



Computation of fluctuating parts of probable distribution function through plasma wave- particle interaction in the Polar Ionosphere

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ABSTRACT :

There are wave-wave interactions and wave-particle interactions through non-linear energy exchange processes takes place in both space and laboratory plasma environment .In this paper we present our study on derivation of fluctuating parts of distribution function due to wave-particle interaction in ionospheric plasma.

Keywords : Plasma Waves, Wave-Particle Interaction, Fluctuating Parts

Introduction :

The Polar ionosphere is one of the special investigative arenas on energy exchange process among different physical entities of the Earth's ionosphere different from other atmospheric zones. Solar radiations in the Far Ultraviolet (FUV) and Extreme Ultraviolet (EUV) wavelength range is one of the major energy and momenta source to this region. This region being illuminated by sun lights continuously in summer season and in winter, it is completely dark up to nearly 6 months. Geomagnetic field is nearly vertical in polar cap region in the northern and southern hemi- spheres, for this the solar wind affect is closely connected with this near space Earth's environment which may provide an additional ionization source with aurora through solar energetic protons(SEP). The variable intensity of solar wind and the rotation of the Earth also affect on the conditions of the Polar ionosphere [1,2]. There are turbulence and instabilities are recorded in this thermodynamically non equilibrium polar ionospheric plasma from ground based observatories and low altitude satellites. This atmospheric domain is supporting different types of electrostatic and electromagnetic plasma waves. Observation of nonlinear stimulating processes in this zone can lead to interesting theoretical investigations on different aspects[3,4,5,6,7,8].

In space plasma , the wave energy of high frequency modes was found to be enhanced in presence of low frequency modes waves. A different type of non-linear wave-particle interaction called plasma maser effect was developed by Nambu [9] on the platform of nonlinear theory of turbulent plasma and it predicts energy up conversion of non-resonant waves in the presence of resonant waves. Plasma maser instability arises from the interaction between particles and the nonlinear modulating electric field which occurs for the coupling between resonant and non-resonant plasma mode. For this nonlinear field, charged particles are accelerated nonlinearly to generate either amplified electrostatic wave or electromagnetic radiation through modulated field.

In this study we wish to present our study on computation of fluctuating parts of distribution function due to high frequency non resonant waves with low frequency resonant waves in presence of plasma particles like electron in open plasma system.

Formulation :

Consider a semi infinite bounded inhomogeneous plasma which is confined by a magnetic field. Let the plasma density decreases in the x-direction is balanced by a magnetic field that increases with x.

For this system, the zeroth order particle distribution function for electron is considered as [10]

$$f_{e0} = f_{e0}(v_x^2, v_y) [1 - \varepsilon'(x + \frac{v_y}{\Omega_e})] \quad (1)$$

$$\varepsilon' = - \frac{1}{f_{e0}} \frac{\partial f_{e0}}{\partial x} \Big|_{x=x_0} = \text{Density gradient}$$

$$\Omega_e = \frac{eB_0}{m_e c} = \text{Electron cyclotron frequency}$$

The interaction of Langmuir wave with ion acoustic wave is generated by the Vlasov-Poisson system of equations which are

$$\left[\frac{\partial}{\partial t} + \vec{v} \cdot \frac{\partial}{\partial \vec{r}} - \frac{e}{m} (\vec{E} + \frac{\vec{v} \times \vec{B}}{c}) \cdot \frac{\partial}{\partial \vec{v}} \right] \bar{F}(\vec{r}, \vec{v}, t) = 0 \tag{2}$$

$$\vec{\nabla} \cdot \vec{E} = -4\pi n_e e \int f(\vec{r}, \vec{v}, t) d\vec{v} \tag{3}$$

The unperturbed distribution function for electrons, the unperturbed electric field and the unperturbed magnetic fields are taken as

$$F_{oe} = f_{oe} + \epsilon f_{1e} + \epsilon^2 f_{2e} \tag{4}$$

$$\vec{E}_{o1} = \epsilon \vec{E}_1$$

$$\vec{B}_{o1} = \hat{z} B_0 (1 + \epsilon^n x) = \vec{B}_0(x)$$

Where

f_{oe} =Space and time averaged part of the distribution function

f_{1e} and f_{2e} =Fluctuating parts due to low frequency ion acoustic turbulence

ϵ = Ordering of the low-frequency ion acoustic wave turbulence field $\vec{E}_1 = (0, 0, E_{1z})$ with propagation vector $\vec{k} = (0, 0, k_z)$

