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TRIBONACCI PRODUCT CORDIAL LABELING ON PATH RELATED GRAPHS

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Abstract:

In this paper we investigate the existence of Tribonacci Product Cordial labeling on path related graphs.

Keywords: Graph Labeling, Tribonacci Number, Cordial labeling, Product Cordial labeling.

1. INTRODUCTION:

The concept of graph labeling was introduced by Rosa in 1967 [5]. Bala et.al., discussed the concept of Tribonacci Product Cordial Labeling [1].An injective function $\delta : R(G) \to \{T_1, T_2, \dots T_m\}$ is said to be Tribonacci product cordial labelling if the induced function δ^*: B(G)→ {0,1} defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod 2)$ satisfies the condition $| b_{\delta^*}(0) - b_{\delta^*}(1)| \leq 1$. A graph which admits Tribonacci Product cordial labelling is called Tribonacci product cordial graph.

Definition 1.1:Bistar

The **Bistar**$B_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end of K_2 and n pendent edges to the other end of K_2. The edge of K_2 is called the central edge of $B_{m,n}$ and the vertices of K_2 are the central vertices of $B_{m,n}$. It has $2m + 2$ vertices and $2m + 1$ edges.

Definition 1.2:Bi-Double star

By attaching m copies of P_2 (path of length 2) in one end of K_2 and n copies of P_2 (path of length 2) in other end of K_2 we get a **Bi-Double Star**. Bi-Double star has $4m + 2$ vertices and $4m + 1$ edges.

Definition 1.3:Triangular snake

The **Triangular snake**T_m is obtained from the path P_m by replacing each edge of the path by a triangle C_3.It has $2m - 1$ vertices*and* $3(m - 1)$edges.

Definition 1.4:Coconut tree

A **Coconut tree** $CT(m, n)$ is the graph obtained from the path P_m by appending new pendent n edges at an end vertex of P_m.It has $m + n$ vertices*and* $m + n + 1$ edges.

Definition 1.5:Total graph of path

Total graph$T(G)$ is a graph with the vertex set $R(G) \cup B(G)$in which two vertices are adjacent whenever they are either adjacent or incident in G.A **Total graph of path** has $2m - 1$ vertices and $4m - 5$ edges, the vertex set and the edge set of $T(P_m)$be $R(T(P_m)) = \{u_1, u_2, \dots, u_m, r_1, r_2, \dots r_{m-1}\}andB(T(P_m)) = \{u_i u_{i+1}; 1 \leq i \leq m - 1\} \cup \{r_i r_{i+1}; 1 \leq i \leq m - 2\} \cup \{r_i u_{i+1}; 1 \leq i \leq m - 1\} \cup \{u_i r_i; 1 \leq i \leq m - 1\}$.Let b be an edge with end vertices as u and r in a graph G. Then by subdivision of the edge b=ur in G, we mean introduction of a new vertex w in G and where the edge $b = ur$ is replaced by two new edges $b' = uw$ and $b'' = wr$ in G. Thus, subdividing a single edge in G increases the cardinality of its vertex and edge set by one.

2. MAIN RESULT

In this section we investigate the existence of Tribonacci Product Cordial Labeling on Total graph of Path, Bistar, Bi-Double star, Triangular Snake graph, Coconut tree graph.

THEOREM 2.1

Total graph of path $T(P_m)$ admits Tribonacci product cordial labeling

Proof

From the structure ofTotal graph of path $T(P_m)$,it is clear that it has $2m-1$ vertices and $4m-5$ edges.

Define the function $\delta: \mathrm{R} \rightarrow \{\{T_1, T_2, T_3, , \ldots . T_n\}\}$ to labelthe vertex as follows:

