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New Approach on the Construction of Maximal Size Antichains Using Set Theoretic Notations

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ABSTRACT

Anti chains play a central role in combinatorics, lattice theory and graphs due to their fundamental properties and application on optimization, information theory and order theory.

This paper provides a new approach to the construction of maximal size antichains by employing some set theoretic concepts and real number system. The research establish a systematic method for deriving maximal size antichain in finite partially ordered sets. Furthermore, the approach highlights new insights in to the structure of Boolean lattices and their extremal properties. The paper also used the structures by defining an arbitrary posets Δ on the structures to establish some algebraic theoretic consequences. The research further used the results to described some new structures such as Modular and semi lattice structures. This research also provides a more transparent representation of elements and their comparability making it easier to characterize family of sets that form antichains of maximum cardinality.

Keywords: *Antichains ,Posets, Boolean Lattice and sperner's theorem*

Introduction

Antichains are important structures used in construction methods in engineering and design of computer softwires as well as used as algebraic tool for exploring broader problems in discrete mathematics (Dmitry, 2023). Antichains of a subsets is a set of subsets such that no subset in the antichain is a proper subset of that set (Patrick, Stefan and Jay,2017). In accordance with

Zaid, Ibrahim and Garba, 2015) for the application of antichain in cement industries, maximum or largest antichain in production is one of which the greatest element in the collection subset of raw materials is possible and the size of the largest antichain is known as the poset"s width.. Freese (1974) reported on the new method of the proof of the Dilworth theorem on the formation of maximal size antichains and obtained some algebraic structures..Ibrahim (2007) described posset as a set together with a binary relationwhich indicates that, for certain pairs of elements in the set, one of the elements precedes the other. Such a relation is called a *partial order.*

A chain C is maximal in a posset (S, ≤)if no element of S-C is comparable to every element of C. An antichain A is maximal if no element of S-A is in comparable to every element of A.A maximum or largest chain (largest antichain) is one of whichS is the greatest possible element.. The size of the longest chain is known as a posset's height andthe size of the S largest antichain is known as the posset's width (Patric, *at el,* 2016.)

Definition of some basic notations used

Definition 1.1

A chain is a totally ordered subset of a poset S, and antichain is a subset of a poset S in which no any two distinct elements are comparable.

Definition 1.2

An poset is a partially odered sets on which the binary relation defined satisfied the reflective, antisymetric and transitive relation.

Definition 1.3

An element x of a Poset (X, R) is called maximal if there is no element $y \in X$satisfying$x < Ry$. Dually, $x$is minimal if no element satisfies$y < Rx$. In a general Poset there may be no maximal element, or there may be more than one. But in a finite Posset there is always at least one maximal element, which can be found as follows: choose any element$x$; if it is not maximal, replace it by an element y satisfying $x < Ry$; repeat until a maximal element is found. The process must terminate, since by the irreflexive and transitive laws the chain can never revisit any element (Garba and Ibrahim,2009)

Definition 1.3

Let (X, R)be a finite Poset. Then there is a partition of Xinto $w(X)$chains. An up-set in a Poset$(X,\ R)$is a subset $XandY$such that, if $y \in Y$and $y \leq R$, then $z \in Y$. The set of minimal elements in an up-set is an antichain. Conversely, if A is an antichain, then $\uparrow (A) = \{x \in X: a \leq Rx for some a \in A\}$is an upset. These two correspondences between up-sets and antichains are mutually inverse; so the numbers of up-sets and anti-chains in a poset are equal. Down-sets are, of course, defined dually. The complement of an up-set is a down-set; so there are equally many up-sets and down-sets (Ibrahim, 2005)

Method of Construction:.

