International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Some Mean Labelings for Extended Triplicate of Bistar Graph

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ABSTRACT

In 2003[7], Somasundaram et.al, introduced the concept of mean labeling. In this paper, we investigate the existence of some mean labelings for extended triplicate graph of bistar graph.

Keywords: Graph labeling, Star graph, Mean labeling.

1. INTRODUCTION

In 1967[8], Rosa introduced the concept of graph labeling. Assigning an integer to the edges or vertices or to both on certain conditions is said to be a graph labeling. In 2023[3], Bala .et.al. introduced the concept of Extended triplicate graph of star ETG($k_{1,p}$).

In 2019[1],Alamelu studied the concept of Even mean labeling. Let $G = (\delta(G), \beta(G))$ be a graph with p vertices and q edges. An injective function $S: \delta(G) \to \{2,4,6, \dots ,2q\}$ is said to be a Even mean labeling . if an induced function $S^*: \beta(G) \to N$ defined as $S^*(bc) = \frac{s(b)+s(c)}{2}$,$\forall\, bc \,\epsilon\, \beta(G)$, in which the resulting each edge labels are distinct. A graph which admits an even mean labeling is called as Even mean graph.

In 2006[6], Manickam and Marudai introduced the concept of Odd mean labeling of a graph. Let $G = (\delta(G), \beta(G))$ be a graph with p vertices and q edges. An injective function $S: \delta(G) \to \{1,3,5, \dots ,2q-1\}$ is said to be a odd mean labeling. If an induced function $S^*: \beta(G) \to N$ is defined as $S^*(bc) = \frac{s(b)+s(c)}{2}$,$\forall\, bc \,\epsilon\, \beta(G)$, with the resulting distinct edge labels. A graph which admits an odd mean labeling is called as Odd mean graph.

In 2017[5], The concept of Root cube mean labeling was introduced by Gowri.et.al.,. Let G be a simple graph with p vertices and q edges. A bijective function S: δ(G) $\to \{1,2, \dots , q+1\}$ is said to be Root cube mean labeling. If an induced function S^*: β(G) $\to N$ is defined by $S^*(bc) = \left\lceil\sqrt{\frac{S(b)^3+S(c)^3}{2}}\right\rceil$ or $\left\lfloor\sqrt{\frac{S(b)^3+S(c)^3}{2}}\right\rfloor$,then the resulting edges are distinct. A graph which admits a root cube mean labeling is called as Root cube mean graph.

Motivated by the preceding study, In this paper we investigate the existence of Even mean labeling, Odd mean labeling and Root cube mean labeling in the context of Extended Triplicate graph of bistar.

2. MAIN RESULT

In this section, we examine the existence of Even mean labeling, Odd mean labeling, Root cube mean labeling in the Extended Triplicate graph of bistar.

2.1 STRUCTURE OF EXTENDED TRIPLICATE OF BISTAR GRAPH

Let G be a bistar graph $B_{(p,l)}$ .The triplicate of bistar graph with the vertex set $\delta'(G)$and edge set $\beta'(G)$is given by $\delta'(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_j \cup d_j' \cup d_j'' / 1 \le i \le p , 1 \le j \le l \}$ and $\beta'(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_j' \cup b_1''d_j' \cup b_1'd_j \cup b_1'd_j''/1 \le i \le p, 1 \le j \le l \}$. Clearly, Triplicate of bistar graph TG($B_{p,l}$) with $3(p + l + 2)$ vertices and $4(p + l + 1)$ edges is disconnected. To make this as a connected graph include a new edge $b'b_1'$ to the edge set of $\beta'(G)$.Thus, we get an Extended triplicate graph of bistar with vertex set $\delta(G) = \delta'(G)$ and edge set $\beta(G) = \beta'(G) \cup b'b_1'$ denoted by ETG($B_{p,l}$). Clearly, ETG($B_{p,l}$) has $3(p + l + 2)$ vertices and $4(p + l) + 5$ edges.

THEOREM 2.1: Extended triplicate of bistar graph is an odd mean graph.

