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Determination of Catastrophic Disease Insurance Premium using Markov Chain

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A B S T R A C T

The high incidence of Catastrophic diseases, especially Chronic Kidney Disease (CKD), Diabetes Mellitus (DM), hypertension, and heart disease, necessitates Long Term Care and process a significant risk of financial loss. To minimize the risk, insurance products designed according to the characteristics of chronic disease using a four state markov chain and calculate the single premium for Long Term Care: Annuity as A RIder Benefit insurance. The data used is sourced from disease prevalence (Indonesia Healty Survey, 2023) and mortality table (TMI, 2019). The multi state model defines four transition states: healty, suffering from Catastrophicdiseases, death due to Catastrophic diseases, and death due to causes other than Catastrophicdiseases, where the transition probabilities between state are defined based on prevalence and mortality rates, assuming one way transition. The results show that the markov chain transition matrix was successfully used to determine the probability of changing state. This probability was then implemented in the formulation of the actuarial value to calculate the single premium. It was found that amount of the single premium is significantly influenced by the age of the insured's age at the policy agreement and the duration of the insurance coverage, where the older the age and the longer the coverage period, the greater the premium that must be paid.

Keywords: Markov Chain, Catastrophic Diseases, Single Premium, Long Term Care: Annuity as a Rider Benefit

1. Introduction

Indonesia is confronted with a high number of non communicable diseases categorized as catastrophic diseases, which are chronic and degenerative diseases. They are called chronic because the diseases are hidden and often unnoticed, requiring a long time for recovery. They are called degenerative because the incidence of the diseases increases with age (Heniwati and Thabrany, 2017). Some diseases included in the catastrophic disease category are Chronic Kidney Disease (CKD), Diabetes Mellitus (DM), hypertension, and heart disease.

One approach to modelling the progression of catastrophic diseases is the Markov Chain. The markov cahin is a stochastic process where future occurrences depend only on the events of today and are independent of past states. A markov chain is defined by it's transition probability matrix. The transition probability matrix is a matrix containing information that regulates the system's movement from one state to another. In the context of catastrophic diseases, disease progression can be modeled through a multi state framework, which allow for the representation of the patient's state from healty, suffering from the disease, to death either due to the catastrophic disease or other causes (Pitacco, 2014).

The application of actuarial calculation with the multi state model is in health insurance. Health insurance is an agreement on providing health protection guarantees against the risk of illness and accident, involving two parties: the insured and the insurer. To minimize the risk of financial loss due to catastrophic diseases, an insurance proct is needed, one of which is Long Term Care (LTC) insurance. LTC insurance provides benefit guarantees for the insured who requires long term medical care (Syamdena *et al*., 2019). The product used in this research is LTC: Annuity as a Rider Benefit insurance, which provides medical care cost benefits for a certain period and death benefits.

2. Literature Review

2.1 Linear Interpolation for Prevalence Rate

Linear interpolation is defined as a method for connecting two points with a straight line (Munir, 2015). The equation of the straight line passing through 2 points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ can be written as follows:

$$y = y_1 + \frac{x-x_1}{x_2-x_1}(y_2 - y_1) \quad (1)$$

with:

x = the age to be interpolated

x_1 = the starting age of the age group where x years is located

x_2 = the second age in the age group after x years is located

y = the prevalence rate of the age to be interpolated

y_1 = the prevalence rate at age group x_1

y_2 = the prevalence rate at age group x_2

2.2 Markov Chain

The markov chain is one mathematical stochastic process with a collection of random variables that move from one state to the next based on probability rules (Walpole *et al.*, 2017). Pendekatan rantai markov digunakan untuk melihat dan memprediksi perubahan yang kemungkinan akan terjadi di masa depan. The Markov Chain approach is used to view and predict changes that are likely to occur in the future. This change is represented by variables that are dynamic over time. Based on this concept, the Markov Chain can be used to analyze certain future events in a structured manner. It is referred to as a Discreate Time Markov Chains (DTMC) model if for every i and j in S, the following equation is satisfied (Kulkarni 2014)

$P(X_{n+1} = j|X_n = i, X_{n-1} = i_{n-1}, \dots, X_o = i_o) = P(X_{n+1} = j|X_n = i)$.