We first begin the constrution by defining the set theoretic notations A_B, B_A, A^B, B^A and then applied the concept of real number system as contained in (Ibrahim and Audu, 2005).We defined as follows:

$$A_B = \{a \in A \big| a \geq b \mid for\ some\ b \in B\}$$
$$= \big|\{a_1 \geq b, a_2 \geq b, a_3 \geq b, a_4 \geq b......\} \mid = A \qquad (1)$$

Equation (1) implies there exist $b \in B$ which is smaller than some $a_{i's} \in A$ but not all a_i

example: If A = {3,5,7,11,13 , 17}, B = {2,4,6,8,10,12,14,16,18}

and then choose an element $3 = a_1, 5 = a_2, 7 = a_3, 11 = a_4$ *and so on in such away that* $3 = a_1 \geq 2 = b\ , a_2 \geq b, a_3 \geq b\ , a_4 \geq b$ ….

$$Hence, \quad A_B = \{3,5,7,11,13...\} \qquad (2)$$

$$Similarly,$$
$$A_B = \{a \in A \mid a \geq b\ for\ some\ b \in B\} \qquad (3)$$
$$B_A = \{b \in B \mid b \geq a\ \ for\ some\ a \in A\} = B \qquad (4)$$

Using the set theoretic concept method: we have the union and intersection represented as $\vee \to \cup, \wedge \to \cap$ and $a, b, c \Rightarrow A, B, C,$ and Sets with a, b, c represents the elements in the set. In the study of partially ordered set with l.u.b and g.l.b to form a lattice, then $a \leq b$ will form a chain and consequently $a \geq b$ will be an antichain. However:

$$A_B = \{a \in A \mid a \geq b\ for\ some \quad b \in B\}\, [b, \infty) \qquad (7)$$
$$A^B = \{a \in A \,| a \leq b\ for\ some \quad b \in B\}\ (-\infty, b] \qquad (8)$$
$$= (-\infty, b] \ \ \cup \ \ [b, \infty)$$
$$= \ (-\infty\ , \infty) = A =$$
$$B_A = \{b \in B \mid b \geq a, \quad for\ some\ a \in A\}, (b, \infty) \qquad (9)$$
$$B^A = \{b \in B \mid b \leq a \quad for\ some\ a \in A\}[-\infty, b] \qquad (10)$$
$$= (-\infty, \infty) = B =$$

$$\therefore A^B = \{a \in A \mid a \leq b \quad for\ some\ b \in B\} \qquad (12)$$
$$example: let \quad A = \{1,10\}, B = \{2,12\}, A \vee B = A$$

$$If \quad a_1 \le b_1, \quad a_2 \le b_2, \quad a_3 \le b_3, \quad and \quad a_1, a_2, a_3 \in A$$
$$also \quad b_1, b_2, b_3 \in B \quad then$$
$$A^B = \{3,5,7,11\} \ne A$$
$$A \le B \Leftrightarrow A^B = A \tag{13}$$

Using the above notations:

$$| A_B \cup B_A | = | A^B | + | B^A | - | A^B \cap B^A | \tag{14}$$

$$| A_B \cup B_A | = | A^B | + B^A | - | A^B \cap B^A || A_B \cup B_A | + | A^B \cup B^A | \tag{15}$$
$$= | A | + | BA | - | A_B - \cap B_A | + A \cap B | + | A^B | B^A | - | A^B \cap B^A |$$
$$= | A_B || B_A | + | A^B | + | B^A | -2 | A \cap B |$$

$$\therefore | A \vee B | + | A \wedge B | = | A_B \cup B_A | + | A^B \cup B^A | \tag{16}$$

$$and\ also, \quad A \vee B = A_B \cup B_A \tag{17}$$
$$A \wedge B = A^B \cup B^A \tag{18}$$

However,

$$| A_B \cup B_A | + | A^B \cup B^A | = | (A^B \cup B^A) | + | B_A \cup B^A | \tag{19}$$
$$= | A | + | B |$$

$$let \quad | A | = n \quad be \max imum\ Antichain$$
$$| B | = n \quad be \max imum\ Antichain$$
$$then, \quad | A \wedge B | \le n, \quad where\ n\ is\ the \max imal\ size\ antichain$$

