Cordial Distance Labeling of P_5 is given **Fig.1**

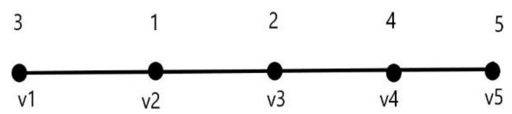

Fig.1

Theorem 3.4

Cycle graph C_7 admits prime cordial distance labeling.

Proof:

Let C_7 be the cycle graph with vertices v_1, v_2, v_3, v_4, …. v_7 and edges $e_1, e_2 \ldots\ldots e_7$ respectively. The prime cordial distance labeling conditions are satisfied by cycle graph. Therefore, the cycle graph C_7 admits a prime cordial distance labeling.

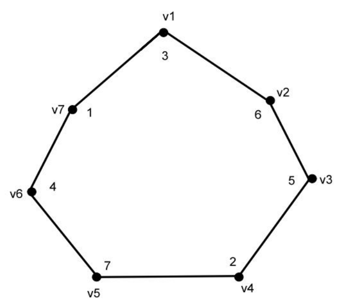

Fig-2

Theorem 3.5

Star graph S_{10} admits prime cordial distance labeling.

Proof:

Consider a star graph with vertices v_1, v_2, v_3, v_4, …. v_{10} and edges $e_1, e_2 \ldots\ldots e_{15}$ respectively. Since the conditions for prime cordial distance labeling are satisfied, it follows that S_{10} admits prime cordial distance labeling.

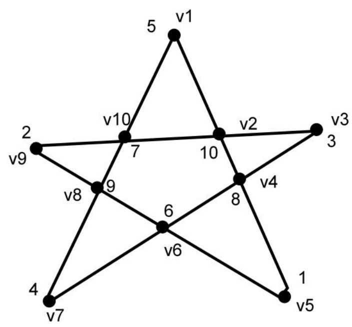

Fig-3

Theorem 3.6

Tree graph T_{11} admits prime cordial distance labeling.

Proof:

Consider the tree graph T_{11} with vertices v_1, v_2, v_3, v_4, …. v_{11} and edges e_1, e_2…... e_{10}. The conditions for prime cordial distance labeling are satisfied and we conclude that the tree graph admits a prime cordial distance labeling.

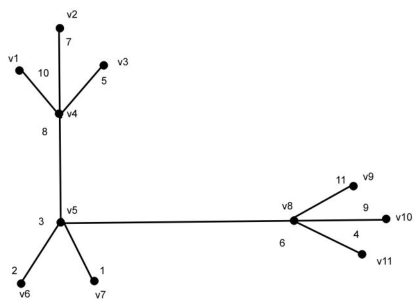

Fig-4

Theorem 3.7

Fish graph F_6 admits prime cordial distance labeling.

Proof:

Let the Fish graph with the vertices v_1, v_2, v_3, v_4, …. v_6 and edges e_1, e_2…... e_7 respectively. The conditions for prime cordial distance labeling are satisfied, it follows that F_6 admits the prime cordial distance labeling.

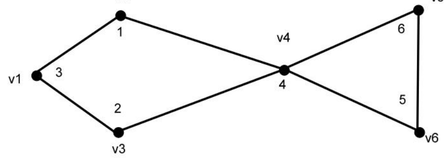

Fig -5

Theorem 3.8

Planar Graph admits prime cordial distance labeling.

Proof:

Consider the planar graph with vertices v_1, v_2, v_3, v_4, …. v_6 and edges e_1, e_2…... e_8 respectively. The conditions for prime cordial distance labeling are satisfied and we conclude that the planar graph P_6 admits a prime cordial distance labeling.

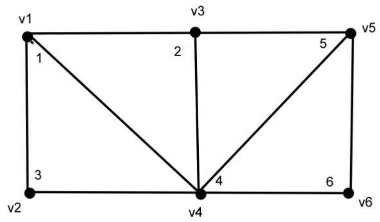

Fig-6

Theorem 3.9

Butterfly Graph BF_7 admits prime cordial distance labeling.