Using Fourier transform and the method of characteristics [10], it has been found from equation (2) as

$$\begin{aligned} f_{1e}(\vec{k}, \omega) &= \sum \frac{e}{m} \int_{-\alpha}^0 (\vec{E}_1 \cdot \frac{\partial}{\partial \vec{v}} f_{oe}) \exp\{i\{\vec{k} \cdot (\vec{r}' - \vec{r}) - \omega \tau\}\} d\tau \\ &= \sum \frac{e}{m} \int_{-\alpha}^0 (E_{1z} \frac{\partial}{\partial v_z} f_{oe}) \exp\{i\{k(z' - z) - \omega \tau\}\} d\tau \\ &= \frac{ie}{m} \frac{E_{1z}}{\omega - k_z v_z + i0^+} \frac{\partial f_{oe}}{\partial v_z} \end{aligned} \tag{5}$$

The quasisteady state of the system is now perturb by the test longitudinal Langmuir wave field $\mu \delta \vec{E}_h$ with wave vector $\vec{K} = (0, K_\perp, K_z)$. The total perturbed electric field, magnetic field and the electron distribution function due to this perturbation are

$$\begin{aligned} \delta \vec{E} &= \mu \delta \vec{E}_h + \mu \epsilon \delta \vec{E}_{lh} + \mu \epsilon^2 \Delta \vec{E} \\ \delta \vec{B} &= 0 \\ \delta f_e &= \mu \delta f_h + \mu \epsilon \delta f_{lh} + \mu \epsilon^2 \Delta f \end{aligned} \tag{6}$$

Where

$\delta \vec{E}_{lh}$ and $\Delta \vec{E}$ are modulating fields

δf_h is the fluctuating part due to high frequency Langmuir wave

δf_{lh} and Δf are particle distribution function corresponds to modulating fields

Applying Fourier transform and integrating along the unperturbed orbit, we can evaluate the fluctuating part δf_h of the distribution function due to high frequency longitudinal Langmuir wave $(0, K_\perp, K_z)$ over the particle trajectories. Here fluctuating part due to high frequency plasma wave

$$\begin{aligned} \delta f_h &= \frac{e}{m} \int_{-\alpha}^0 \delta \vec{E}_h \cdot \frac{\partial f_{oe}}{\partial \vec{v}} \exp[i\{\vec{K} \cdot (\vec{r}' - \vec{r}) - \omega \tau\}] d\tau \\ &= \frac{e}{m} \int_{-\alpha}^0 (\delta E_{hy} \frac{\partial}{\partial v_y} + \delta E_{hz} \frac{\partial}{\partial v_z}) f_{oe} \exp[i\{K_\perp(y' - y) + K_z(z' - z) - \Omega \tau\}] d\tau \end{aligned}$$

After lengthy calculations it has been found

$$\begin{aligned} \delta f_h &= -\sum \frac{e}{m} f_{oe} \delta E_{hy} \frac{m}{i T_e K_\perp} [1 + i(\Omega - K_z v_z + \frac{\epsilon' K_\perp T_e}{m \Omega_e}) \\ &\quad J_a(\frac{v_\perp K_\perp}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}}) J_b(\frac{v_\perp K_\perp}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}}) \exp\{i(a-b)(\theta - \frac{\pi}{2})\} \\ &\quad \sum_{a,b} \frac{1}{i(a(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_\perp K_\perp}{2\Omega_e} + K_z v_z - \Omega)} \\ &\quad J_a(\frac{v_\perp K_\perp}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}}) J_b(\frac{v_\perp K_\perp}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}}) \exp\{i(a-b)(\theta - \frac{\pi}{2})\} \\ &+ \sum \frac{e}{m} \delta E_{hz} \frac{\partial f_{oe}}{\partial v_z} \sum_{a,b} \frac{1}{i(a(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_\perp K_\perp}{2\Omega_e} + K_z v_z - \Omega)} \end{aligned}$$

Using Fourier transform and the method of characteristics it has been found

$$\begin{aligned} \delta f_{ih} &= \sum_{m=-\alpha}^0 \frac{e}{m} \int (\delta \bar{E}_{ih} \frac{\partial}{\partial v} f_{oe} + \delta \bar{E}_h \frac{\partial}{\partial v} f_{ie} + \bar{E}_l \frac{\partial}{\partial v} \delta f_h) \cdot \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\tau\}] d\tau \\ &= I_{ih}^1 + I_{ih}^2 + I_{ih}^3 \end{aligned} \tag{7}$$