PROOF: Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex set

$\delta(G) = \{b \cup b' \cup b'' \cup b_1 \cup b_1' \cup b_1'' \cup c_i \cup c_i' \cup c_i'' \cup d_i \cup d_i' \cup d_i''/ 1 \le i \le p \ \}$ and edge set

$\beta(G) = \{bc_i' \cup b''c_i' \cup b'c_i \cup b'c_i'' \cup bb_1' \cup b''b_1' \cup b'b_1 \cup b'b_1'' \cup b_1d_i' \cup b_1''d_i' \cup b_1'd_i \cup b_1'd_i''/1 \le i \le p \}$. Clearly, It has $6(p + 1)$ vertices and $(8p + 5)$ edges.

To show that ETG($B_{p,p}$) is an odd mean graph.

Define an injective function $S: \delta(G) \to \{1,3,5, \ldots, (2(8p+5)-1)\}$ to label the vertices as follows.

$s(b) = 4(3p+2)+1$	$s(b') = 1$	$s(b'') = 2(7p+4)+1$
$s(b_1) = 4(3p+2)-1$	$s(b_1') = 4p+3$	$s(b_1'') = 2(7p+4)-1$
For $1 \leq i \leq p$		
$s(c_i) = i+2$	$s(c_i') = 4\left(2p+\frac{i}{2}\right)+3$	$s(c_i'') = 2(p+i)+1$
$s(d_i) = 2(2p+i)+3$	$s(d_i') = 2(7p+i)+9$	$s(d_i'') = 2(3p+i)+3$

Define an induced function $S^*: \beta(G) \to N$ by $S^*(bc) = \frac{s(b)+s(c)}{2}$, $\forall\, bc\, \epsilon \beta(G)$ to obtain the labels of edges as follows.

$S^*(bb_1') = 2(4p+3)$	$S^*(b'b_1'') = 7p+4$
$S^*(b''b_1') = 3(3p+2)$	$S^*(b'b_1') = 2(p+1)$
$S^*(b'b_1) = 2(3p+2)$	
For $1 \leq i \leq p$	
$S^*(b'c_i) = i+1$	$S^*(b_1d_i') = 13p+i+8$
$S^*(bc_i') = 2(5p+3)+i$	$S^*(b_1'd_i) = 4p+i+3$
$S^*(b''c_i') = 11p+i+6$	$S^*(b_1'd_i'') = 5p+i+3$
$S^*(b'c_i'') = (p+1)+i$	$S^*(b_1''d_i') = 2(7p+4)+i$.

Thus, the resulting edge labels are distinct.

Hence, Extended triplicate of bistar graph is an odd mean graph.

EXAMPLE 2.1: ETG($B_{3,3}$) and its odd mean labeling is shown in figure 1.

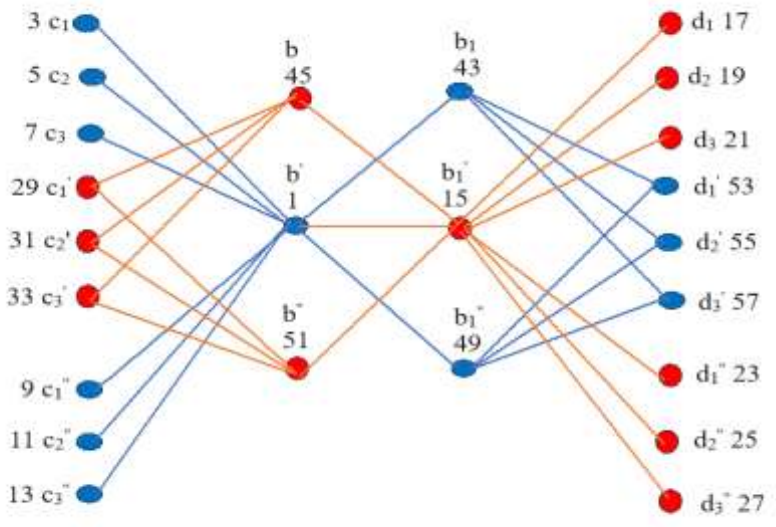

FIGURE - 1

THEOREM 2.2: Extended triplicate of bistar graph is Even mean graph.

