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DIFFERENTIAL TRANSFORM METHOD FOR FIN TEMPERATURE DISTRIBUTION ANALYSIS WITH CONVECTIVE-RADIATIVE HEAT TRANSFER USING MATLAB

[1]OGUNMOLA B.Y., Ph. D., [2]ADEBOWALE Peter Adeoye, [3]NWEKE Gideon Chukwudi

[1]Senior Lecturer, Department of Mechanical Engineering, University of Lagos, Nigeria. bayemi@gmail.com

[2] M. Sc. Mechanical Engineering University of Lagos, Nigeria. padebowale17@gmail.com

[3]PhD Student, Federal University of Technology, Minna, Niger State, Nigeria. askgideon09@gmail.com

ABSTRACT :

Fins are typically tiny extended surfaces found on the external region of devices and equipment. Their primary purpose is to enhance the rate of heat transfer to or out of a device or an object by adjusting convective heat transfer coefficient. They are available in different shapes and configurations depending on the required applications: there are straight fins which may be rectangular, triangular and trapezoidal; curved fins such as convex fins; radial fins; annular fins and pin fins, to mention a few. The alteration in convective heat transfer coefficient is made possible by adjusting such parameters as surface area of the body, the material which the body is made of, the geometry of the body, characteristics of fluid flowing across the material, and so on. Research findings also show that it is more cost effective, economical, easy and less time consuming to alter the shape of the device then to increase heat transfer coefficient of the body. In this research, various configurations of fins will be presented; differential transform method will be employed to provide a semi-analytical solution to the general heat equation; geometries of fins are obtained using SOLIDWORKs; with the aid of MATLAB, temperature distributions are obtained for each geometry under varying conditions of thermal conductivity, radiation and internal heat source in turns. The results are then presented and discussed for further recommendation.

Keywords: fins, differential transformation, matlab.

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1.0 INTRODUCTION :

Fins can be described as a form of elongation on outer surfaces of materials, equipment, devices or machines. Their sole aim is to improve the heat transfer through an appliance by increasing convection. They present a more suitable and economical solution to the process of enhancing heat transfer and are thus globally utilized in wide range of devices or equipment. In addition, they help to rapidly get heat transfer to or from a body. Increasing heat transfer rate can be achieved in two ways as earlier mentioned (increasing h or increasing As). Since h depends on several parameters increasing h may be costly as it may require the fitting of pumps/fan, or adjusting power or size of the pump/fan which is of unarguably a costly solution. Increasing As is found to the cheap and economical process comparatively. In this research, temperature distribution through fins of various geometries is obtained for varying conditions of thermal conductivity, internal heat source and radiation. The nonlinear heat equation will be solved with the aid of a semi-analytical method (differential transform method), the thermal conductivity of the material is assumed to be variable, the material is being subjected to internal heat generation coupled with the effect of radiation at the boundary.

The concept of internal heat generation in fins was studied by Minkler and Rouleau (1960), they were among the pioneer researchers who obtained fin temperature distribution and heat removal rate as a function of some dimensionless parameters, they expressed the internal heat generation in dimensionless form using non-dimensionalization techniques. Another early work on fins was carried out by Starner and Mcmanus (1963), they carried out an experiment to investigate the thermal performance of rectangular fin arrays. In the experiment, a number of fin arrays (four sets) were scrutinized to figure out the effect of free heat convection parameters. Three different types of arrays (vertical, inclined and horizontal) are positioned with guard and main heaters in order to reduce the strength of the net heat loss, it was gathered that inclined arrays show the lowest heat transfer rate compared to vertical and horizontal arrays. They were also able to successfully predict the effect of fin height and spacing on heat transfer performance. In yet another ASME journal.

One of the concrete mathematical approach to solving fin problems was contained in the work of Zhou (1986) who used DTM (a near-analytical method) to solve the initial value problems in electrical circuits to obtain definite nth derivative values. Two-dimensional DTM used to solve the differential equation was developed by Chen and Ho (1999).

Sobamowo (2017) investigated the effect of internal heat generation and temperature-dependent heat conduction. Heat transfer and temperature distribution in circular convective radiative porous fins of different shapes were analyzed by Pasha (2018). In recent years, thermal enhancement flow problems have been analyzed by a few authors. The differential transform method was applied by Pasha. et al (2018), in the study of heat flow though some selected longitudinal fins under unsteady conditions of heat transfer. The thermal conductivity was assumed to vary with temperature. Variable thermal behavior of straight fins was discussed by Ndlovu and Moitsheki (2020). Sowmya et al. (2021) examined the heat performance in longitudinal fins with a heat source due to natural convection. Shi (2021) studied the bioconvection flow of magneto-cross nanofluid containing gyrostatic microorganisms with activation energy. Sabu et al. (2021) fathomed the significance of nanoparticles' shape and thermohydrodynamic slip constraints on MHD alumina-water nanoliquid flows over a rotating heated disk. Karus et al. (2022) gave a concise generic review of fins. Using the above concepts, Gireesha and Sowmya (2022) dealt with fin complications with heat distribution in an inclined fin. Electric filaments or nuclear reactors may generate electric current when exposed to temperature thereby causing internal heat generation in some systems. The nonlinearity of this process makes it rigorous or almost impossible to solve analytically. It can thus be solved using numerical or semi-analytical methods.