Here, after lengthy calculations it has been found

$$\begin{aligned} I_{ih}^1 &= \sum_{m=-\alpha}^0 \frac{e}{m} \int \delta E_{ih} \frac{\partial}{\partial v} f_{oe} \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\}] d\tau \\ &= \sum \frac{e}{im} \frac{\delta E_{ih}}{k_{\perp}} \frac{m}{T_e} [1 + i\{(\Omega - \omega) - (K_{\perp} - k_{\perp})v_{\perp} - \frac{\epsilon' T_e k_{\perp}}{\Omega_e m}\}] \\ &\quad \frac{J_a \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) J_b \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) \exp\{i(a-b)(\theta - \frac{\pi}{2})\}}{i\{a(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_{\perp} K_{\perp}}{2\Omega_e} + (K_{\perp} - k_{\perp})v_{\perp} - (\Omega - \omega)\tau\}} \\ &\quad \sum \frac{J_s \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) J_t \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) \exp\{i(s-t)(\theta - \frac{\pi}{2})\}}{i\{s(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_{\perp} K_{\perp}}{2\Omega_e} + (K_{\perp} - k_{\perp})v_{\perp} - (\Omega - \omega)\tau\}} \end{aligned} \tag{8}$$

$$\begin{aligned} I_{ih}^2 &= \sum_{m=-\alpha}^0 \frac{e}{m} \int \delta \bar{E}_h \frac{\partial}{\partial v} f_{ie} \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\}] d\tau \\ &= \sum_{m=-\alpha}^0 \frac{e}{m} \int (\delta E_{iy} \frac{\partial}{\partial v_y} + \delta E_{iz} \frac{\partial}{\partial v_z}) f_{ie} \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\}] d\tau \\ &= \sum \left(\frac{e}{m} \right)^2 \frac{\delta E_h}{|\bar{K}|} E_l \left[K_{\perp} \frac{\frac{\partial f_{oe}}{\partial v_{\perp}}}{\omega - k_{\perp} v_{\perp} + i0^+} \left(\frac{m}{k_{\perp} T_e} \right) \left(1 + i\{(\Omega - \omega) - (K_{\perp} - k_{\perp})v_{\perp} - \frac{\epsilon' T_e k_{\perp}}{\Omega_e}\} \right) + iK_{\perp} \frac{\partial}{\partial v_{\perp}} \left(\frac{\frac{\partial f_{oe}}{\partial v_{\perp}}}{\omega - k_{\perp} v_{\perp} + i0^+} \right) \right] \\ &\quad \frac{J_s \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) J_t \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) \exp\{i(s-t)(\theta - \frac{\pi}{2})\}}{i\{s(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_{\perp} K_{\perp}}{2\Omega_e} + (K_{\perp} - k_{\perp})v_{\perp} - (\Omega - \omega)\tau\}} \end{aligned} \tag{9}$$

$$\begin{aligned} I_{ih}^3 &= \sum_{m=-\alpha}^0 \frac{e}{m} \int \bar{E}_l \frac{\partial}{\partial v} \delta f_h \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\tau\}] d\tau \\ &= \sum_{m=-\alpha}^0 \frac{e}{m} \int E_l \frac{\partial}{\partial v_{\perp}} \delta f_h \exp[i\{(\bar{K} - \bar{k}) \cdot (\bar{r}' - \bar{r}) - (\Omega - \omega)\tau\}] d\tau \\ &= \sum \frac{e}{m} E_l \frac{\partial}{\partial v_{\perp}} \left[\frac{e}{m} \left(\frac{\delta E_{iy}}{iT_e K_{\perp}} f_{oe} \{1 + i(\Omega - K_{\perp} v_{\perp} + \frac{\epsilon' K_{\perp} T_e}{m \Omega_e})\} + \delta E_{iz} \frac{\partial f_{oe}}{\partial v_{\perp}} \right) \times \right. \\ &\quad \frac{J_a \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) J_b \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) \exp\{i(a-b)(\theta - \frac{\pi}{2})\}}{i \left(a(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_{\perp} K_{\perp}}{2\Omega_e} + K_{\perp} v_{\perp} - \Omega \right)} \times \\ &\quad \left. \frac{J_s \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) J_t \left(\frac{v_{\perp} K_{\perp}}{1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}} \right) \exp\{i(s-t)(\theta - \frac{\pi}{2})\}}{i \left(s(1 + \epsilon'' x - \frac{\epsilon'' v_y}{\Omega_e}) - \frac{\epsilon'' v_{\perp} K_{\perp}}{2\Omega_e} + K_{\perp} v_{\perp} - \Omega \right)} \right] \end{aligned} \tag{10}$$

Conclusion :

From these fluctuating parts of distribution function and with the help of continuity equation, momentum equations and using quasi neutrality condition [11] it has been found dispersion relation from which the approximate growth rate of high frequency wave can be estimated from coupling terms. By using observational data from ground and space based observatories the approximate growth rate of high frequency wave can be evaluated in polar ionospheric region.

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