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Simulation and Performance Evaluation of Extended Kalman Filter in Angular Coordinate Tracking of Highly Maneuvering Targets

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A B S T R A C T

This paper presents the simulation and performance evaluation of the Extended Kalman Filter (EKF) in estimating and tracking the angular coordinates of a highly maneuvering target. The target model uses a state-space representation including angle, angular velocity, and angular acceleration, where the acceleration varies over time and is affected by random noise. Three different process noise covariance parameter sets are used to analyze the impact of dynamic characteristics on the accuracy and stability of the filter. Simulation results show that different parameter configurations lead to distinct filtering behaviors in terms of delay, oscillation, and convergence error. Evaluation metrics such as RMSE, bias, and standard deviation (std) under a sliding window are applied to assess tracking quality over time. The results indicate that selecting appropriate dynamic parameters is key to achieving high filtering performance in fast-varying target environments.

Keywords: Extended Kalman Filter, EKF, angle tracking, state estimation, maneuvering target, RMSE, simulation, real-time

1. Introduction

In positioning and control systems such as radar, missile guidance, or target motion monitoring, accurately tracking the angular rotation of a target is crucial to ensure effective tracking and control. However, in practice, targets often exhibit strong maneuverability with irregular and noisy acceleration, leading to significant errors when using simple linear filtering methods. The Extended Kalman Filter (EKF) is a nonlinear filtering technique capable of updating system states in real-time under noisy and nonlinear environments. EKF uses local linearization to process the state and measurement functions, thereby improving prediction and correction capabilities. However, the performance of EKF strongly depends on the selection of dynamic parameters such as the process noise covariance matrix $\boldsymbol{Q}$ and measurement noise $\boldsymbol{R}$, as well as the mechanical behavior of the target. This study models the target with three states: angular position, angular velocity, and angular acceleration, where the acceleration is influenced by input noise. Three different $\boldsymbol{Q}$ matrices are configured to simulate different levels of inertia and system oscillations. Based on that, the EKF algorithm is applied and evaluated using standard error metrics. The paper aims to address the question: How should EKF parameters be chosen to balance between response speed and accuracy for fast and irregularly maneuvering targets.

2. Algorithm Synthesis

The target is modeled as a second-order nonlinear system with three states: angle, angular velocity, and angular acceleration. The state vector at time step k is represented as:

$$x_k = \begin{bmatrix} \theta_k \\ \dot{\theta}_k \\ \ddot{\theta}_k \end{bmatrix} \qquad (1)$$

Where:

θ_k - angle at time k;

$\dot{\theta}_k$ - angular velocity at time k;

$\ddot{\theta}_k$ - angular acceleration at time k.

Assuming uniformly accelerated motion, the nonlinear model predicting the next state is:

$$x_k = f(x_{k-1}) + \mathrm{w}_{k-1} \qquad (2)$$

Where:

$f(.)$ - is the nonlinear transition function;

$\mathrm{w}_{k-1} \sim N(0,\boldsymbol{Q})$ - is process noise.

The nonlinear function is:

$$f(x_{k-1}) = \begin{bmatrix} \theta_{k-1} + \dot{\theta}_{k-1}\Delta t + \frac{1}{2}\ddot{\theta}_{k-1}\Delta t^2 \\ \dot{\theta}_{k-1} + \ddot{\theta}_{k-1}\Delta t \\ \ddot{\theta}_{k-1} \end{bmatrix} \qquad (3)$$

Only the angular position θ is measured, with measurement noise $\upsilon_k \sim N(0,\boldsymbol{R})$. The measurement model is:

$$z_k = h(x_k) + \upsilon_k = \theta_k + \upsilon_k \qquad (4)$$

The Jacobian of the measurement function is:

$$\boldsymbol{H}_k = \frac{\partial h}{\partial x} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \qquad (5)$$

The Extended Kalman Filter (EKF) is an effective tool for estimating the state in nonlinear systems, by locally linearizing the state and measurement functions at the current operating point. The EKF algorithm consists of two main steps: prediction and update.

Prediction Step:

In this step, the system state at time k is predicted from the state at time $(k-1)$ using the nonlinear function $f(.)$. At the same time, the predicted error covariance matrix is updated based on the Jacobian of the state function.

$$\hat{x}_{k/k-1} = f(\hat{x}_{k-1/k-1}) \qquad (6)$$

Where:

$\hat{x}_{k/k-1}$ - is the predicted state at time k based on the information at time $(k-1)$.