The solution for a system of differential equations by the DTM was expounded by Fatma (2004). Fatma (2003) proved that DTM is better to solve a nonlinear problem than the Taylor series method. The DTM has been applied to various problems in applied mathematics and physics such as systems of differential equations as shown in the work of Kanth (2008). Fallo et al. (2018) applied the 3D DTM for the first time to study heat transfer in a cylindrical spine fin with variable thermal properties. Chiba et al. (2014) solved the one-dimensional phase change problem in a slab of finite thickness using the DTM. The finite Taylor series and the repetition described by the modified equations obtained from the original equation applying differential transformation operations can be utilized to assess the approximating solution. More recently, Ananth et al. (2022) used DTM to solve a problem analogous to what is contained in this thesis. A review of the above literature shows no attempt has been made to analyze the heat transfer for the above-considered profiles except for the work of Ananth et al (2022). This thesis gives a vivid approach to how the DTM was applied to solve a similar problem by Ananth et al and other researchers.

3.0 METHODOLOGY :

The differential transform method will be applied to solving the heat equation coupled with the following stated assumptions:

- Temperature depends of x and remains constant over time
- Temperature variation due to fin thickness is neglected
- Steady condition of temperature and heat flow.
- Dynamic equilibrium between solid and fluid
- Thermal conductivity varies with temperature

The basic governing equation is also non-dimensionalized prior to the application of DTM as presented below:

$$\frac{d}{dx}\left[k(T) \times P(x)\frac{dT}{dx}\right] - \varepsilon\sigma(T^4 - T_a^4) - h(T - T_a) + Q^* = 0 \qquad (3.00)$$

$$Where\ k(T) = k_a[1 + \omega(T - T_a)], \qquad \omega\ being\ a\ constant\ and\ T_a\ is\ ambient\ temperature$$

$$P(x) = b\tau_x, \ \ \tau_x\ depends\ on\ the\ geometry\ of\ fin$$

$$For\ a\ rectangular\ fin\ \tau_x = b\tau_b = A_{b,}\ b = girth\ and\ \tau_b = thickness\ along\ fin\ length$$

$$\varepsilon\sigma(T^4 - T_a^4) = Radiation\ heat\ transfer\ per\ unit\ area$$

$$h(T - T_a) = convective\ heat\ transfer\ per\ unit\ area$$

$$Q^* = Internal\ heat\ generation\ term.$$

Boundary conditions $\frac{dT(0)}{dx} = 0\ and\ T(L) = T_b$

The first term in Equ. (3.02) can be simplified using product rule as follows:

$$\frac{d}{dx}\left[k_a[1 + \omega(T - T_a)] \times b\tau_b\frac{dT}{dx}\right] = k_aA_b\frac{d}{dx}\left[[1 + \omega(T - T_a)] \times \frac{dT}{dx}\right]$$

$$\frac{d}{dx}\left[[1 + \omega(T - T_a)] \times \frac{dT}{dx}\right] = \frac{d^2T}{dx^2}[1 + \omega(T - T_a)] + \frac{d}{dT}[1 + \omega(T - T_a)]\frac{dT}{dx} \times \frac{dT}{dx}\ OR$$

$$\frac{d^2T}{dx^2}[1 + \omega(T - T_a)] + \frac{d}{dT}[1 + \omega(T - T_a)]\left(\frac{dT}{dx}\right)^2\ OR\ \frac{d^2T}{dx^2}[1 + \omega(T - T_a)] + \omega\left(\frac{dT}{dx}\right)^2$$

Now Equation 3.00 becomes

$$k_aA_b\frac{d^2T}{dx^2}[1 + \omega(T - T_a)] + \frac{d}{dT}[1 + \omega(T - T_a)]\left(\frac{dT}{dx}\right)^2 + \varepsilon\sigma(T^4 - T_a^4) + h(T - T_a) + Q^* = 0 \qquad 3.01$$

Introducing the dimensionless parameters

$$\theta = \frac{T}{T_b} \xrightarrow{yields} T = \theta T_b; \;\; \theta_a = \frac{T_a}{T_b} \xrightarrow{yields} T_a = \theta_a T_b; \;\; X = \frac{x}{L} \xrightarrow{yields} x = XL; \;\; B = \frac{\varepsilon\sigma L^2 T_b^3}{A_b k_a};$$

$$A^2 = \frac{hL^2}{A_b k_a}; \quad \phi = \frac{L^2 Q^*}{A_b k_a T_b}.$$

$$k_a A_b \frac{d^2(\theta T_b)}{d(XL)^2}[1 + \omega T_b(\theta - \theta_a)] + \omega\left(\frac{d\theta T_b}{dXL}\right)^2 + \varepsilon\sigma T_b^4(\theta^4 - \theta_a^4) + hT_b(\theta - \theta_a) + Q^* = 0$$

$$\frac{k_a A_b T_b}{L^2}\frac{d^2\theta}{dX^2} + \frac{\omega k_a A_b T_b{}^2}{L^2}(\theta - \theta_a)\frac{d^2\theta}{dX^2} + \frac{\omega T_b{}^2}{L^2}\left(\frac{d\theta}{dX}\right)^2 + \varepsilon\sigma T_b^4(\theta^4 - \theta_a^4) + hT_b(\theta - \theta_a) + Q^* = 0$$

Dividing through by $\frac{k_a A_b T_b}{L^2}$ and introducing the dimensionless constants we have:

$$\alpha\left(\frac{d\theta}{dX}\right)^2 + \frac{d^2\theta}{dX^2} + \alpha\theta\frac{d^2\theta}{dX^2} - \alpha\theta_a\frac{d^2\theta}{dX^2} - A^2\theta + A^2\theta_a - B\theta^4 + B\theta_a^4 + \phi = 0$$