Case(i): $m \equiv 1(mod2)$

For $1 \leq i \leq \frac{m+1}{2}$

(i) $\delta(r_i) = T_{4i-3}$

For $1 \leq i \leq \frac{m-1}{2}$

(ii) $\delta\left(r_{\frac{m-1}{2}+i+1}\right) = T_{4i-1}$

(iii) $\delta(r_{n+i}) = T_{4i-2}$

(iv) $\delta\left(r_{\frac{3m+i}{2}}\right) = T_{4i}$

Case(ii): $m \equiv 0(mod2)$

For $1 \leq i \leq \frac{m}{2}$

(i) $\delta(r_i) = T_{4i-3}$

(ii) $\delta\left(r_{\frac{m}{2}+i}\right) = T_{4i-1}$

(iii) $\delta(r_{n+i}) = T_{4i-2}$

For $1 \leq i \leq \frac{m}{2}-1$

(iv) $\delta\left(r_{\frac{3m}{2}+i}\right) = T_{4i}$

To obtain the edge labels,define the induced function$\delta^*: \mathrm{B} \rightarrow \{0,1\}$defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$. Thus using the induced function the edges receive the labels as follows:

Case(i): $m \equiv 1(mod2)$

For $1 \leq i \leq \frac{m-1}{2}$

(i) $\delta^*(r_i r_{i+1}) = 1$

(ii) $\delta^*(r_i r_{m+1}) = 1$

(iii) $\delta^*(r_{i+1} r_{m+1}) = 1$

(iv) $\delta^*\left(r_{\frac{m-1}{2}+i} r_{\frac{m-1}{2}+i+1}\right) = 0$

(v) $\delta^*\left(r_{\frac{m-1}{2}+i} r_{\frac{3m+i}{2}}\right) = 0$

(vi) $\delta^*\left(r_{\frac{m-1}{2}+i+1} r_{\frac{3m+i}{2}}\right) = 0$

For $1 \le i \le \frac{m-3}{2}$

(i) $\delta^*(r_{i+m} r_{m+1+i}) = 1$

For $\frac{m-1}{2} \le i \le m-3$

(ii) $\delta^*(r_{i+m} r_{m++i+2}) = 0$

Case(ii): $\boldsymbol{m \equiv 0(mod2)}$

For $1 \le i \le \frac{m-2}{2}$

(i) $\delta^*(r_i r_{i+1}) = 1$
(ii) $\delta^*(r_i r_{n+1}) = 1$
(iii) $\delta^*(r_{i+1} r_{n+1}) = 1$
(iv) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{m}{2}+i+1}\right) = 0$
(v) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{3m}{2}+i}\right) = 0$
(vi) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{3m}{2}+i}\right) = 0$

For $1 \le i \le \frac{m-2}{2}$

(vii) $\delta^*(r_{i+m} r_{m+1+i}) = 1$

For $\frac{m}{2} \le i \le m-2$

(viii) $\delta^*(r_{i+m} r_{m++i+2}) = 0$

OUTPUT:

cases	TS_n	Label '0'	Label '1'
1	$\boldsymbol{m \equiv 1(mod2)}$	$\frac{4m-4}{2}$	$\frac{4m-6}{2}$
2	$\boldsymbol{m \equiv 0(mod2)}$	$\frac{4m-4}{2}$	$\frac{4m-6}{2}$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the Total graph of path $T(P_m)$admits Tribonacci product cordial labeling.

EXAMPLE 2.1

Tribonacci product cordial labeling forTotal graph of path $T(P_6)$ and$T(P_5)$are shown in the figure 2.1.1 and 2.1.2respectively.