Similarly,

$$B_A = \{b \in B \mid b \ge a,\ for\ some\ a \in A\} = [a, \infty) \tag{20}$$
$$B^A = \{b \in B \mid b \le a,\ for\ some\ a \in A\} = (-\infty, a] \tag{21}$$
$$\therefore [-\infty, a) \cup [a, \infty)$$
$$= (-\infty, \infty) = B =$$
$$where, \quad A_B \cup A^B = A, \quad B_A \cup B^A = B \tag{22}$$

$$A_B \cap B_A = [b, \infty) \cap [a, \infty) \tag{23}$$
$$= [a, \infty) \cap [b, \infty)$$
$$= A \cap B$$

$$A^B \cap B^A = (-\infty, b] \cap (-\infty, a] \tag{24}$$
$$= (-\infty, a] \cap [-\infty, b)$$
$$= A \cap B$$

$$A_B \cap A^B = [b, \infty) \cap (-\infty, b] \tag{25}$$
$$= (-\infty, \infty] = A =$$

$$B_A \cap B^A = [a, \infty) = (-\infty, a] \tag{26}$$
$$= (-\infty, \infty) = A =$$

$$and\ \ finally, (A_B \cap A^B) \cap (B_A \cap B^A) = A \cap B \tag{27}$$

.

$nsidering\ the\ \ above\ structures\ we\ defined\ as\ follows{:}\ Let\ \ Type\ equation\ here..$

RESULTS AND DISCUSSIONS

By examine the structures of maximal size antichains constructed , we obtained the following algebraic theoretic consequences.

Proposition 1.1

Let ς be a partially ordered sets defined on set of antichains and G be group of homomorphism of elements, then the relation forms a semi lattice structure.

Proof: Suppose Δ is the set of all antichains ordered by $r \leq s\ \forall r \in R\ and\ s \in S\ \ such\ that\ r, s \in \Delta$. Then a partial ordered of the set $(\Delta, \leq)$ where every doubleton (r, s) has a $l.u.b\ \ and\ \ g.l.b$ respectively. Since both satisfied the associative, commutative and idempotent properties. Then ($r \wedge s$) would be the greatest lower bond and $(r \vee s)$ will be the maximum. If a power set σ is injected in Δ by ordered inclusion relation, it forms the upper semi lattices with $\vee$ and lower semi lattices with $\wedge$. Hence, it forms a semi lattices.

Proposition 1.2

Let ς be a partially ordered set defined on set of antichains., then the set of elements in the poset forms an equivalence relation .

Proof: By the definition of partially ordered set, the elements in the poset satisfied the reflexive, symmetric and transitive properties by the union and intersection of their elements , then the relation is an equivalence relation

Proposition 1.3

The set of elements of the finite posets forms a maximal Antichains.

Proof:.

Let Δ be the set of maximal elements of ς , and then that any augmentation of Δ form an antichain .. Also, consider the distinct elements $x, y \in \Delta$ and if x is maximal then $y \neq x$ but x and y are guarantees for $x \leq y.$ Thus $x\ and\ y$ are antichains

The proof for minimal elements proceeds along similar lines.

Proposition 1.4.

Let ς be a partially ordered set defined on set of antichains, then L forms a modular lattices

Proof: suppose that r, s, t $\in$ L, then it is clear that the associative property holds in L. That is That is $r \wedge (s \vee t) = (r \wedge s) \vee t.$ Hence it is modular lattices

Proposition 1.5.

Let ς be a partially ordered set defined on set of antichains, then the Lattice L forms composition relation

Proof: Suppose that $\rho \subseteq r \times s, \sigma \subseteq s \times t$ and $\gamma \subseteq t \times y\ \ \ \forall r, s, t, y \in L$ and also let $(r, y) \in (\rho \circ \sigma) \circ \gamma$ in the set of maximal antichains,then that there exist $s \in S$ and $t \in \sigma$ ssuch that $(a, t) \in \rho \circ \sigma\ and\ (t, y) \in \sigma$ $(r, s) \in \rho, (s, t) \in \sigma\ \ \ and\ (t, y) \in \gamma$ This implies that: $(r, s) \in \rho, and\ (s, y) \in (\sigma \circ \gamma)\ \ \ \Rightarrow (r, y) \in \rho \circ (\sigma \circ \gamma)$

$\therefore (\rho \circ \sigma) \circ \gamma \subseteq \rho \circ (\sigma \circ \gamma)\ and\ hence\ \ \ (\rho \circ \sigma) \circ \gamma = \rho \circ (\sigma \circ \gamma)$ as a composition relation.