Applying the properties of differential transform we have:

$$\frac{d^2\theta}{dX^2} = (n+1)(n+2)W(n+2); \;\; \theta\frac{d^2\theta}{dX^2} = \sum_{m=0}^{n} W(m)(n-m+1)(n-m+2)W(n-m+2);$$

$$\left(\frac{d\theta}{dX}\right)^2 = \sum_{m=0}^{n}(m+1)W(m+1)(n-m+1)\,W(n-m+1); \quad \theta^4 = \sum_{m=0}^{n}\sum_{p=0}^{n-m}\sum_{q=0}^{m-p} W(m)W(n-m)W(m-p)W(p-q)$$

Now putting the transforms in the non-dimensionalized differential equation we have:

$$\begin{aligned}(n+1)(n+2)W(n+2) &+ \alpha\sum_{m=0}^{n}(m+1)W(m+1)(n-m+1)\,W(n-m+1)\\ &+ \alpha\sum_{m=0}^{n} W(m)(n-m+1)(n-m+2)W(n-m+2) - \alpha\theta_a(n+1)(n+2)W(n+2) - A^2W(n)\\ &+ B\sum_{m=0}^{n}\sum_{p=0}^{n-m}\sum_{q=0}^{m-p} W(m)W(n-m)W(m-p)W(p-q) + (B\theta_a^4 + A^2\theta_a + \phi)\delta(m)\\ &= 0 \end{aligned} \quad (3.04)$$

$Where\ \delta(m) = \begin{cases}1, & m = n\\ 0, & n \neq m\end{cases}$

Boundary conditions are W(1) = 0, $\sum_{n=0}^{\infty} \mathrm{W(m)} = 1 \;\; and\ W(0) = b$

When n=0, Equation 3.04 becomes

$W(2) = \frac{-\phi + bA^2 + b^4B - A^2\theta_a - B\theta_a^4}{2(1 + b\beta - \beta\theta_a)}$ (3.03)

$$W(3) = 0,$$

$W(4) = \frac{A^2W(2) - 6\beta W(2)^2}{12(1 + b\alpha - \alpha\theta_a)}$ (3.04)

$$W(5) = 0$$

$W(6) = \frac{W(4) - 30\beta W(2)W(4)}{30((1 + b\alpha - \alpha\theta_a))}$ (3.05)

$$W(7) = 0 \ldots$$

Writing out the solution up to k=7 we have:

$$\theta(X) = \sum_{k=0}^{7} W(K)X^K = b + W(2)X^2 + W(4)X^4 + W(6)X^6$$

Hence $\theta(X) = b + \frac{-\phi + bA^2 + b^4B - A^2\theta_a - B\theta_a^4}{2(1 + b\alpha - \alpha\theta_a)}X^2 + \frac{A^2W(2) - 6\alpha W(2)^2}{12(1 + b\alpha - \alpha\theta_a)}X^4 + \frac{W(4) - 30\alpha W(2)W(4)}{30((1 + b\alpha - \alpha\theta_a))}X^6 + \cdots$ 5

Exponential Profile

$$\tau(x) = \tau_b e^{b\left(\frac{x}{L}\right)} \ldots\ldots\ldots \frac{x}{L} = X$$

$$\tau(x) = \tau_b e^{bX} \ldots\ldots\ldots \tau_b = A_b$$

$$\tau(x) = A_b e^{aX}$$

$\frac{d}{dx}\left[k_a[1+\omega(T-T_a)]A_b e^{bX}\times\frac{dT}{dx}\right]= k_a A_b\frac{d}{dx}\left[[1+\omega(T-T_a)]e^{bX}\times\frac{dT}{dx}\right]$

$\frac{d}{dx}\left[[1+\omega(T-T_a)]e^{aX}\times\frac{dT}{dx}\right]=e^{aX}\frac{d^2T}{dx^2}[1+\omega(T-T_a)]+\frac{d}{dT}e^{aX}[1+\omega(T-T_a)]\frac{dT}{dx}\times\frac{dT}{dx}+\frac{d}{dx}e^{bX}[1+\omega(T-T_a)]\frac{dT}{dx}\ \ OR$

$$e^{aX}\frac{d^2T}{dx^2}[1+\omega(T-T_a)]+e^{aX}\frac{d}{dT}[1+\omega(T-T_a)]\left(\frac{dT}{dx}\right)^2+be^{bX}[1+\omega(T-T_a)]$$

Introducing dimensionless parameters and simplifying the differential equation becomes:

$$e^{bX}\frac{d^2\theta}{dX^2}[1+\omega(\theta-\theta_a)]+e^{bX}\omega\left(\frac{d\theta}{dX}\right)^2+be^{bX}[1+\omega(\theta-\theta_a)]\frac{d\theta}{dX}-B(\theta^4-\theta_a^4)-A^2(\theta-\theta_a)+\phi=0$$

The equation thus becomes:

$$e^{bX}\theta''+\omega\theta e^{aX}\theta''-\omega\theta_a e^{bX}\theta''+\omega e^{bX}(\theta')^2+be^{bX}\theta'+b\omega e^{bX}\theta'-b\theta_a e^{bX}\theta'-B\theta^4+B\theta_a^4-A^2\theta+A\theta_a+\phi=0$$

Applying differential transform to all terms we have:

$\sum_{m=0}^{n}\frac{b^m}{m!}(n-m+1)(n-m+2)W(n-m+2)+\alpha\sum_{m=0}^{n}\frac{b^m}{m!}\sum_{p=0}^{n-m}W(m)(n-m-p+1)\,(n-m-p+2)W(n-m-p+2)-\alpha\theta_a\sum_{m=0}^{n}\frac{b^m}{m!}(n-m+1)(n-m+2)W(n-m+2)+b\sum_{m=0}^{n}\frac{b^m}{m!}\,(n-m+1)W(n-m+1)+b\alpha\sum_{m=0}^{n}\frac{a^m}{m!}\sum_{p=0}^{n-m}W(m)(n-m-p+1)\,W(n-m-p+1)-b\alpha\theta_a\sum_{m=0}^{n}\frac{b^m}{m!}\sum_{p=0}^{n-m}(n-m-p+1)\,W(n-m-p+1)+\alpha\sum_{m=0}^{n}\frac{b^m}{m!}\sum_{p=0}^{n-m}(m+1)W(m+1)\,(n-m-p+1)W(n-m-p+1)+B\sum_{m=0}^{n}\sum_{p=0}^{n-m}\sum_{r=0}^{m-p}W(m)\,W(n-m)\,W(m-p)W(p-r)+A^2W(n)+(B\theta_a^4+N^2\theta_a+\phi)\delta(m)=0$ (3.06)

Upon applying the boundary conditions evoked: the following solutions are obtained:

$W(2)=\frac{-\phi+bA^2+b^4B-A^2\theta_a-B\theta_a^4}{2(1+b\alpha-\alpha\theta_a)}$ (3.07)

$W(3)=\frac{-2bW(2)-b\alpha W(2)-b^2\alpha W(2)+2a\alpha\theta_a W(2)}{3(1+b\alpha-\alpha\theta_a)}$ (3.08)

$W(4)=\frac{-3b^2W(2)+A^2W(2)-2b\alpha W(2)+3b^2\alpha\theta_a W(2)-4b\alpha\theta_a W(2)^2-b^2\alpha W(2)^2-9bW(3)-6b\alpha W(3)-3b^2\alpha W(3)+9b\alpha\theta_a W(3)+\cdots}{12(1+b\alpha-\alpha\theta_a)}$

Hence:

$$\theta(X)=W(0)+W(1)X+W(2)X^2+W(3)X^3+W(4)X^4+\cdots$$

$\theta(X)=b+\frac{-\phi+bA^2+b^4B-A^2\theta_a-B\theta_a^4}{2(1+b\alpha-\alpha\theta_a)}X^2+\frac{-2bW(2)-b\alpha W(2)-b^2\alpha W(2)+2a\alpha\theta_a W(2)}{3(1+b\alpha-\alpha\theta_a)}X^3+\frac{-3b^2W(2)+A^2W(2)-2b\alpha W(2)+3b^2\alpha\theta_a W(2)-4b\alpha\theta_a W(2)^2-b^2\alpha W(2)^2-9bW(3)-6b\alpha W(3)-3b^2\alpha W(3)+9b\alpha\theta_a W(3)+\cdots}{12(1+b\alpha-\alpha\theta_a)}X^4$ (3.09)

Triangular Fin

According to Ikram Ullah et al, neglecting porosity, the governing equation is given as:

$$X\,\theta''+\theta'+X\,Pe\,\theta'-(A^2+B)\theta-A^2\theta_a+B\theta_s+QX=0$$

Where Pe = peclet number as given by literature; A = conduction-convection constant; B = convection-radiation constant; the radiation effect is modified by writing:

The boundary conditions are:

1. At x = L, T = T_b (fin base temperature)
2. At x = 0, $\frac{dT}{dx}=1$

Equ. 3.13 becomes:

$W(m+1)=\frac{1}{m(m+1)^2}[(A^2+B)W(m)-Pe\sum_{m=0}^{n}\delta(m-1)(n-m+1)W(n-m+1)-\phi\sum_{m=0}^{n}\delta\,(m-1))-(A^2\theta_a+B\theta_s)\delta(m)]$ (3.14)

From the boundary conditions W(1) = 0 and W(0) = a, We have the following solutions:

$W(2) = \frac{\phi}{6}$ (3.10)

$$W(3) = \frac{1}{12}\left[(-2Pe + A^2 + B)\frac{\phi}{6} - \phi\right] = \frac{(-2Pe + A^2 + B - 6)\phi}{72}$$

$$W(4) = \frac{1}{20}[A^2 + B - 3PeW(3) - \phi]$$

$$W(5) = \frac{1}{30}[A^2 + B - 4PeW(4) - \phi]$$

The complete solution is therefore:

$$\theta(X) = \sum_{m=0}^{n} W(m)\, X^m$$

$$\theta(X) = W(0) + W(1)X + W(2)X^2 + W(3)X^3 + W(4)X^4 + W(5)X^5 + \cdots$$

$$\theta(X) = p + W(2)X^2 + W(3)X^3 + W(4)X^4 + W(5)X^5 + \cdots$$

Using the boundary condition $\theta(1) = 0$, when X = 1, $\theta = 0$

$$p + \frac{\phi}{6} + \frac{(-2Pe + A^2 + B - 6)\phi}{72} + \frac{1}{20}[A^2 + B - 3PeW(3) - \phi] + \frac{1}{30}[A^2 + B - 4PeW(4) - \phi] = 0$$

Using MATLAB, for A = 0.80, B = 0.30, $\phi = 0.25$, Pe = 0.15…….The constant p is obtained as 0.96144.

Putting the value of p in the series solution, we have:

$\theta(X) = 0.96144 + W(2)X^2 + W(3)X^3 + W(4)X^4 + W(5)X^5 + \cdots$ (3.11)

The temperature distribution can thus be obtained for known values of A, B, ϕ and Pe.