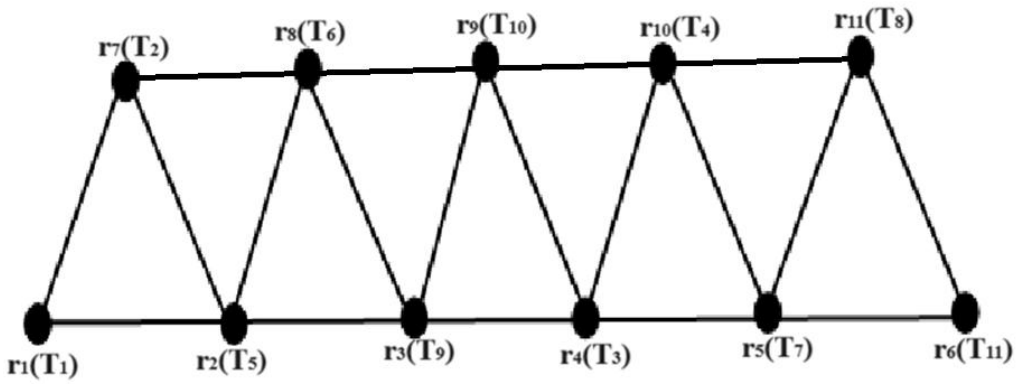

Figure2.1.1

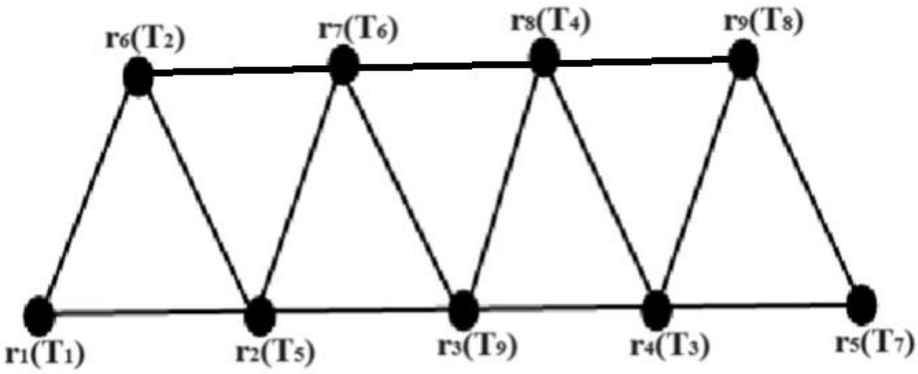

Figure2.1.2

THEOREM 2.2

BiStar graph $K_{m,m}$ admits Tribonacci product cordial labeling

Proof

From the structure of Double Star graph $K_{m,m}$,it is clear that it has $2m+2$ vertices and $2m+1$ edges.

Define the function δ: R → $\{\{T_1, T_2, T_3,, \dots . T_n\}\}$ to label the vertex as follows:

$$\boldsymbol{Case(i): m = 0(mod 2)}$$

(i) $\delta(r) = T_1$

(ii) $\delta(u) = T_3$

For $1 \leq i \leq \frac{m}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-2}$

(ii) $\delta(r_{2i}) = T_{4i+1}$

(iii) $\delta(u_{2i-1}) = T_{4i}$

For $1 \leq i \leq \frac{m-2}{2}$

(iv) $\delta(u_{2i}) = T_{4i+3}$

(v) $\delta(u_n) = T_{2n+2}$

$$\boldsymbol{Case(ii)}: \boldsymbol{m = 1(mod 2)}$$

(i) $\delta(r) = T_1$

(ii) $\delta(u) = T_3$

For $1 \le i \le \frac{m+1}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-2}$

(ii) $\delta(u_{2i-1}) = T_{4i}$

For $1 \le i \le \frac{m-1}{2}$

(i) $\delta(u_{2i}) = T_{4i+3}$

(ii) $\delta(r_{2i}) = T_{4i+1}$

To obtain the edge labels,define the induced function$\delta^* : B \to \{0,1\}$defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod 2)$. Thus using the induced function the edges receive the labels as follows:

For $1 \le i \le m$

(iii) $\delta^*(rr_i) = 1$

(iv) $\delta^*(uu_i) = \delta^*(ru) = 0$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the BiStar graph $K_{m,m}$ admits Tribonacci product cordial labeling

EXAMPLE2.3

Tribonacci product cordial labeling for $K_{4,4}, K_{5,5}$are shown in the figure 2.2.1 and 2.2.2 respectively.