Thus, the formulation for the one-step transition probability is written as follows.

$p_{i,j} = P(X_{n+1} = j|X_n = i)$ dengan $i, j = 1, 2, \dots, N$ (2)

The multi step transition probability is a conditional probability that determines the system will be in state j exactly after several unit time steps, if the system started in state i at time t. These steps can be symbolized by h steps. Therefore, the transition steps can be denoted by ${p_{i,j}}^{(h)}$. Because ${p_{i,j}}^{(h)}$ is a conditional probability, it must be non negative and thus must satisfy the properties:

1. $0 \leq p_{i,j} \leq 1$, for all i and j
2. $\sum_{j=1}^{\infty} p_{i,j} = 1$, for all i

Therefore, the h step transition probability is for formulated as follows, with the matrix notation.

$${p_{i,j}}^{(h)} = \sum_{k=1}^{N} {p_{i,k}}^{(h-1)} p_{k,j} \quad (3)$$

The theorem regarding multi step transition probability is known as the Chapman Kolmogorov equation as follows.

$${p_{i,j}}^{(n+m)} = \sum_{k=1}^{N} {p_{i,k}}^{(n)} {p_{k,j}}^{(m)} \quad (4)$$

Equation (4) describes how a markov chain can transition from state i (initial) to state j (final) through state k (intermediate) in a total of h steps, with the assumption $h = n + m$, then Equation (4) can be modified to:

$${p_{i,j}}^{(h)} = \sum_{k=1}^{N} {p_{i,k}}^{(h-m)} {p_{k,j}}^{(m)} \quad \text{with } m < h \quad (5)$$

2.3 Long Term Care (LTC) Insurance

Long Term Care (LTC) insurance is insurance that guarantees health protection for people who need medical care. LTC insurance is often associated with the elderly. However, in reality, this insurance can be started at any age (Haberman and Pitacco, 1998). LTC insurance provides income support in the form of benefits paid as an annuity or through reimbursement of medical and care costs.

LTC insurance is also influenced by several types of risk, but insurance companies only focus on risks originating from morbidity and duration of life (longevity). LTC insurance benefits can be grouped into three categories (Haberman and Pitacco, 1998):

1. A number of benefits in the form of an annuity offered to healty people.
2. A number of benefits in the form of an annuity offered to the elderly at the time of entering or currently entering the care period.
3. Reimbursement of care and medical expenses.

Four states are defined as follows (Baione and Levantesi, 2014):

1 : Healty status

2 : Suffering from catastrophic diseases status

3 : Death due to catastrophic diseases status

4 : Death due to causes other than catastrophic diseases status

The illustration of the four state transition can be described as follows:

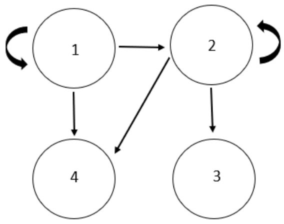

Fig. 1 - Four State Transition Illustration

2.4 Probability for Discrete Time Model

Let x be the age of the insured at the policy agreement and the status space $S = \{1,2,3,4\}$ as a discrete markov chain (Kulkarni, 2014). The illustration of the four state model can be seen in Figure 1 and can be presented in the form of the transition matrix as follows.

$$\boldsymbol{P_x} = \begin{bmatrix} p_x^{1,1} & p_x^{1,2} & 0 & q_x^{1,4} \\ 0 & p_x^{2,2} & q_x^{2,3} & q_x^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

with:

$p_x^{1,1}$ = probability that a person aged x years in a healthy state will remain in a healthy state at age $x + 1$ years.

$p_x^{1,2}$ = probability that a person aged x ears in a healthy state will move to a state of suffering from catastrophic diseases at age $x + 1$ years.

