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Modeling the Memory Effects during the Charging and Discharging Dynamics of RC Circuits

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ABSTRACT

This paper discusses the fractional-order model of the charging and discharging of RC circuits. Charging and Discharging process of an RC circuits with a resistor 39 kiloohms and a capacitor 1200 microfarad. The work determines the classical and fractional-orders method of model reduction of the behavior of the circuit. In the classical approach, the charging and discharging processes are studied with the natural logarithms which give standard exponential responses. The fractional-order methodology, however, uses non-integral derivatives, which accounts for the slow response to, and non-ideal behavior of, real world circuits. A comparative study of the two techniques shows that fractional modeling gives a more realistic and versatile input of the dynamic response of the circuit, which is effective in emphasizing memory effects in the circuit. These results indicate that fractional calculus may be a useful tool to model real-world RC circuits which are not constrained by classical-exponential model

Keywords: Fractional Calculus, RC Circuits, Memory Effects, Charging Dynamics, Discharging Dynamics.

1. Introduction

Classical exponential functions (using integers-order calculus) are often used to model RC circuits [1]. Non-ideal behavior is however common in practical circuits, as the memory effects and retention of long-term charge cannot be represented well in standard models [2]. Such non-exponential and history-dependent responses o electrical systems can be better modeled using a more appropriate framework offered by the fractional calculus [3].

2. Methodology

2.1 RC Circuits Law

A circuit containing only two elements, a resistor (R), opposing the current flow, and a capacitor (C), storing electrical energy in the form of an electric field between the plates, is called an RC circuit. Resistor and capacitor are called as time constant (τ). Time constant are:

$\boldsymbol{\tau = R * C = 39\, k\Omega * 1200\, \mu F = 46.8\, sec}$.

2.2 Fractional-Order RC Circuit Model

Applying Kirchhoff's voltage law:

$$\boldsymbol{\varepsilon = V_R + V_c}$$

$$\boldsymbol{\varepsilon = R\frac{dQ}{dt} + \frac{Q}{C}}$$

(Integral order differential equation)

$$\boldsymbol{\frac{d^{\alpha}Q}{dt_{\alpha}} = \frac{C\varepsilon - Q}{RC}}$$

Since,

$$\boldsymbol{D^{\alpha}I(x) = \frac{\Gamma(\beta)}{\Gamma(\beta - \alpha + 1)}x^{1-\alpha}\frac{dI}{dx}}$$

$$\boldsymbol{\int_0^{Q_o}\frac{-dQ}{Q - C\varepsilon} = \mu_o\frac{\Gamma(\beta - \alpha + 1)}{\Gamma(\beta)}\int_0^t\frac{dt}{t^{1-\alpha}}}$$

$At \quad t = 0, Q = 0 \; \& \; t = t_{max}, Q = Q_o :$

$$ln\,|1 - \frac{Q}{Q_o} = -\frac{t^{\alpha}}{\Gamma(\alpha+1)}$$

The above equation represents the Mittag-Leffler function [4] in τ, where $\tau = RC$:

$$[1 - \frac{Q}{Q_o}] = exp\,[-\tau\,\frac{t^{\alpha}}{\Gamma(\alpha+1)}]$$

$$Q = Q_o[1 - exp\,(-\tau\,\frac{t^{\alpha}}{\Gamma(\alpha+1)})]$$

For $\alpha = 1$:

$$\Gamma(2) = 1$$

$$Q(t) = Q_o[\,1 - exp(-\frac{t}{RC})]$$

Fractional-order equation for Charging of capacitor:

$$V(t) = V_o[\,1 - exp(-\,\frac{t}{RC})]$$

And the capacitor begins to discharge across the resistor when the external supply is removed. The discharge process can be represented as a fractional order equation, which is given as:

$$V(t) = V_o[exp(-\frac{t}{RC})]$$

3. Results and Discussion

3.1 Voltage across the Capacitor during the Charging and Discharging:

In the experimental setup, the capacitor and resistor were attached to the bread board in a series configuration. The DC power supply ensured the constant supply of 9 V voltage. In the voltage across the capacitor V(t), the multimeter was used in the measurement at some periods during the charging and discharging. The findings show that the voltage across the capacitor during charging and discharging are:

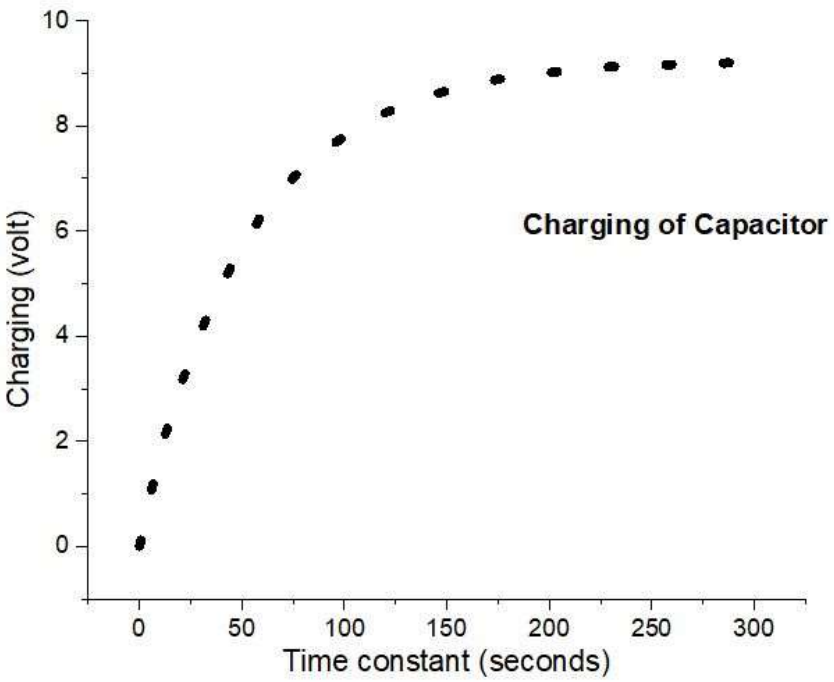

Figure 1: Voltage V(t) across the Capacitor during the Charging.

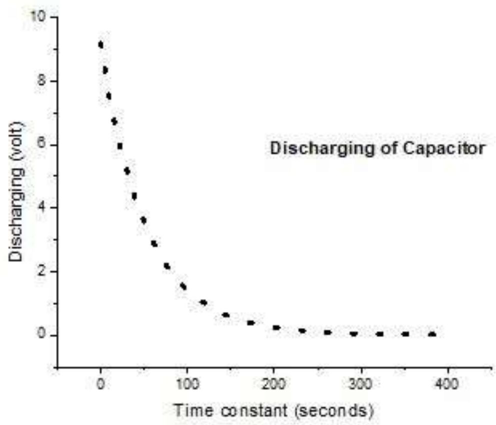

Figure 2: Voltage V(t) across the Capacitor during the Discharging.

3.2 Graphical Determination of Time Constants for Charging and Discharging

An alternative method to determine the time constant involves plotting functions of the normalized voltage difference on the y-axis versus time (t) on the x-axis. The key difference lies in the transformation used for charging and discharging are:

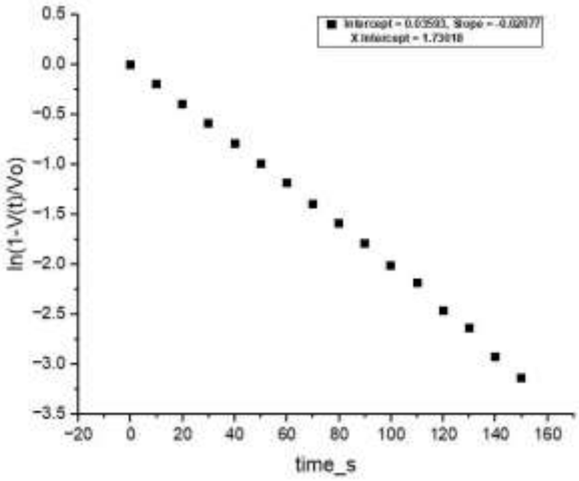

Figure 3: Plotting ln (1 − V (t)/V o) by time for the Charging of capacitor.

The slope of this linear plot for charging provides the negative inverse of the charging time constant $\tau_{charging}$, which can be calculated as

$$\tau_{charging} = -\frac{1}{slope}$$

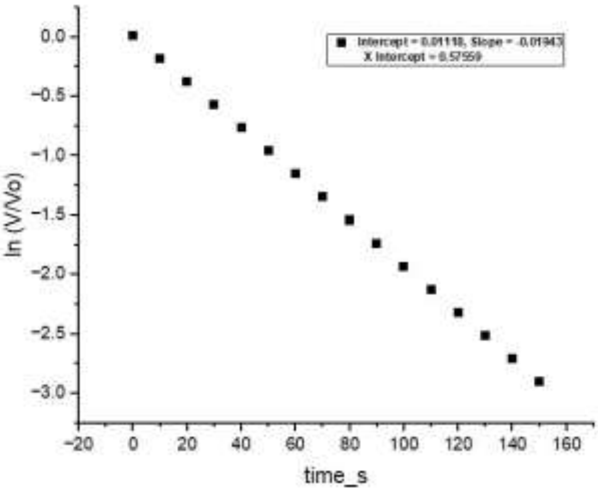

Figure 4: Plotting ln (V (t)/V o) by time for the Discharging of capacitor.

The slope of this linear plot for discharging provides the negative inverse of the discharging time constant $\tau_{discharging}$, which can be calculated as:

$$\boldsymbol{\tau_{discharging} = -\frac{1}{slope}}$$

4. Conclusions

According to the study, in conclusion, this study has presented useful information on how the modeling of the RC circuits can be further refined using fractional calculus especially in the explanation of the memory effects which the other integer-order models fail to explain. By simple adjustment of the theoretical time constant τ through the use of the fractional order α, we managed to optimize the theoretical time constant to experimentally measured values $39k\Omega, 1200\mu F$. The findings showed that the fractional calculus provides a higher level of flexibility in the modeling of the time constant that has the potential to model the behavior of the system more accurately particularly in circuits that are characterized by the presence of memory effects and long-term dynamics.

5. References

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