Next, compute the Jacobian matrix of f denoted as $\boldsymbol{F}_k$ used to approximate linearization in the error covariance propagation step:

$$\boldsymbol{F}_k = \left.\frac{\partial f}{\partial x}\right|_{\hat{x}_{k-1/k-1}} \qquad (7)$$

Update the predicted error covariance matrix:

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{F}_k \boldsymbol{P}_{k-1/k-1} \boldsymbol{F}_k + \boldsymbol{Q} \qquad (8)$$

Where:

$\boldsymbol{P}_{k/k-1}$ - predicted error covariance matrix;

$\boldsymbol{P}_{k-1/k-1}$ - covariance after the previous update;

$\boldsymbol{Q}$ - process noise covariance matrix.

Update Step:

Once the measurement z_k at time k is available, the EKF updates the estimated state by computing the residual y_k (residual) between the actual measurement and the predicted measurement from the estimated state. Residual (measurement innovation):

$$y_k = z_k - h(\hat{x}_{k/k-1}) \qquad (9)$$

Where:

$h(.)$ - measurement function;

z_k - actual measurement k ;

$\hat{x}_{k/k-1}$ - predicted state at time k .

Linearize the measurement function by calculating the Jacobian $\boldsymbol{H}_k$:

$$\boldsymbol{H}_k = \left.\frac{\partial h}{\partial x}\right|_{\hat{x}_{k/k-1}} \tag{10}$$

Measurement residual covariance matrix:

$$\boldsymbol{S}_k = \boldsymbol{H}_k \boldsymbol{P}_{k/k-1} \boldsymbol{H}_k^T + \boldsymbol{R} \tag{11}$$

Compute Kalman Gain:

$$\boldsymbol{K}_k = \boldsymbol{P}_{k/k-1} \boldsymbol{H}_k^T \boldsymbol{S}_k^{-1} \tag{12}$$

Update the state and error covariance:

$$\begin{aligned} \hat{x}_{k/k} &= \hat{x}_{k/k-1} + \boldsymbol{K}_k y_k \\ \boldsymbol{P}_{k/k} &= (\boldsymbol{I} - \boldsymbol{K}_k \boldsymbol{H}_k)\boldsymbol{P}_{k/k-1} \end{aligned} \tag{13}$$

Where:

$\hat{x}_{k/k}$ - final estimated state at time k ;

$\boldsymbol{P}_{k/k}$ - error covariance after update;

$\boldsymbol{I}$ - identity matrix.

To evaluate the performance of the EKF in the problem of angular coordinate tracking, we analyze the error quantities between the estimated and the true states. The evaluation metrics include:

- Instantaneous error at each time step;

- Root Mean Square Error (RMSE);

- Bias;

- Standard Deviation (Std).

These metrics are computed separately for each state component, including: angular position (θ) , angular velocity $(\dot{\theta})$ and angular acceleration and are compared across different parameter sets.Instantaneous error is defined as the difference between the estimated and the true value of each state:

$$e_i(k) = \hat{x}_i(k) - x_i(k) \tag{14}$$

Where:

$i = 1,2,3$ corresponds to angular position, angular velocity, and angular acceleration, respectively;

k - is the discrete-time index.

- Root Mean Square Error (RMSE) over the entire time sequence is calculated as:

$$RMSE_i = \sqrt{\frac{1}{N}\sum_{1}^{N} e_i(k)^2} \tag{15}$$

RMSE reflects the average deviation between the estimated and actual values. A lower RMSE indicates better tracking quality. To analyze the dynamic behavior of the error over time (especially during phases when the target changes rapidly), a sliding window technique is applied with a fixed length of W samples (typically W=50).

- Sliding window RMSE:

$$RMSE_i^k = \sqrt{\frac{1}{\mathrm{W}}\sum_{j=k}^{k+\mathrm{W}-1} e_i(j)^2} \tag{16}$$

- Bias (average error within the window):

$$Bias_i^{(k)} = \frac{1}{\mathrm{W}}\sum_{j=k}^{k+\mathrm{W}-1} e_i(j) \tag{17}$$

- Local Standard Deviation**:**

$$Std_i^k = \sqrt{\frac{1}{\text{W-1}} \sum_{j=k}^{k+\text{W}-1} e_i(j) - Bias_i^{(k)2}} \qquad (18)$$

3. Simulation and Evaluation

To conduct the simulation and evaluate the developed algorithm, the following parameter sets are selected:

Process noise is configured through the covariance matrix:

$$\boldsymbol{Q}^{(i)} = \begin{bmatrix} \sigma^2_{\theta^{(i)}} & 0 & 0 \\ 0 & \sigma^2_{\dot{\theta}^{(i)}} & 0 \\ 0 & 0 & \sigma^2_{\ddot{\theta}^{(i)}} \end{bmatrix} \begin{bmatrix} \deg^2 \\ (\deg/s)^2 \\ (\deg/s^2)^2 \end{bmatrix} \quad (19)$$

$\sigma^2_{\theta^{(i)}}$ - process noise variance of angular position $\deg^2$;

$\sigma^2_{\dot{\theta}^{(i)}}$ - process noise variance of angular velocity $(\deg/s)^2$;

$\sigma^2_{\ddot{\theta}^{(i)}}$ - process noise variance of angular acceleration $(\deg/s^2)^2$.

Set 1*:*

$$\boldsymbol{Q}^{(1)} = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} \deg^2 \\ (\deg/s)^2 \\ (\deg/s^2)^2 \end{bmatrix}$$

Set 2*:*

$$\boldsymbol{Q}^{(2)} = \begin{bmatrix} 0.005 & 0 & 0 \\ 0 & 0.05 & 0 \\ 0 & 0 & 0.2 \end{bmatrix} \begin{bmatrix} \deg^2 \\ (\deg/s)^2 \\ (\deg/s^2)^2 \end{bmatrix}$$

Set 3:

$$\boldsymbol{Q}^{(3)} = \begin{bmatrix} 0.02 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 1.0 \end{bmatrix} \begin{bmatrix} \deg^2 \\ (\deg/s)^2 \\ (\deg/s^2)^2 \end{bmatrix}$$

- Measurement noise covariance:

$$\boldsymbol{R} = \sigma_z^2 = 0.25\left[\deg^2\right]$$

- Acceleration input noise:

$$\sigma_{input} = 0.8[\deg/s^2]$$

This value is used to model the random noise affecting the target's acceleration process. Simulation Results:

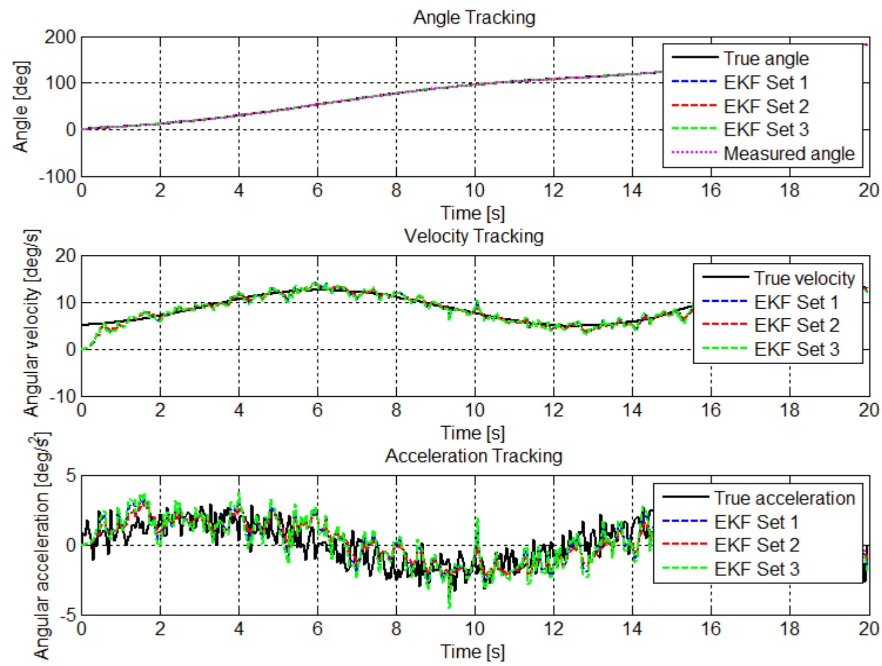

Figure 1. Illustrates the tracking results for the three system states using three EKF configurations with different process noise parameters $\boldsymbol{Q}$

It can be observed that all filters track the true state well. However, the tracking quality varies significantly among the parameter sets. The filter with smaller process noise provides smoother signals but responds more slowly to sudden changes. Conversely, filters with larger noise parameters respond faster but exhibit more oscillation and noise in the estimated results. Some filters show clear delay in the acceleration state, while others respond faster but with greater fluctuations. The simulation results clarify the role of dynamic parameters in tuning filter behavior and emphasize that there is no universally optimal configuration. Instead, the choice of $\boldsymbol{Q}$ should be appropriate for the target's dynamic characteristics and the specific accuracy or responsiveness requirements of each application.