$q_x^{1,4}$ = probability that a person aged x years in a healthy state will move to a state of death due to causes other than catastrophic diseases at age $x + 1$ years.

$p_x^{2,2}$ = probability that a person aged x years in a state of suffering from catastrophic diseases will remain in a state of suffering from catastrophic diseases at age $x + 1$ years.

$q_x^{2,3}$ = probability that a person aged x t years in a state of suffering from catastrophic diseases will move to a state of death due to catastrophic diseases at age $x + 1$ years.

$q_x^{2,4}$ = probability that a person aged x years in a state of suffering from catastrophic diseases will move to a state of death due to causes other than catastrophic diseases at age $x + 1$ years.

The h step transition probability based on the markov chain can be written as follows.

$p_x^{1,1} = P(x_1 = 1|x_0 = 1)$

$p_x^{1,2} = P(x_1 = 2|x_0 = 1)$, and so on (7)

Thus, the Chapman Kolmogorov equation is obtained as follows.

$p_x^{1,1(h)} = P(x_h = 1|x_0 = 1)$

$p_x^{1,2(h)} = P(x_h = 2|x_0 = 1)$ (8)

Equation (7) and (8) can be written in matrix form, as follows.

$$\boldsymbol{P_x}^{(\boldsymbol{h})} = \prod_{z=x}^{x+h-1} \begin{bmatrix} p_z^{1,1} & p_z^{1,2} & 0 & q_z^{1,4} \\ 0 & p_z^{2,2} & q_z^{2,3} & q_z^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P_x}^{(\boldsymbol{h})} = \begin{bmatrix} p_x^{1,1} & p_x^{1,2} & 0 & q_x^{1,4} \\ 0 & p_x^{2,2} & q_x^{2,3} & q_x^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{x+1}^{1,1} & p_{x+1}^{1,2} & 0 & q_{x+1}^{1,4} \\ 0 & p_{x+1}^{2,2} & q_{x+1}^{2,3} & q_{x+1}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{x+h-1}^{1,1} & p_{x+h-1}^{1,2} & 0 & q_{x+h-1}^{1,4} \\ 0 & p_{x+h-1}^{2,2} & q_{x+h-1}^{2,3} & q_{x+h-1}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

2.5 Annuity as a Rider Benefit

One of the Long Term Care (LTC) insurance products is *Annuity as a Rider Benefit*, which provides medical care cost benefits for a certain period and death benefits if the insured dies due to illness or dies without having been sick first. In the *Annuity as a Rider Benefit* product, there is no transition from the sick state to the healthy state, meaning there is no assumption of recovery. (Haberman and Pitacco, 1998).

In this four state model, the death benefit is denoted by c which is a sum of money given when the insured dies. Meanwhile, the annual payment is denoted by b which is a benefit paid routinely every year if the insured undergoes a period of care. It is assumed that $b = \frac{c}{r}$ with the notation r as the maximum value of the benefit annuity payment time (in years) if the insured is in the care period.

The Net Single Premium value for LTC insurance is:

$$B^{LTC}_{x:r|} = c\sum_{e=1}^{r} v^e {}_{e-1}p_x^{1,1} q^1_{x+e-1} + \sum_{e=1}^{r}\left[v^e {}_{e-1}p_x^{1,1} p^{1,2}_{x+e-1} \times \left(b\ddot{a}^{2,2}_{x+e:r} + \sum_{h=1}^{\frac{c}{b}}(c-hb)v^h {}_{h-1}p^{2,2}_{x+e} q^2_{x+e+h-1}\right)\right]$$

$$= c\sum_{e=1}^{r} v^e {}_{e-1}p_x^{1,1} q^1_{x+e-1} + b\sum_{e=1}^{r} v^e {}_{e-1}p_x^{1,1} p^{1,2}_{x+e-1}\ddot{a}^{2,2}_{x+e:r} + \sum_{e=1}^{r} v^e {}_{e-1}p_x^{1,1} p^{1,2}_{x+e-1}\sum_{h=1}^{c/b}(c-hb)v^h {}_{h-1}p^{2,2}_{x+e} q^2_{x+e+h-1} \quad (10)$$