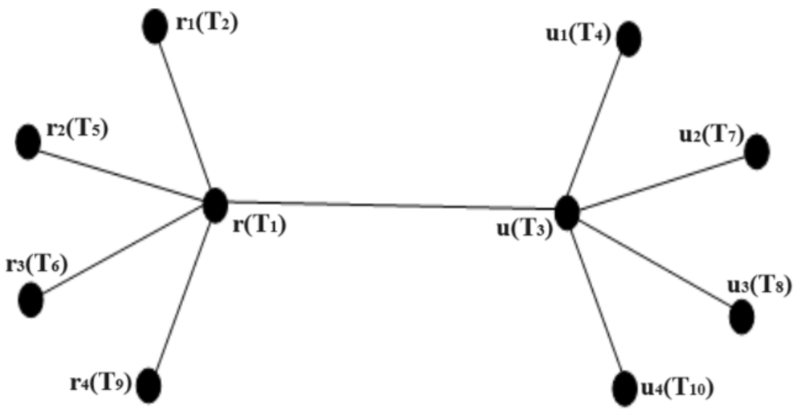

Figure 2.2.1

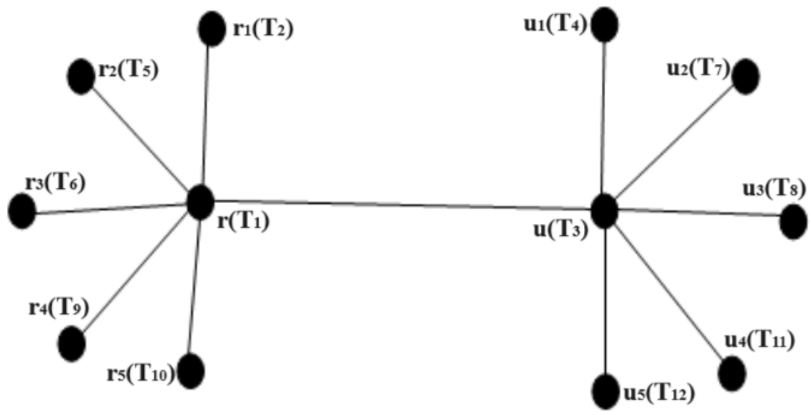

Figure 2.2.2

THEOREM 2.3

Bi-Double star graph $BDS_{m,}$ admits Tribonacci product cordial labeling

Proof

From the structure ofBi-Double star graph $BDS_{m,}$,it is clear that it has $4m+2$ vertices and $4m+1$ edges.

Define the function δ: R → $\{\{T_1, T_2, T_3, , \ldots . T_n\}\}$ to labelthe vertex as follows:

(i) $\delta(r) = T_1$

(ii) $\delta(u) = T_2$

For$1 \le i \le m$

(i) $\delta(r_i) = T_{4i-1}$

(ii) $\delta(r_{m+i}) = T_{4i}$

(iii) $\delta(u_i) = T_{4i+1}$

(iv) $\delta(u_{m+i}) = T_{4i+2}$

To obtain the edge labels,define the induced functionδ^* : B → {0,1}defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod 2)$. Thus using the induced function the edges receive the labels as follows:

(i) $\delta^*(ur) = 1$

For $1 \le i \le m$

(ii) $\delta^*(ru) = 0$

(iii) $\delta^*(r_i r_{m+i}) = 0$

(iv) $\delta^*(uu_i) = 1$

(v) $\delta^*(u_i u_{m+i}) = 1$

So, $|B_{\delta^*}(0) - B_{\delta^*}(1)| = |2m - (2m+1)| = 1$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, theBi-Double star graph $BDS_{m,}$, admits Tribonacci product cordial labeling.

EXAMPLE 2.3

Tribonacci product cordial labeling foroftheBi-Double star graph $BDS_{4,}$, is shown in the figure 2.3.1.