Proof:

Let BF_7 be the Butterfly graph with vertices v_1, v_2, v_3, v_4, …. v_7 and edges e_1, e_2…... e_{10} respectively. The prime cordial distance labeling conditions are satisfied by the butterfly graph. Therefore, the butterfly graph BF_7 admits a prime cordial distance labeling.

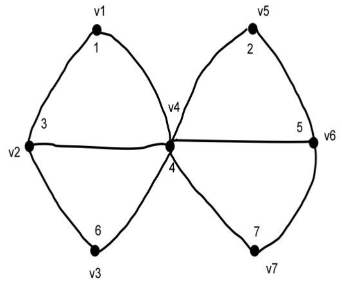

Fig-7

Theorem 3.10

Helm Graph H_{13} admits prime cordial distance labeling.

Proof:

Consider the Helm graph H_{13} with a vertices v_1, v_2, v_3, v_4, …. v_{13} and edges e_1, e_2…... e_{18} respectively. The conditions for prime cordial distance labeling are satisfied by the helm graph and we conclude that the helm graph H_{13} admits a prime cordial distance labeling.

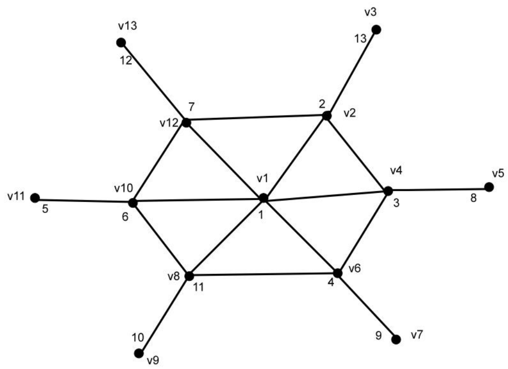

Fig-8

Theorem 3.11

Diamond graph admits prime cordial distance labeling.

Proof:

Let the Diamond graph with the vertices v_1, v_2, v_3, v_4, …. v_{18} and edges e_1, e_2…... e_{33} respectively. since the conditions for prime cordial distance labeling are satisfied, it follows that admits the prime cordial distance labeling.

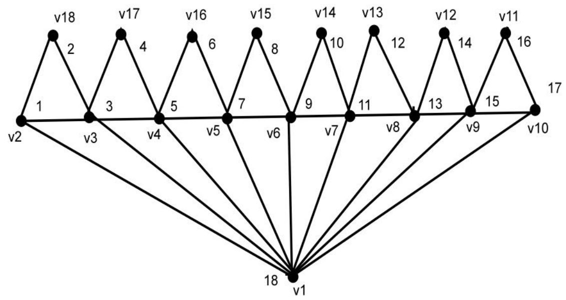

Fig-9

Theorem 3.12

Lotus graph L_{13} admits prime cordial distance labeling.

Proof:

Consider the Lotus graph L_{13} with vertices $v_1, v_2, v_3, v_4, \ldots. v_{13}$ and edges $e_1, e_2\ldots\ldots e_{24}$.The conditions for prime cordial distance labeling are satisfied and we conclude that the Lotus graph L_{13} admits a prime cordial distance labeling.

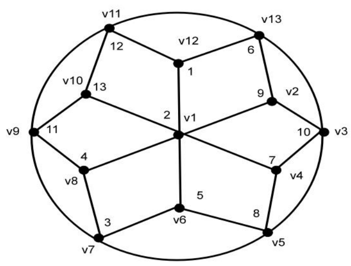

Fig-10

Theorem 3.13

Let G be a graph of order n≥ 2 with a vertex of degree n-1. Then G is a prime if and only if G$\cong p_2$ or p_3

Proof:

Since G has a vertex of degree n-1 = 2n-1 or 2m, it follows that m=1. It is obvious that, G$\cong p_2$ or p_3.

CONCLUSION:

This paper investigated the concept of prime cordial distance labeling for various graphs and underscores the importance of Prime Cordial Distance Labeling in graph Theory. This study has enhanced for deeper explorations and broader applications for various graphs such as Path graph, Cycle graph, Star graph, Tree graph, Planar graph, Fish graph, Butterfly graph, Diamond graph, Lotus graph.

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