It is assumed that the premium is paid at the beginning of each year for m years while the insured is still in a healthy state. P is denoted as the annual premium, and is obtained:

$$P = \frac{B^{LTC}_{x:r|}}{\ddot{a}^{11}_{x:m}} \qquad \text{where } \ddot{a}^{11}_{x:m} = \sum_{e=0}^{m-1} v^e {}_{e}p_x^{1,1} \quad (11)$$

3. Research Methodology

The type of data used in this study is secondary data obtainded from the publication of the Indonesia Health Survey (SKI) 2023 for chronic kidney disease, diabetes mellitus, hypertension, and heart disease based on Doctor's Diagnosis in the Population Aged ≥ 15 years. Mortality data was obtained from the Indonesia Mortality Table IV (TMI IV) 2019 released by the Indonesian Life Insurance Association. The interest rate (α) is assumed to be fixed according to the BI rate in 2025 at 4,75% annually.

The obtained data was then processed using Microsoft Excel and R Program. The steps of the data analysis in this study can be outlined as follows:

a. Inputting Indonesia Mortality Table IV data.

b. Inputting disease prevalence rate data. At this stage, the process of collecting prevalence data for chronic kidney disease, diabetes mellitus, hypertension, and heart disease is carried out.

c. Interpolating disease prevalence data. At this stage, the linear interpolation process is carried out for the prevalence data of chronic kidney disease, diabetes mellitus, hypertension, and heart disease.

d. Comiling the h step transition probability matrix for the multi-state model.

e. Calculating the net premium for the LTC insurance product Annuity as A Rider Benefit.

4. Results And Discussion

In this discussion, premium calculations are performed for two cases where a male insured is 25 years old and 50 years old, taking LTC insurance for a duration of 10 years with coverage against catastrophic diseases. The requested insurance benefit is Rp100.000.000. In the agreement, the premium is paid for 10 years, with a fixed interest rate according to the 2025 BI rate of 4,75% annually.

The following is the prevalence rate data used:

Table 1. Data of Catastrophic Disease Prevalence Rate

Type of Disease	Prevalence Rate (in %) Age Group						
	15-24	25-34	35-44	45-54	55-64	65-74	75+
	%	%	%	%	%	%	%
CKD	0,02	0,07	0,11	0,26	0,40	0,45	0,57
DM	0,00	0,20	1,00	3,50	6,60	6,70	4,80
Hypertension	0,30	1,80	5,20	11,80	18,70	23,80	26,10
Heart Disease	0,11	0,15	0,46	1,34	2,65	4,05	4,60

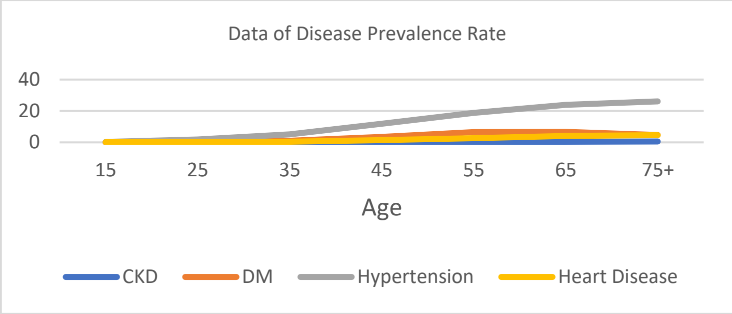

Fig. 2 - Graph of Catastrophic Disease Prevalence Rate Data

Based on Figure 2, it is seen that the catastrophic disease prevalence rate tends to increase with age. Hypertension shows the highest prevalence compared to the other three diseases. Meanwhile, diabetes mellitus experiences a fairly rapid increase starting from the age of 35 years and reaches its peak in the age range of 55 – 65 years. Heart disease also shows a stable increasing pattern since the age of 35 years, with a lower prevalence than hypertension. Chronic kidney disease has the lowest prevalence among the four diseases. Overall, this pattern illustrates that age is an important factor in the increasing prevalence of non-communicable diseases, where the highest risk is found in hypertension, followed by diabetes mellitus, heart disease, and finally chronic kidney disease.