Trapezoidal fins

The DTM coupled with pade approximant as proposed by Jayaprakash et al. (2021) will be applied in this analysis. The thermal conductivity is assumed to vary in accordance with power law. The same governing equation is used as with the case of rectangular fins but the thermal conductivity k and the convective heat transfer coefficient h are non-linear with respect to temperature as shown:

$k(T) = k_r\left(\frac{T-T_r}{T_b-T_r}\right)^n$ (3.12)

$$Where\ T_r = Ambient\ or\ surrounding\ temperature,$$

$$k_r = thermal\ conductivity\ under\ ambient\ conditions\ and\ n\ is\ a\ constant.$$

$h(T) = h_r\left(\frac{T-T_r}{T_b-T_r}\right)^p$ (3.13)

It should be noted that n and p are positive numbers. If n and p are equal to zero, linearity arises in the sense that the thermal conductivity and convective heat transfer coefficient each vary linearly with temperature.

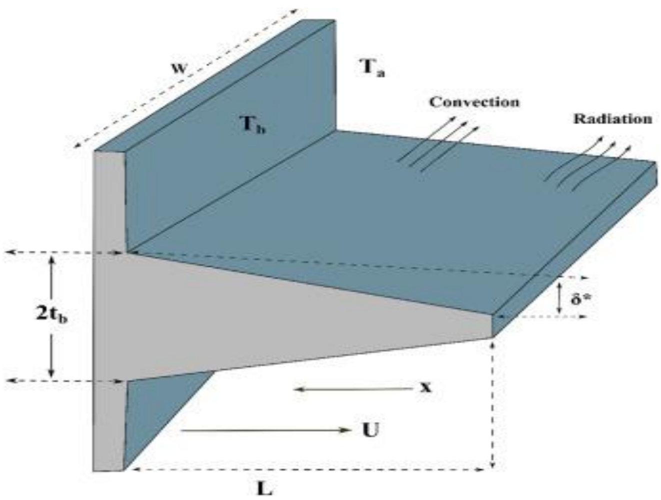


Figure1.3 Trapezoidal Fin….M.C Jayaprakash et al.

According to Aziz et al. (2009), the one dimensional heat equation which governs heat transfer through a trapezoidal fin is given by:

$$\frac{d}{dx}\left[k^*(T)\, t(x)\frac{dT}{dx}\right] + \rho c_p t(x) U \frac{dT}{dx} + q(T)t(x) = h^*(T)(T - T_a) + \sigma\varepsilon^*(T^4 - T_a^4) \quad (3.14)$$

In Equ. 3.19 above:

k * (T) = variable thermal conductivity in (W/mK)

t (x) = local semi fin thickness (m) = $\left(t_b + \delta^*\left[\left(\frac{x}{L}\right) - 1\right]\right)$ (3.15)

t_b = semi-base thickness, δ^*= semi-offset(m)

U = velocity at which the fin is moving (m/s)

x = axial distance(m) ; L = length of the fin(m) ; q(T) = internal heat generated per unit volume(W/m^3)

$q(T) = q_0[1 + \nu(T - T_a)]$ (3.16)

Note: The internal heat generated per unit volume varies linearly with temperature

The following dimensionless parameters are invoked into Equ. 3.14

$$\theta = \frac{T - T_a}{T_b - T_a}; \quad \beta = \varphi(T_b - T_a); \quad \gamma = \nu(T_b - T_a); \quad X = \frac{x}{L}; \quad N_a = \frac{h_b L^2}{k_a t_b};$$

$$N_r = \frac{\sigma\epsilon^* L^2 (T_b - T_a)^3}{k_a t_b}; \quad Pe = \frac{\rho c_p UL}{k_a}; \ N_t = \frac{T_a}{T_b - T_a}; \quad Q = \frac{L^2 q_o}{T_b - T_a}; \ C = \frac{\delta^*}{t_b}$$

Now, substituting t (x) and dimensionless parameters in Equ. 3.14 then applying differential transform, we have:

W (0) = p and W(1) = 0. We have:

$W(2) = -\frac{p^4 N_r + 4p^3 N_r N_t + 6p^2 N_r N_t^2 + 4p N_r N_t^3 - pQ\gamma + pN_c + CQ - Q}{2B(C-1)}$

$W(3) = -\frac{2p^5 CN_r + 8p^4 CN_r N_t + 12p^3 CN_r N_t^3 + 8p^2 CN_r N_t^3 - p^2 CQ\gamma + 2p^2 CN_c + pC^2 Q - pCQ}{6B^2(C-1)^2}$

And so on…

Hence:

$$\theta(X) = W(0)X^0 + W(1)X^1 + W(2)X^2 + W(3)X^3 + \cdots$$

$\theta(X) = p - \left(\frac{p^4 N_r + 4p^3 N_r N_t + 6p^2 N_r N_t^2 + 4p N_r N_t^3 - pQ\gamma + pN_c + CQ - Q}{2p(C-1)}\right) X^2 -$
$\left(\frac{2p^5 CN_r + 8p^4 CN_r N_t + 12p^3 CN_r N_t^3 + 8p^2 CN_r N_t^3 - p^2 CQ\gamma + 2p^2 CN_c + pC^2 Q - pCQ}{6p^2(C-1)^2}\right) X^3 + \cdots$ (3.17)

The value of p can be obtained using pade approximant an approximate value of p is 0.9929. Now using assumed values of 0.91, 0.92, 0.15, 0.35, 0.98 and 0.95 for parameters N_r, N_c N_t, C, Q and γ respectively. We have:

$\theta(X) = 0.9929 + 3.755X^2 - 0.4385X^3 + \cdots$ (3.18)

When Nr is raised to 1.5 and other parameter fixed, Equ. 3.25 becomes:

$\theta(X) = 0.9929 + 1.29X^2 - 0.7013X^3 + \cdots$ (3.19)

Convex Fins

Referring to Equation 25 with $\tau(x) = A_b\left(\frac{x}{L}\right)^{\frac{1}{2}}$. Upon substituting and introducing dimensionless constants the equation becomes

$\beta\left(\frac{d\theta}{dX}\right)^2 + [1 + \beta(\theta - \theta_a)]\frac{d^2\theta}{dX^2} - 4BX(\theta^4 - \theta_a^4) - 4XA^2(\theta - \theta_a) + 4\phi X = 0$ (3.20)

After applying differential transform and invoking the boundary conditions we have:

$$W(0) = d; \quad W(1) = 0 \quad (Boundary\ condition)$$

$$W(2) = W(4) = W(5) = W(7) = W(8) = 0$$

$$W(3) = \frac{2(\phi - dA^2 + dA^2\theta_a + aB\theta_a^4)}{3(1 + d\beta - \beta\theta_a)}$$

$$W(6) = \frac{4([A^2 - 4A^2\theta_a - 4B\theta_a^4]W(3) - 15\beta W(3)^2)}{30(1 + d\beta - \beta\,\theta_a)}$$

The dimensionless temperature profile is thus given as:

$$\theta(X) = d + W(3)X^3 + W(6)X^6 + \cdots$$

$$\theta(X) = d + \left(\frac{2(\phi - dA^2 + dA^2\theta_a + aB\theta_a^4)}{3(1 + d\beta - \beta\theta_a)}\right)X^3 + \left(\frac{4([A^2 - 4A^2\theta_a - 4B\theta_a^4]W(3) - 15\beta W(3)^2)}{30(1 + d\beta - \beta\,\theta_a)}\right)X^6 + \cdots \quad (3.29)$$

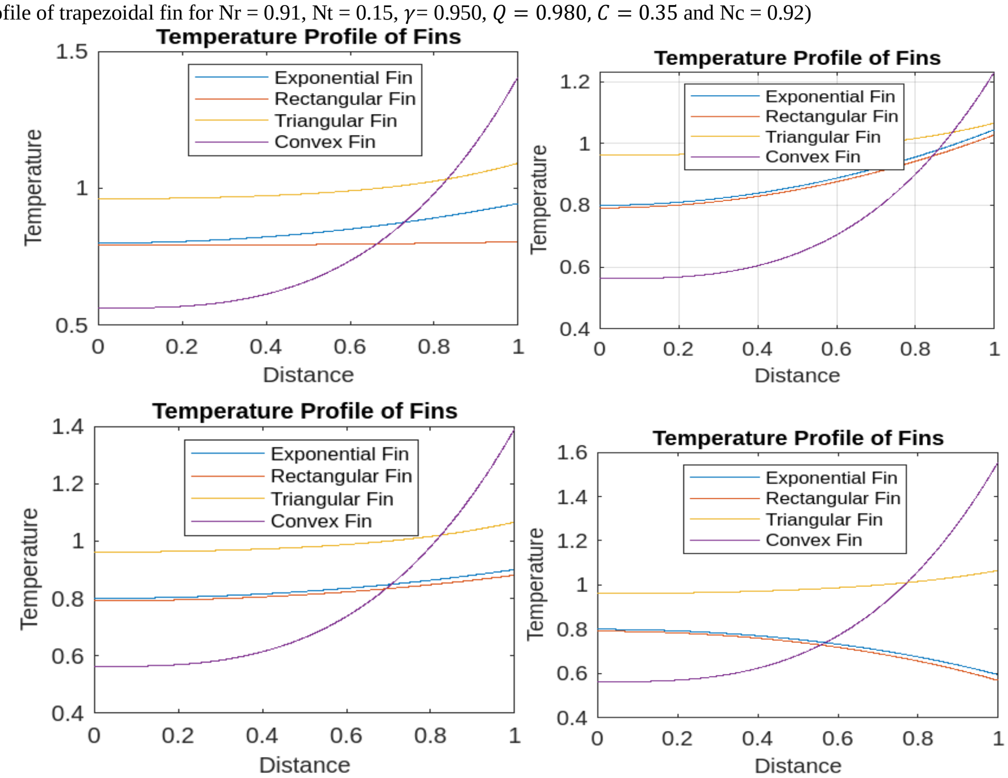
Using guess values of 0.2, 0.85, 0.90, 0.45 and 0.20 for the dimensionless parameters ϕ, A, B, β and θ_a respectively, the value of constant d is obtained by invoking the boundary condition $\theta(1) = 1$. Using MATLAB, the only real solution of d is 0.5626.