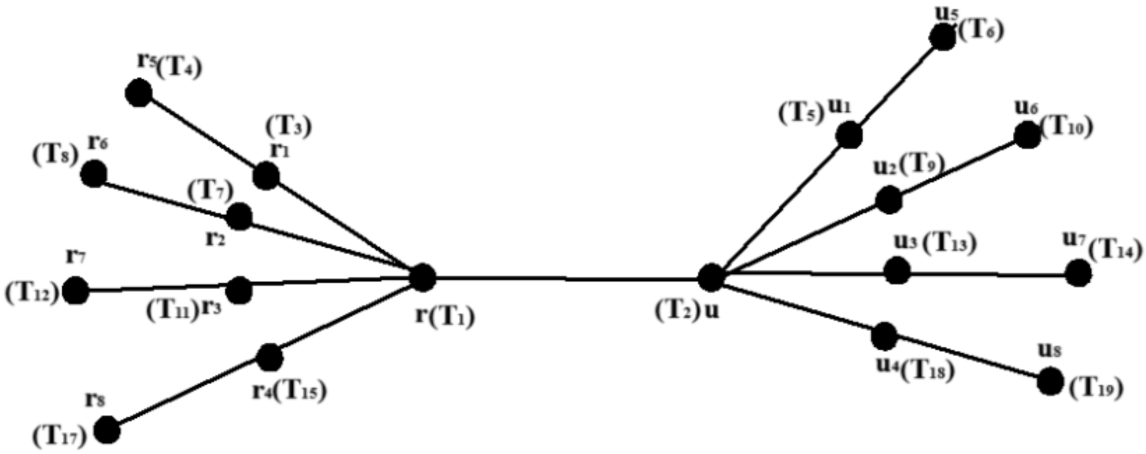

Figure2.3.1

THEOREM 2.4

Coconut tree graph $CT(m,n)$ admits Tribonacci product cordial labeling

Proof

From the structure of Cononut tree graph $CT(m,n)$,it is clear that it has $2m$ vertices and $2m$ edges.

Define the function $\delta: \mathrm{R} \to \{\{T_1, T_2, T_3, , \ldots . T_n\}\}$ to labelthe vertex as follows:

$$\boldsymbol{Case(i): m, n \equiv 1(mod2)}$$

For $1 \leq i \leq \frac{m+1}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-3}$

For $1 \leq i \leq \frac{m-1}{2}$

(ii) $\delta(r_{2i}) = T_{4i-2}$

(iii) $\delta(u_{2i-1}) = T_{4i-1}$

(iv) $\delta(u_{2i}) = T_{4i}$

(v) $\delta(u_n) = T_{2n}$

$$\boldsymbol{Case(ii): m, n \equiv 0(mod2)}$$

For $1 \leq i \leq \frac{m}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-3}$

(ii) $\delta(r_{2i}) = T_{4i-2}$

(iii) $\delta(u_{2i-1}) = T_{4i-1}$

(iv) $\delta(u_{2i}) = T_{4i}$

$$\boldsymbol{Case(iii): m \neq n, n = m-1}$$

$$\boldsymbol{Subcase\ (i): m \equiv 1(mod2)}$$

For $1 \leq i \leq \frac{m+1}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-3}$

For $1 \leq i \leq \frac{m-1}{2}$

(i) $\delta(r_{2i}) = T_{4i-2}$
(ii) $\delta(u_{2i-1}) = T_{4i-1}$
(iii) $\delta(u_{2i}) = T_{4i}$

$Subcase\ (ii): m \equiv 0(mod2)$

For $1 \leq i \leq \frac{m}{2}$

(i) $\delta(r_{2i-1}) = T_{4i-3}$
(ii) $\delta(r_{2i}) = T_{4i-2}$
(iii) $\delta(u_{2i-1}) = T_{4i-1}$

For $1 \leq i \leq \frac{m-2}{2}$

(iv) $\delta(u_{2i}) = T_{4i}$

To obtain the edge labels,define the induced function$\delta^* : B \rightarrow \{0,1\}$defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod2)$. Thus using the induced function the edges receive the labels as follows:

$Case(i): m = n$

For $1 \leq i \leq m-1$

(i) $\delta^*(r_i r_{i+1}) = 1$

For $1 \leq i \leq m$

(ii) $\delta^*(r_n u_i) = 0$

$Case(ii): m \neq n, n = m-1$

For $1 \leq i \leq m-1$

(i) $\delta^*(r_i r_{i+1}) = 1$
(ii) $\delta^*(r_n u_i) = 0$

OUTPUT:

Cases	$CT(m,n)$	Label '0'	Label '1'
1	$\boldsymbol{m, n \equiv 0(mod2)}$	$m-1$	m
2	$\boldsymbol{m, n \equiv 1(mod2)}$	m	$m-1$
3	$\boldsymbol{m \neq n, n = m-1}$	n	n

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \leq 1$ is satisfied.

Therefore, the Coconet tree graphs admits Tribonacci product cordial labeling.

EXAMPLE 2.4

Tribonacci product cordial labeling for $CT(5,5)$, $CT(4,4)$ and $CT(5,4)$are shown in the figure2.4..1,2.4.2 and 2.4..3 respectively.