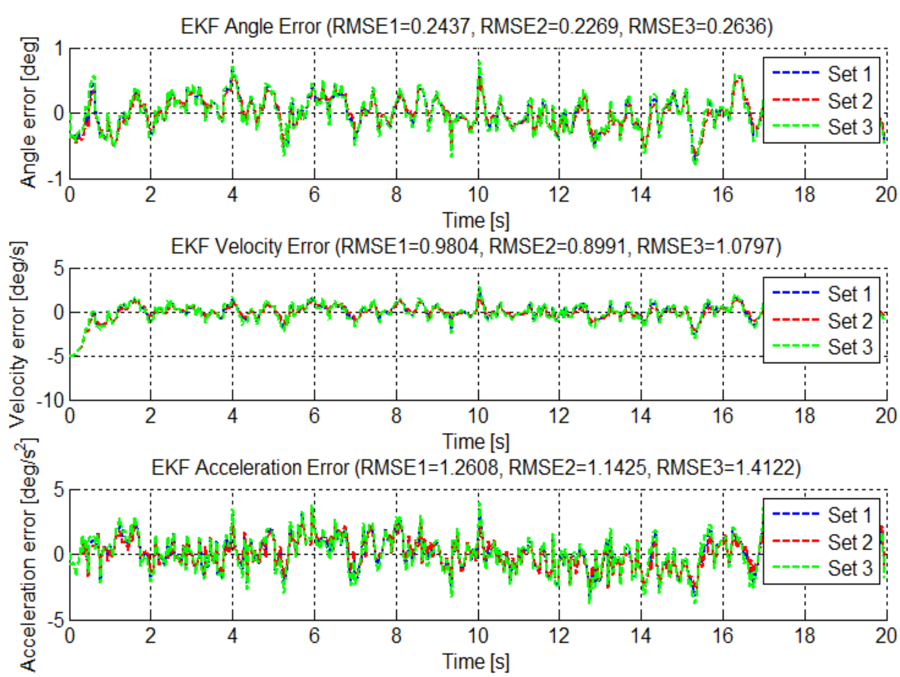

Figure 2. Errors in angular position, angular velocity and angular acceleration of the target using EKF with three different process noise covariance parameter sets $\boldsymbol{Q}$

From the graphs, we can observe that the error varies over time and differs significantly across the $\boldsymbol{Q}$ parameter configurations, especially during periods of strong system dynamics. For the angular position state, the error remains small and more stable with parameter sets having lower process noise, indicating better tracking stability. In contrast, filters using larger $\boldsymbol{Q}$ values tend to show stronger error fluctuations, due to their high responsiveness to both signal and noise. In the velocity and acceleration states, the differences are even more pronounced: the error from the filter using $\boldsymbol{Q}^{(3)}$ increases rapidly during maneuvering phases, but significantly decreases once the system stabilizes reflecting fast convergence but less smooth performance. These results suggest that filtering performance depends not only on the absolute magnitude of the error but also on the stability and responsiveness of the filter across different operating phases. In particular, the choice of $\boldsymbol{Q}$ directly influences the convergence and oscillation characteristics of the error, and thereby determines the real-time tracking quality.