Based on the catastrophic disease prevalence data, the interpolation results will be determined using the interpolation formula in Equation (1)..

1. Case 1 Data Interpolation

x = 25 years

x_1 = 20 years

x_2 = 30 years

2. Case 2 Data Interpolation

x = 50 years

x_1 = 45 years

x_2 = 55 years

After obtaining the interpolation results for the prevalence rate, the calculation of the h step transition probability matrix is continued with the following matrix multiplication.

$$\boldsymbol{P_x}^{(h)} = \begin{bmatrix} p_x^{1,1} & p_x^{1,2} & 0 & q_x^{1,4} \\ 0 & p_x^{2,2} & q_x^{2,3} & q_x^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{x+1}^{1,1} & p_{x+1}^{1,2} & 0 & q_{x+1}^{1,4} \\ 0 & p_{x+1}^{2,2} & q_{x+1}^{2,3} & q_{x+1}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{x+h-1}^{1,1} & p_{x+h-1}^{1,2} & 0 & q_{x+h-1}^{1,4} \\ 0 & p_{x+h-1}^{2,2} & q_{x+h-1}^{2,3} & q_{x+h-1}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

with:

$p_x^{1,1} = 1 - \left(p_x^{1,2} + q_x^{1,4}\right)$

$p_x^{2,2} = 1 - \left(q_x^{2,3} + q_x^{2,4}\right)$

$q_x^{2,4} = q_x^{1,4}$

Example for Case 1**,** for a 25 year old male, the one step transition matrix is desired:

1. The 'total' result of catastrophic disease prevalence interpolation for a 25 year old for the value $p_{25}^{1,2}$ is 0,02910
2. The mortality rate for a 25 year old male for the value $q_{25}^{1,4}$ is 0,00052
3. $p_{25}^{1,1} = 1 - \left(p_{25}^{1,2} + q_{25}^{1,4}\right) = 0{,}97038$
4. The mortality rate for a 25 year old male multiplied by the catastrophic disease prevalence interpolation result for the value $q_{25}^{2,3}$ is 0,00002
5. $q_{25}^{2,4} = q_{25}^{1,4} = 0{,}00052$
6. $p_{25}^{2,2} = 1 - \left(q_{25}^{2,3} + q_{25}^{2,4}\right) = 0{,}99946$

The one step transition probability matrix is obtained as follows.

$$\boldsymbol{P_{25}} = \begin{bmatrix} 0{,}97038 & 0{,}02910 & 0 & 0{,}00052 \\ 0 & 0{,}99946 & 0{,}00002 & 0{,}00052 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, using the same calculation method, the one-step transition matrices for ages 26 to 34 are obtained.

After obtaining the one-step transition matrices, the h step transition matrix calculation is performed using Equation (9). In Case 1, a 10 year insurance duration is used, so the transition matrix up to 9 steps ahead is calculated. The 9-step transition matrix calculation for a 25 year old male is as follows.

$$\boldsymbol{P_{25}}^{(9)} = \prod_{z=25}^{33} \begin{bmatrix} p_z^{1,1} & p_z^{1,2} & 0 & q_z^{1,4} \\ 0 & p_z^{2,2} & q_z^{2,3} & q_z^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P_{25}}^{(9)} = \begin{bmatrix} p_{25}^{1,1} & p_{25}^{1,2} & 0 & q_{25}^{1,4} \\ 0 & p_{25}^{2,2} & q_{25}^{2,3} & q_{25}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{26}^{1,1} & p_{26}^{1,2} & 0 & q_{26}^{1,4} \\ 0 & p_{26}^{2,2} & q_{26}^{2,3} & q_{26}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{33}^{1,1} & p_{33}^{1,2} & 0 & q_{33}^{1,4} \\ 0 & p_{33}^{2,2} & q_{33}^{2,3} & q_{33}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P_{25}}^{(9)} = \begin{bmatrix} 0{,}68006 & 0{,}31354 & 0{,}00004 & 0{,}00636 \\ 0 & 0{,}99337 & 0{,}00027 & 0{,}00636 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Example for Case 2**,** for a 50 year old male, the one step transition matrix is desired:

1. The 'total' result of catastrophic disease prevalence interpolation for a 50 year old for the value $p_{50}^{1,2}$ is 0,22625
2. The mortality rate for a 50-year-old male for the value $q_{50}^{1,4}$ is 0,00508
3. $p_{50}^{1,1} = 1 - \left(p_{50}^{1,2} + q_{50}^{1,4}\right) = 0{,}76867$
4. The mortality rate for a 50-year-old male multiplied by the catastrophic disease prevalence interpolation result for the value $q_{50}^{2,3}$ is 0,00115
5. $q_{50}^{2,4} = q_{50}^{1,4} = 0{,}00508$
6. $p_{50}^{2,2} = 1 - \left(q_{50}^{2,3} + q_{50}^{2,4}\right) = 0{,}99377$

The one step transition probability matrix is obtained as follows.

$$\boldsymbol{P_{50}} = \begin{bmatrix} 0{,}76867 & 0{,}22625 & 0 & 0{,}00508 \\ 0 & 0{,}99377 & 0{,}00115 & 0{,}00508 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, using the same calculation method, the one step transition matrices for ages 51 to 59 are obtained.

After obtaining the one-step transition matrices, the h langkah ke depan menggunakan Persamaan (9). step transition matrix calculation is performed using Equation (9). In Case 2, a 10 year insurance duration is used, so the transition matrix up to 9 steps ahead is calculated. The 9 step transition matrix calculation for a 50 year old male is as follows.

$$\boldsymbol{P_{50}}^{(9)} = \prod_{z=50}^{58} \begin{bmatrix} p_z^{1,1} & p_z^{1,2} & 0 & q_z^{1,4} \\ 0 & p_z^{2,2} & q_z^{2,3} & q_z^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P_{50}}^{(9)} = \begin{bmatrix} p_{50}^{1,1} & p_{50}^{1,2} & 0 & q_{50}^{1,4} \\ 0 & p_{50}^{2,2} & q_{50}^{2,3} & q_{50}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_{51}^{1,1} & p_{51}^{1,2} & 0 & q_{51}^{1,4} \\ 0 & p_{51}^{2,2} & q_{51}^{2,3} & q_{51}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdots \begin{bmatrix} p_{58}^{1,1} & p_{58}^{1,2} & 0 & q_{58}^{1,4} \\ 0 & p_{58}^{2,2} & q_{58}^{2,3} & q_{58}^{2,4} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boldsymbol{P_{50}}^{(9)} = \begin{bmatrix} 0{,}05424 & 0{,}87100 & 0{,}01145 & 0{,}06310 \\ 0 & 0{,}91967 & 0{,}01726 & 0{,}06310 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The examples to be carried out are 2 cases in determining the net premium value for a person participating in the Long Term Care: Annuity as a Rider Benefit insurance product at ages 25 and 50, who wishes to receive a benefit of Rp 100,000,000 if they pass away. The insured also desires other benefits in the form of an annuity payment for a maximum of 10 years if they undergo care (state 2). The premium is paid at the beginning of each year for 5 years, while the insured is in a healthy state (state 1), with an effective interest rate of 4.75%. The annual premium that must be paid by the insured during the agreement period to receive the benefits according to the agreement will be calculated.