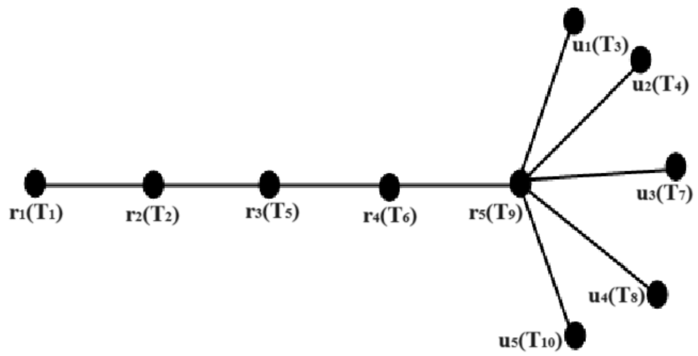

Figure 2.4.1

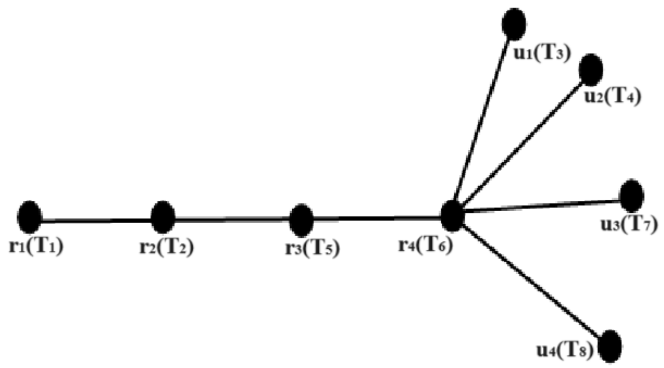

Figure 2.4.2

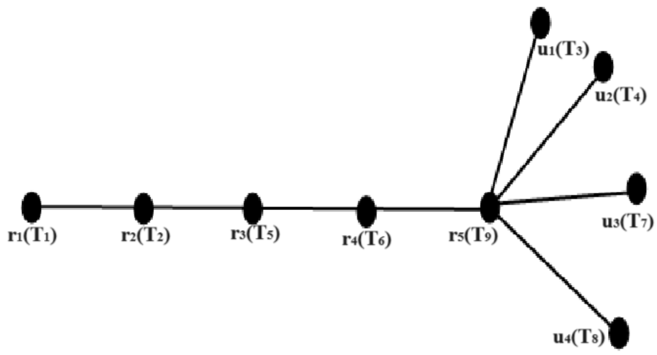

Figure 2.4.3

THEOREM 2.5

Triangular Snake graph TS_m admits Tribonacci product cordial labeling

Proof

From the structure of Triangular Snake graph TS_m,it is clear that it has $2m-1$ vertices and $3(m-1)$ edges.

Define the function $\delta: \mathrm{R} \rightarrow \{\{T_1, T_2, T_3, , \ldots . T_n\}\}$ to labelthe vertex as follows:

Case(i): $\boldsymbol{m \equiv 1(mod 2)}$

For $1 \leq i \leq \frac{m+1}{2}$

(v) $\delta(r_i) \quad = T_{4i-3}$

For $1 \leq i \leq \frac{m-1}{2}$

(vi) $\delta\left(r_{\frac{m-1}{2}+i+`}\right) \quad = T_{4i-1}$

(vii) $\delta(r_{n+i}) \ = T_{4i-2}$

(viii) $\delta\left(r_{\frac{3m+1}{2}}\right) = T_{4i}$

Case(ii): $\boldsymbol{m \equiv 0(mod 2)}$

For $1 \leq i \leq \frac{m}{2}$

(v) $\delta(r_i) \quad = T_{4i-3}$

(vi) $\delta\left(r_{\frac{m}{2}+i}\right) = T_{4i-1}$

(vii) $\delta(r_{n+i}) \ = T_{4i-2}$

For $1 \leq i \leq \frac{m}{2} - 1$

(viii) $\delta\left(r_{\frac{3m}{2}+i}\right) \quad = T_{4i}$

To obtain the edge labels,define the induced function$\delta^* : \mathrm{B} \rightarrow \{0,1\}$defined by $\delta^*(r_i r_j) = (\delta(r_i)\delta(r_j))(mod 2)$. Thus using the induced function the edges receive the labels as follows:

Case(i): $\boldsymbol{m \equiv 1(mod 2)}$

For $1 \leq i \leq \frac{m-1}{2}$

(vii) $\delta^*(r_i r_{i+1}) \quad = 1$

(viii) $\delta^*(r_i r_{n+1}) \quad = 1$

(ix) $\delta^*(r_{i+1} r_{n+1}) \quad = 1$

(x) $\delta^*\left(r_{\frac{m-1}{2}+i} r_{\frac{m-1}{2}+i+1}\right) \quad = 0$

(xi) $\delta^*\left(r_{\frac{m-1}{2}+i} r_{\frac{3m+i}{2}}\right) \quad = 0$

(xii) $\delta^*\left(r_{\frac{m-1}{2}+i+1} r_{\frac{3m+i}{2}}\right) \quad = 0$

Case(i): $\boldsymbol{m \equiv 0(mod 2)}$

For $1 \leq i \leq \frac{m-2}{2}$

(ix) $\delta^*(r_i r_{i+1}) \quad = 1$

(x) $\delta^*(r_i r_{n+1}) \quad = 1$

(xi) $\delta^*(r_{i+1} r_{n+1}) \quad = 1$

(xii) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{m}{2}+i+1}\right) \quad = 0$

(xiii) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{3m}{2}+i}\right) \quad = 0$

(xiv) $\delta^*\left(r_{\frac{m}{2}+i} r_{\frac{3m}{2}+i}\right) \quad = 0$

OUTPUT:

cases	TS_m	Label '0'	Label '1'
1	$\boldsymbol{m \equiv 1(mod2)}$	$\frac{3(m-1)}{2}$	$\frac{3(m-1)}{2}$
2	$\boldsymbol{m \equiv 0(mod2)}$	$\frac{3m-5}{2}$	$\frac{3m-3}{2}$

Hence the condition $|B_{\delta^*}(0) - B_{\delta^*}(1)| \le 1$ is satisfied.

Therefore, the Triangular Snake graph TS_m admits Tribonacci product cordial labeling.

EXAMPLE 2.5:

Tribonacci product cordial labeling forTriangular Snake graph $TS_5$and TS_6 are shown in the figure 2.5.1 and 2.5.2respectively.

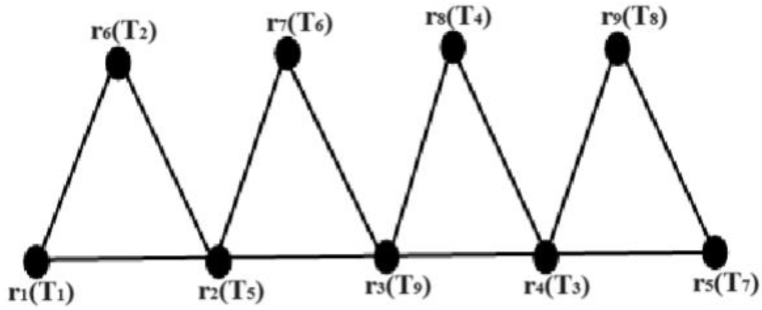

Figure 2.5.1

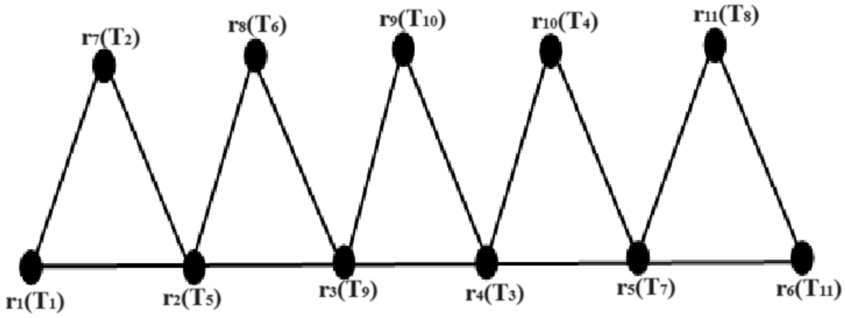

Figure 2.5.2

Conclusion:

This paper, we have confirmed the existence of Tribonacci Product Cordial Labeling for Total graph of path,BiStar, Bi-Double star, and Triangular Snake graphs.

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