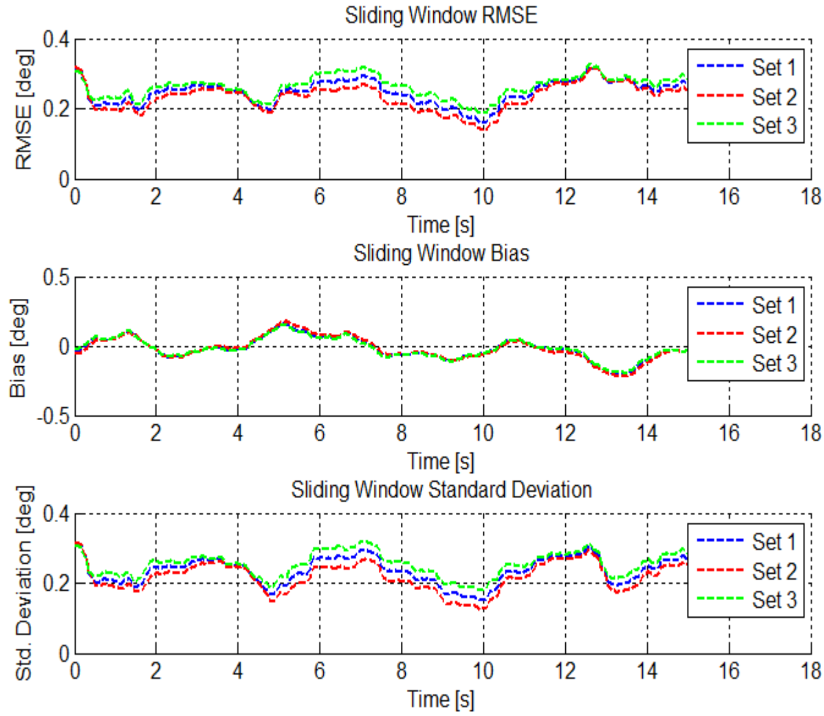

Figure 3. Evaluation of dynamic errors over time via sliding window RMSE, bias and standard deviation for angular position tracking

This is an important basis for assessing the stability and adaptability of the filter during each phase of motion. Regarding dynamic RMSE, the filter using $\boldsymbol{Q}^{(2)}$ maintains the lowest and most stable RMSE throughout most of the simulation. Especially in the period from $t = 6 \div 14(s)$, when the target is oscillating strongly, the RMSE of $\boldsymbol{Q}^{(2)}$ fluctuates around 0.1 (deg), while $\boldsymbol{Q}^{(1)}$ increases to 0.15 (deg), and $\boldsymbol{Q}^{(3)}$ exceeds 0.25 (deg). This indicates that $\boldsymbol{Q}^{(2)}$ not only produces smaller errors but also ensures better stability under adverse conditions.

Regarding bias, all three filters fluctuate around zero, indicating that there is no long-term systematic deviation. However, the local bias at certain moments in the $\boldsymbol{Q}^{(3)}$ filter exceeds ±0.1 (deg), especially during abrupt acceleration phases reflecting high responsiveness but limited noise suppression.

As for standard deviation (Std), which represents the instantaneous fluctuation level in the error, the $\boldsymbol{Q}^{(2)}$ filter again shows a clear advantage with low and stable amplitude. Meanwhile, the $\boldsymbol{Q}^{(3)}$ filter has Std exceeding 0.2 (deg) in many segments, indicating large error oscillations, which can potentially disrupt control systems that rely on EKF outputs.

Overall, the results in Figure 3 confirm that the parameter set $\boldsymbol{Q}^{(2)}$ not only yields lower average error but also maintains smooth and stable performance throughout the tracking process. Conversely, $\boldsymbol{Q}^{(3)}$ tends to trade off between fast response and high fluctuation, which may be suitable for short-term maneuvering problems but not optimal for continuous tracking.The $\boldsymbol{Q}^{(1)}$ set, though not outstanding, provides relatively balanced effectiveness.

4. Conclusion

This paper presented the modeling of a second-order maneuvering target and the application of the Extended Kalman Filter (EKF) to track three states: angular position, angular velocity, and angular acceleration under noisy and highly dynamic conditions.By selecting three different process noise covariance configurations, the dynamic characteristics of the filter were comprehensively evaluated in terms of accuracy, delay, and stability.Simulation results demonstrated that EKF can accurately track target states in nonlinear environments with white noise.Among them, the parameter set $\boldsymbol{Q}^{(2)}$ provided the lowest RMSE and the most stable errors, especially during periods of strong target variation.The $\boldsymbol{Q}^{(3)}$ configuration showed faster responsiveness but introduced more oscillation, while $\boldsymbol{Q}^{(1)}$ yielded intermediate results between the other two.The analysis of evaluation indicators such as RMSE, bias, and standard deviation over time using a sliding window provided deeper insights into the instantaneous behavior of each filtering configuration.This confirms that choosing the process noise matrix $\boldsymbol{Q}$ plays a critical role in balancing the sensitivity and stability of the filter in real-world applications.Future research may explore automatic optimization of $\boldsymbol{Q}$ parameters using adaptive algorithms or machine learning techniques, as well as extend EKF application to multidimensional tracking problems with non-Gaussian or non-white noise to enhance applicability in practical control and navigation systems.

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