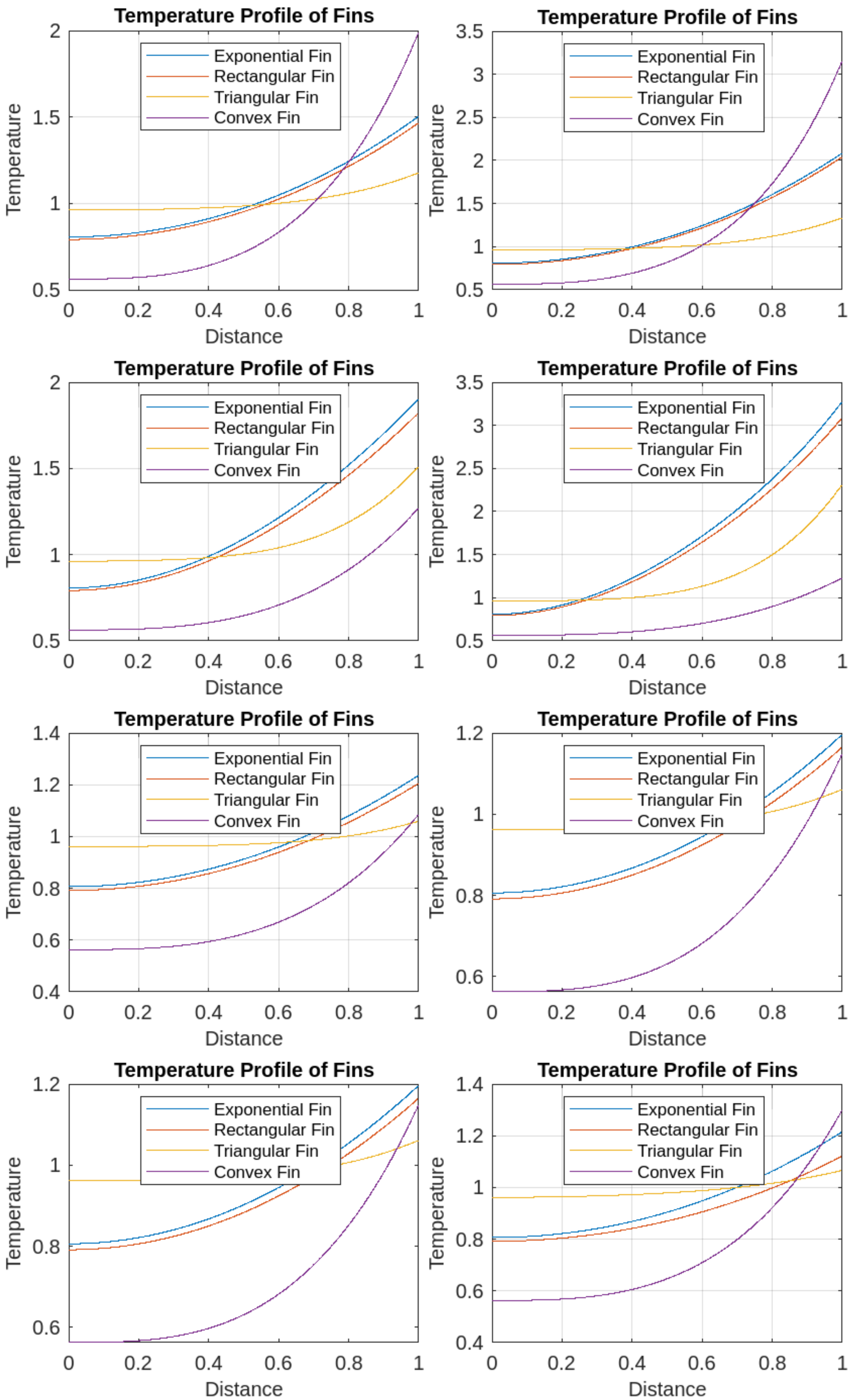
3.0 RESULTS AND DISCUSSIONS :

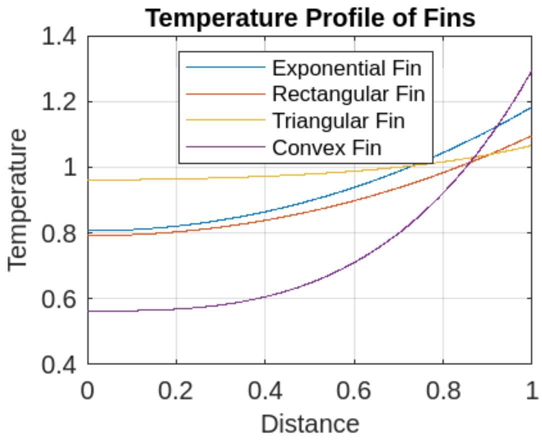
The results of the profiles obtained by varying each parameter in turn are presented in this chapter. Fig. 4.0 shows the result obtained for assumed random values of A, B, β, θ_a and ϕ.

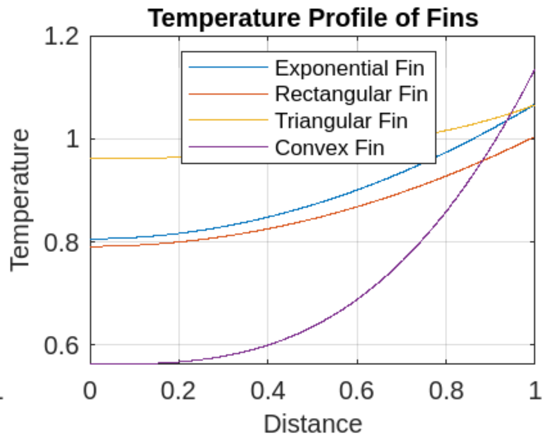
- In fig 4.01 – 4.03, all other parameters are fixed with β varied (0.1, 0.5 and 1.0).
- In fig 4.04 and 4.05, all other parameters are fixed while convection term A varies (1.50 and 2.00)
- In fig 4.06 and 4.07, only the radiation term B is varied (6.00 and 15.00)
- In Fig 4.08 – 4.10, the heat source term varies (0.10, 0.20 and 0.30)
- In fig 4.11 – 4.13, $\theta_a$varies (0.10, 0.20 and 0.80)