PROOF: Extended triplicate of bistar graph ETG($B_{p,p}$) has vertex sethas $6(p+1)$ vertices and $(8p+5)$ edges.

To show that ETG($B_{p,p}$) is an Even mean graph.

Define a function $S: \delta(G) \to \{2,4,6, \ldots, 2(8p+5)\}$ to label the vertices as follows.

$s(b) = 2(4p + 3)$	$s(b') = 2$	$s(b'') = 2(5p + 3)$
$s(b_1) = 2(6p + 5)$	$s(b_1') = 4(p + 1)$	$s(b_1'') = 2(7p + 5)$
For, $1 \leq i \leq p$		
$s(c_i) = 2(i + 1)$	$s(c_i') = 2(5p + i)$	$s(c_i'') = 2(p + i + 1)$
$s(d_i) = 2(i + 2p + 2)$	$s(d_i') = 2(7p + i + 5)$	$s(d_i'') = 2(3p + i + 2)$

Define an induced function $S^*: \beta(G) \to N$ by $S^*(bc) = \frac{s(b)+s(c)}{2}$, $\forall\ bc\ \epsilon\beta(G)$ to obtain the edge labels as follows.

$S^*(bb_1'') = 6p + 5$	$S^*(b'b_1'') = 7p + 6$
$S^*(b'b_1) = 6(p + 1)$	$S^*(b'b_1') = 2p + 3$
$S^*(b''b_1') = 7p + 5$	
For, $1 \leq i \leq p$	
$S^*(bc_i') = 9p + i + 6$	$S^*(b_1d_i') = 13p + i + 10$
$S^*(b'c_i) = 2 + i$	$S^*(b_1'd_i) = 4(p + 1) + i$
$S^*(b'c_i'') = p + i + 2$	$S^*(b_1'd_i'') = 5p + i + 4$
$S^*(b''c_i') = 10p + i + 6$	$S^*(b_1''d_i') = 2(7p + 5) + i$

Thus, the edges receives distinct labels.

Hence, Extended triplicate of bistar graph is an Even mean graph.

EXAMPLE 2.2: Figure 2 shows the ETG($B_{3,3}$) and its Even mean labeling.

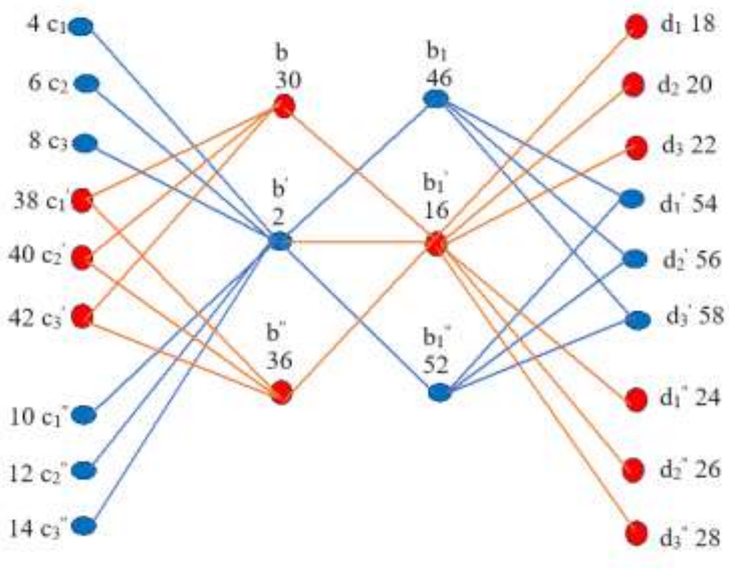

FIGURE - 2

THEOREM 2.3: Extended triplicate of bistar graph is a root cube mean graph.

PROOF: Extended triplicate of bistar graph ETG($B_{p,p}$) has $6(p + 1)$ vertices and $(8p + 5)$ edges.