1. Premium Calculation Case 1

$$B_{x:r|}^{LTC} = c\sum_{e=1}^{r} v^{e}{}_{e-1}p_x^{1,1}\, q_{x+e-1}^{1} + b\sum_{e=1}^{r} v^{e}{}_{e-1}\, p_x^{1,1} p_{x+e-1}^{1,2} \ddot{a}_{x+e:r}^{2,2} + \sum_{e=1}^{r} v^{e}{}_{e-1}p_x^{1,1}\, p_{x+e-1}^{1,2} \sum_{h=1}^{c/b} (c-hb) v^{h}{}_{h-1}p_{x+e}^{2,2}\, q_{x+e+h-1}^{2}$$

$$B_{25:10|}^{LTC} = 100.000.000 \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{25}^{1,1} q_{25+e-1}^{1,4} \quad + 10.000.000 \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{25}^{1,1} p_{25+e-1}^{1,2} \ddot{a}_{25+e:r}^{2,2}$$

$$+ \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{25}^{1,1} p_{25+e-1}^{1,2} \sum_{h=1}^{10} (c-hb) \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{h-1}p_{25+e}^{2,2}\, q_{25+e+h-1}^{2,3}$$

$$B_{25:10|}^{LTC} = 100.000.000(0{,}004742) + 10.000.000(2{,}235472) + 6982{,}93157$$

$$B_{25:10|}^{LTC} = 22.835.903$$

The periodic annual premium amount is calculated using Equation (11).

$$P = \frac{B_{25:10|}^{LTC}}{\ddot{a}_{25:5}^{11}} = \frac{22.835.903}{4{,}31370} = 5.293.805$$

Therefore, if the insured wants to pay the premium periodically at the beginning of each year for 5 years, the annual premium amount that must be paid by the insured is Rp 5.293.805.

2. Premium Calculation Case 2

$$B_{x:r|}^{LTC} = c\sum_{e=1}^{r} v^{e}{}_{e-1}p_x^{1,1}\, q_{x+e-1}^{1} + b\sum_{e=1}^{r} v^{e}{}_{e-1}\, p_x^{1,1} p_{x+e-1}^{1,2} \ddot{a}_{x+e:r}^{2,2} + \sum_{e=1}^{r} v^{e}{}_{e-1}p_x^{1,1}\, p_{x+e-1}^{1,2} \sum_{h=1}^{c/b} (c-hb) v^{h}{}_{h-1}p_{x+e}^{2,2}\, q_{x+e+h-1}^{2}$$

$$B_{50:10|}^{LTC} = 100.000.000 \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{50}^{1,1} q_{50+e-1}^{1,4} \quad + 10.000.000 \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{50}^{1,1} p_{50+e-1}^{1,2} \ddot{a}_{50+e:r}^{2,2}$$

$$+ \sum_{e=1}^{10} \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{e-1}p_{50}^{1,1} p_{50+e-1}^{1,2} \sum_{h=1}^{10} (c-hb) \left(\frac{1}{1+0{,}0475}\right)^{e} {}_{h-1}p_{50+e}^{2,2}\, q_{50+e+h-1}^{2,3}$$

$$B_{50:10|}^{LTC} = 100.000.000(0{,}019946) + 10.000.000(6{,}3216295) + 604675{,}048$$

$$B_{50:10|}^{LTC} = 65.815.529$$

The periodic annual premium amount is calculated using Equation (11).

$$P = \frac{B_{50:10|}^{LTC}}{\ddot{a}_{50:5}^{11}} = \frac{65.815.529}{2{,}90490} = 22.656.730$$

Therefore, if the insured wants to pay the premium periodically at the beginning of each year for 5 years, the annual premium amount that must be paid by the insured is Rp 22.656.730.

5. Conclusion

The single premium for LTC: *Annuity as A Rider Benefit* insurance can be determined by implementing the markov chain transition matrix that has been constructed. The amount of the LTC: *Annuity as A Rider Benefit* single premium shows that the annual net premium value increases with the age of the insured, meaning the older the age, the greater the single premium that must be paid. Furthermore, the longer the desired coverage period, the greater the single premium that must be paid by the insured.

In this discussion, premium calculations are performed for two cases where a male insured is 25 years old and 50 years old, taking LTC insurance for a duration of 10 years with coverage against catastrophic diseases. The requested insurance benefit is Rp100.000.000. In the agreement, the premium is paid for 10 years, with a fixed interest rate according to the 2025 BI rate of 4,75% annually.

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