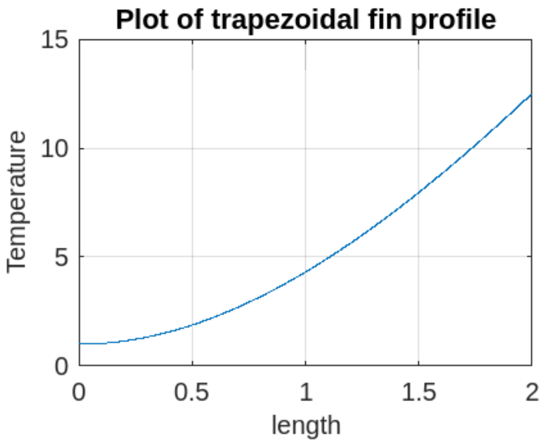
The figures are arranged from left to right in the following manner:
Figure 4.00 (Profile of fins for A = 0.98, B = 1.00, β= 0.40, $\theta_a = 0.60$ and $\phi = 0.42$)
Figure 4.01 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.60$ and $\phi = 0.42$)
Figure 4.02 (Profile of fins for A = 1.00, B = 1.00, β= 0.50, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.03 (Profile of fins for A = 1.00, B = 1.00, β= 1.0, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.04 (Profile of fins for A = 1.50, B = 1.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.05 (Profile of fins for A = 2.00, B = 1.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.06 (Profile of fins for A = 1.00, B = 6.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.07 (Profile of fins for A = 1.00, B = 15.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.42$)
Figure 4.08 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.1$)
Figure 4.09 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.2$)
Figure 4.10 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.30$ and $\phi = 0.3$)
Figure 4.11 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.10$ and $\phi = 0.420$)
Figure 4.12 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.20$ and $\phi = 0.420$)
Figure 4.13 (Profile of fins for A = 1.00, B = 1.00, β= 0.10, $\theta_a = 0.80$ and $\phi = 0.420$)
Figure 4.14 (Profile of trapezoidal fin for Nr = 0.91, Nt = 0.15, γ= 0.950, $Q = 0.980$, $C = 0.35$ and Nc = 0.92)




Temperature Profile of Fins
Exponential Fin
Rectangular Fin
Triangular Fin
Convex Fin
Temperature
Distance







From the results obtained so far it is evident that:

- The heating style gets more enhanced as the parameter A gets raised. For constant values of other parameters, an increase in A improves the thermal performance of all the profiles with the triangular profile showing a comparatively better performance (though going through only a slight temperature change across the length of the fin). The thermal profile obtained from ANSYS also show similar result.
- There is a steady decrease in the temperature profile of the fin as the radiation parameter (B) increases. This may be attributed to the loss of temperature to the surrounding by radiation effect.
- As θ_a increases, the temperature of the surrounding air increases, this in turn alters the rate at which heat is transmitted to the surface until the value of θ_a attains a particular value.
- The internal heat generation term, ϕ also enhance the fin temperature. This may be due to the fact that larger heat dissipation triggers the fin to release more heat to the environment.

Table 4.0 below also shows a comparison between recent research results endemic to the present work. From the table it is evident that the accuracy of differential transform method is much higher than that of variational iteration method (VIM), adomian decomposition method (ADM), homotopic perturbation method (HPM), galerkin method of weighted residual (GMWR) and variation of parameter method (VOP).

It should be noted that the parameters used are A = 1.0, B = 0.0, ϕ = 0.0 and β = 0.0

X	Exact Solution	ADM Arglanturk et al. (2016)	HPM Pranab et al. (2018)	VIM Languri et al. (2018)	GMWR Sobamowo (2015)	DTM Ananth et al. (2022)	DTM (This work)
0.0	0.6480541	0.6480541	0.6480540	0.6480541	0.6480540	0.6480541	0.6480541
0.1	0.6512970	0.6512970	0.6512970	0.6512970	0.6512970	0.6512970	0.6512970
0.2	0.6610588	0.6610588	0.6610590	0.6610590	0.6610590	0.6610588	0.6610588
0.3	0.6774361	0.6774361	0.6774361	0.6774360	0.6774360	0.6774360	0.6774361
0.4	0.7005940	0.7005940	0.7005940	0.7005940	0.7005940	0.7005940	0.7005940
0.5	0.7307630	0.7307631	0.7307631	0.7307630	0.7307630	0.7307630	0.7307632
0.6	0.7682460	0.7682460	0.7682460	0.7682460	0.7682460	0.7682460	0.7682460
0.7	0.8134180	0.8134180	0.8134180	0.8134180	0.8134180	0.8134180	0.8134180
0.8	0.8667310	0.8667310	0.8667310	0.8667310	0.8667310	0.8667310	0.8667310

0.9	0.9287180	0.9287182	0.9287181	0.9287180	0.9287180	0.9287180	0.9287181
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Table 4.0: Temperature distribution through a rectangular fin as obtained by researchers

The temperature profile obtained from MATLAB is also found to be much similar to the profile obtained by researchers.

CONCLUSION AND RECOMMENDATION :

In this research, temperature distribution in fins subjected to variable thermal conductivity, radiation at boundary and internal heat generation is carried out for five different configurations of longitudinal fins (rectangular, triangular, convex, trapezoidal and exponential). The heat transfer model is solved by differential transform method and the results are compared with the ones obtained by other researchers. The results are found to be of considerable similarity to what is found in literature, this shows that differential transform method is an efficient semi analytical approach to solving differential equations in nonlinear form. When compared to HPM and ADM, its syntax is lucid enough to grasp and the solution to the ODE can be obtained faster.

I therefore recommend that researchers apply the differential transform method for the analysis of fins of other geometries (radial, inclined, etc.) as it minimizes the lead time required to obtain final results and also truncates errors obtained by HPM, ADM and other numerical methods

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