To show that ETG($B_{p,p}$) is a root cube mean graph.

Define the bijective function $S: \delta(G) \to \{1,2,3,4, \dots, (8p + 5) + 1\}$ to label the vertices as follows.

$s(b) = 4$	$s(b') = 2$	$s(b'') = 8p + 5$
$s(b_1) = 3$	$s(b_1') = 1$	$s(b_1'') = 8p + 6$
For, $1 \leq i \leq p$		
$s(c_i) = 2i + 5$	$s(c_i') = 2(i + 2p + 3)$	$s(c_i'') = 2(p + i) + 5$
$s(d_i) = 2(i + 3)$	$s(d_i') = 2(i + 2p) + 5$	$s(d_i'') = 2(p + i + 3)$

Define an induced function $S^*: \beta(G) \rightarrow N$ by $S^*(bc) = \left\lfloor \sqrt{\frac{s(b)^3 + s(c)^3}{2}} \right\rfloor or \left\lceil \sqrt{\frac{s(b)^3 + s(c)^3}{2}} \right\rceil$, $\forall\, bc\, \epsilon \beta(G)$ to get the edge labels as follows.

$S^*(bb_1') = 5$	$S^*(b'b_1') = 2$
$S^*(b'b_1) = 4$	$S^*(b''b_1') = \left\lfloor \sqrt{\frac{[8p+5]^3 + (1)^3}{2}} \right\rfloor$
$S^*(b'b_1'') = \left\lfloor \sqrt{\frac{(2)^3 + [8p+6]^3}{2}} \right\rfloor$	
For, $1 \leq i \leq p$	
$S^*(bc_i') = \left\lfloor \sqrt{\frac{[4]^3 + (2(i+2p+3))^3}{2}} \right\rfloor$	$S^*(b_1d_i') = \left\lfloor \sqrt{\frac{[3]^3 + (2(i+2p)+5)^3}{2}} \right\rfloor$
$S^*(b'c_i) = \left\lfloor \sqrt{\frac{[2]^3 + (2i+5)^3}{2}} \right\rfloor$	$S^*(b_1'd_i) = \left\lfloor \sqrt{\frac{[1]^3 + (2(i+3))^3}{2}} \right\rfloor$
$S^*(b'c_i'') = \left\lfloor \sqrt{\frac{[2]^3 + (2(p+i)+5)^3}{2}} \right\rfloor$	$S^*(b_1'd_i'') = \left\lfloor \sqrt{\frac{[1]^3 + (2(p+i+3))^3}{2}} \right\rfloor$
$S^*(b''c_i') = \left\lfloor \sqrt{\frac{[8p+5]^3 + (2(i+2p+3))^3}{2}} \right\rfloor$	$S^*(b_1''d_i') = \left\lfloor \sqrt{\frac{[8p+6]^3 + (2(i+2p)+5)^3}{2}} \right\rfloor$

Thus, the resulting edge labels are distinct. Hence, Extended triplicate of bistar graph is a root cube mean graph.

EXAMPLE 2.3: ETG($B_{3,3}$) and its root cube mean labeling is shown in figure 3.

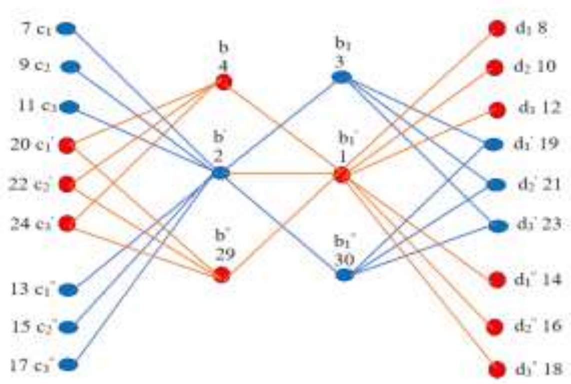

FIGURE – 3

CONCLUSION

In this paper, we have investigated that extended triplicate of bistar graph admits Odd mean labeling, Even mean labeling and Root cube mean labeling.

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