CONCLUSION:

In this paper, some useful results of maximal size antichains were provided. The research examined the structures by defining the finite poset on the structures and obtained some useful algebraic theoretic consequences. The research further used the new structures to described the formation of the modular lattice as well as semi lattice structures. Finally, the work provide a derivation that the elements in the set of antichain forms a composition as well as equivalence relation relevant in order and the optimization theory

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CONSTRUCTION OF GROUP ACTION:

Let Δ be the set of maximal size antichains generated from $|A| \quad and \quad |B|$ of three elements sample satisfying reflexive, antisymetric and transitive algebraic property of poset. Let G be a group of automomorphism of these groups on each element of the poset Δ.To test the associative group action on each element of the poset, we begin as follows:

$$Let\ \ G = g_n = \left\{ \begin{matrix} (1) & (123) & (132) & (12) & (13) & (23) \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ g_1 & g_2 & g_3 & g_4 & g_5 & g_6 \end{matrix} \right\} \quad for\ n = \ 1...6$$

We shall test the action of G on each element of the poset, since

$$\Delta = \alpha_n = \left\{ \begin{matrix} 1 & 2 & 3 \\ \downarrow & \downarrow & \downarrow \\ \alpha_1 & \alpha_2 & \alpha_3 \end{matrix} \right\} \qquad for \quad n = 1...3$$

$$\begin{aligned} for \quad \Delta^G &= \alpha_1{}^{g_1} = (1)^{(1)} = 1 \\ &= \alpha_1{}^{g_2} = (1)^{(12)} = 2 \\ &= \alpha_1{}^{g_3} = (1)^{(13)} = 3 \\ &= \alpha_1{}^{g_4} = (1)^{(23)} = 1 \\ &= \alpha_1{}^{g_5} = (1)^{(123)} = 2 \end{aligned} \qquad \{1 \quad 2 \quad 3\}$$

$$\begin{aligned} P^G &= \alpha_2{}^{g_1} = (2)^{(1)} = 2 \\ &= \alpha_2{}^{g_2} = (2)^{(12)} = 1 \\ &= \alpha_2{}^{g_3} = (2)^{(13)} = 2 \\ &= \alpha_2{}^{g_4} = (2)^{(23)} = 3 \\ &= \alpha_2{}^{g_5} = (2)^{(123)} = 3 \\ &= \alpha_2{}^{g_6} = (2)^{(132)} = 1 \end{aligned} \qquad \{2 \quad 3 \quad 1\}$$

$$\begin{aligned} P^G &= \alpha_3{}^{g_1} = (3)^{(1)} = 3 \\ &= \alpha_3{}^{g_2} = (3)^{(12)} = 3 \\ &= \alpha_3{}^{g_3} = (3)^{(13)} = 1 \\ &= \alpha_3{}^{g_4} = (3)^{(23)} = 2 \\ &= \alpha_3{}^{g_5} = (3)^{(123)} = 1 \\ &= \alpha_3{}^{g_6} = (3)^{(132)} = 2 \end{aligned} \qquad \{3 \quad 1 \quad 2\}$$

FORMATION OF ORBIT STRUCTUR

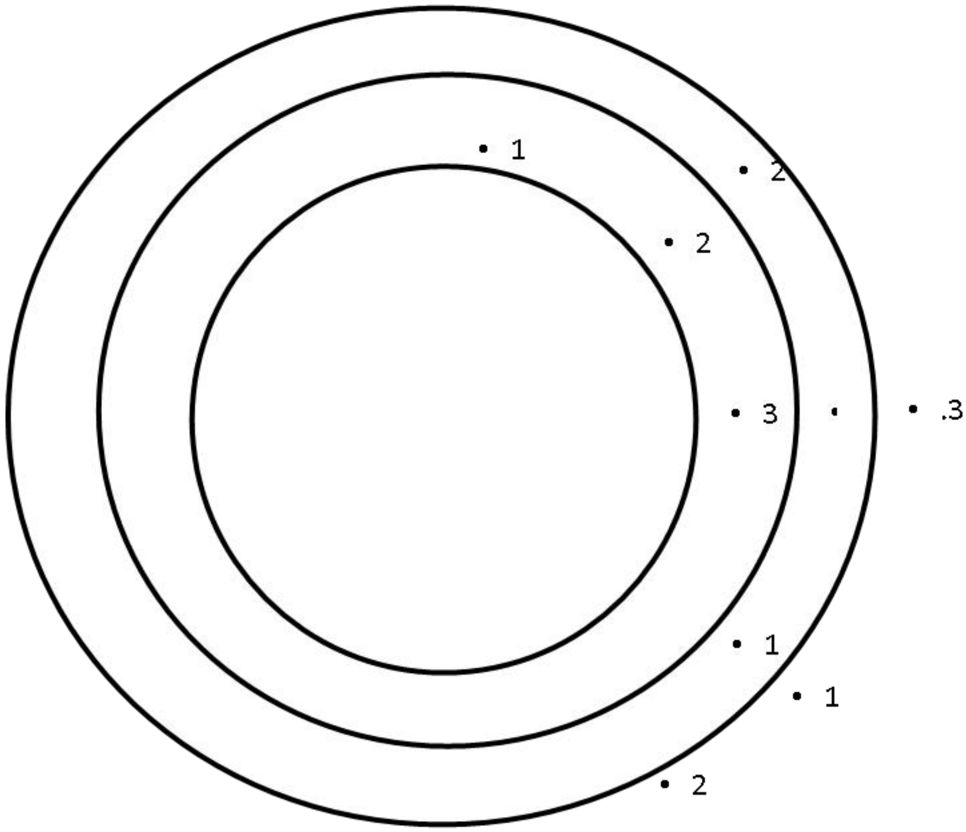

Figure1 : 3-points orbit structure

Some algebraic theoretic properties obtained

1. **Lemma 1.**The action group on any element of the poset gives rise to the formation of orbits of equal length.

Proof: It is clear that $|G| = |G_\alpha| \cdot |\alpha^G|, \quad where\,|\alpha^G| \quad represent\ the\ length\ of\ orbit.$

2. The structure of orbits formed defined the stabilizer of the action group as $G_\alpha = \eta(\alpha) = \{g \in G \mid \alpha^g = \alpha\}$ where η is the stabliser
3. Formation of 3-point orbit structure with a smooth curves which is relevant to the study of isometry groups of space forms in Riemanian manifold.

Theorem 1 (Abubakar and Ibrahim,2012)

Let G be a finite group and H be a proper subgroup of G of index n. Then there is a normal subgroup K of G contain in H Such that G/K is isomorphic to a subgroup S_n. In particular $|G:K|$ divides n! and it is atleast n.

Proof: let *K*be the kernel of action of *G* on the element of *P*. Where *P* is

the left cosets of *H* from a subgroup of S_n.From the Lagranges

theorem, the sets of left cosets implies a sub-group of S_n Which

has order dividing n! Since $K \leq H$ and H is of index n in G

then $|G/K| \geq n$ .Now assume that it is a non labelian simple group,

then $G \cong G^p$ which is a subgroup of S_n. But A_n is a normal subgroup of S_n and so $G^p \bigcap A_n$ is normal. It there for implies that $G^p = \{e\}$ or $G^p \bigcap A_n = G^p$ .In the first case the second

implies $G^p \mid (G^p \bigcap A_n) \cong (G^p A_{n)} / A_n \leq S_n \mid A_n$ and hence

$\mid G^p \mid \leq 2$ since $G^p \bigcap A_n = G^p$ a contradiction. Finally, we have $n \geq 4$ since has a non abelian simple subgroups.

4. Let G be a finite group. Then $1 = \sum 1/\mid CG(x)\mid$, summing over disinctt conjugacy classes.

Proof: We count the element of G that is

$\mid G \mid = \sum \mid CCL_G(x) \mid$, so $\mid G \mid = \sum \frac{\mid G \mid}{\mid C_G(x) \mid}$ $since$ $\mid CCL_G(x) = \mid G \mid / \mid C_G(x) \mid$ Which divides $\mid G \mid$

CONCLUSION

This paper provides an alternative method in the proof of Dilworth's theorem to strengthenthe result of the other proof and also to described the formation of some algebraic structures associated with the new construction.The research also used the set of generated maximal antichains to test the

action of group on the arbitrary finite set of cardinality three which helped to established a new algebraic structure resembling to the structure of a lattice. Further more,the research also helped to established some algebraic theoretic consequence of some certain algebraic structures which are relevant to the study of the theory of group and its applications.

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