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Identifying Students' Problems in Solving Quadratic Equation Using Newman Error Analysis Model

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ABSTRACT

This study explores students' errors in solving quadratic equation. The study employed a qualitative approach. Specifically, a case study. The population of the study consists of all the senior secondary schools in Bauchi metropolis. Purposive sampling was used to select a senior secondary school within Bauchi metropolis for the conduct of the study. A total of 34 senior secondary school III students participated in the study. A Quadratic Equation Diagnostic Test (QEDT) was used as instrument for data collection. A semi-structured interview was conducted thereafter to ascertain the students' errors. The students' errors were determined using the Newman's Mathematical Error Analysis Model. Miles, Huberman and Saldana's Interactive Data Analysis Model was used for qualitative content analysis. Findings from the study revealed that, the most frequent error committed by the students in solving quadratic equation problems is the Transformation Error, followed by Process Skill Error. Then comprehension error, Encoding Error was found to be the least error committed by students while solving quadratic equation. No Reading Error was found to be committed by the students. The study recommended that: School administrators should organize seminars and workshops on alternative teaching strategies that will improve learning of quadratic equation; researchers should investigate effective teaching strategies that will improve students' understanding of quadratic equation.

Keywords: Quadratic Equation, Students' Error, Newman's Mathematical Error Analysis, Miles, Huberman and Saldana's Interactive Qualitative Content Analysis.

Introduction

Quadratic equation is important for students' learning of future topics in mathematics such as logarithms, indices, statistics, calculus and other higher-level polynomials. It is also important in science subjects such as physics, engineering, architecture and structural design. (Didiş & Erbaş, 2015; Harripersaud, 2021; Kim-How et al., 2022; Susac et al., 2014; Guner, 2017).

In spite of the importance of quadratic equation, report and studies have shown that, students experience difficulties in learning quadratic equation. For instance, the chief examiners' annual report for the West African Senior School Certificate Examinations (WASSCE) indicated that, students consistently (2010-2021) show weaknesses on quadratic graph, and generally, so many studies both in Nigeria and globally reported students' difficulties in solving quadratic equations (Usman, 2018; Baring & Alegre, 2019; Utami & Jupri, 2021; Harripersaud, 2021). Although so many studies investigated students' problems in solving quadratic equation, no such study was found to have been conducted in Bauchi metropolis. This study is an attempt to fill in this gap.

Literature Review

Students' understanding of quadratic equation generally has to do with the understanding of two types of knowledge regarding quadratic equation, which are: the procedural knowledge of quadratic equation and the conceptual knowledge of quadratic equation.

The procedural knowledge of quadratic equation is the process of applying the step-by-step procedures, algorithms and rules required to successfully solve a given quadratic equation. It includes: using the discriminant; factorization; completing the square; quadratic formula; plotting quadratic graph. It involves effectively and appropriately using rules, methods or specific procedures associated with quadratic equations. Including using the formula to determine the delta and simple or double roots, steps to solve quadratic equations, the rule for finding two numbers when knowing their sum and product, solving the equations that can be reduced to quadratic equations, etc.

On the other hand, conceptual knowledge of quadratic equation involves content mastery where knowledge can be generated and established through many relations between existing and prior knowledge and transferred through reconstruction of procedures (Donevska-Todorova, 2016). Conceptual

knowledge of quadratic equations includes: the concept of degree, which refers to the student's ability to know that a quadratic equation is an algebraic equation of degree two; identifying the nature of the roots without actually solving the equation; the relationship between sum and product of solutions to coefficients of quadratic equations; the knowledge of the point where the graph crosses the *y-axis* is the *y*-intercepts of the equation and the knowing that, the point where the graph crosses the *x-axis* are the *x*-intercepts and as well the roots (solution) of the equation; the knowledge that, a graph with a minimum vertex with the curve looking upward then such a graph is obtained when the value of the coefficient of x^2 is greater than zero and a graph with maximum vertex with the curve looking downward is obtained when the value of the coefficient of x^2 is less than zero.

It is worth noting that the conceptual and procedural knowledge are not mutually exclusive, as Didiş (2018) suggested that, conceptual understanding of quadratic equation is developed through the help of procedural understanding and procedural understanding is developed through the help of conceptual understanding of quadratic equation. Therefore, in understanding quadratic equation, integrating conceptual and procedural knowledge is an indispensable approach.

Some studies attributed students' problems in understanding quadratic equations with the lack of these two forms of knowledge. For instance, Tall et al. (2014) propounded that, students' lack of procedural and conceptual knowledge of quadratic equations often influences their capability to solve quadratic equations. Other studies however, attributed students understanding of quadratic equation with teachers teaching approaches: Harripersaud (2021) reported that, teachers are unable to provide concept related to real life problems; Sosa-Moguel and Aparicio-Landa (2021) provided that, teachers are confused in choosing the appropriate approach to teach quadratic equation; Sari and Jailani (2019) revealed that, teachers are too fast in delivering their lessons on quadratic equation.

Many studies were conducted to identify students' difficulties in solving quadratic equation and it was found that, students have problems in understanding quadratic equation, including basic knowledge such as the meaning of quadratic equation (Didis-Kabar, 2018; Morales-Carballo et al., 2022). In another study, Harripersaud (2021) found that, students struggle particularly with solving for x in problems of the form $x^2 = k$ (k>0) and $(x - r)(x - s) = 0$ where r and s are any real numbers. More so, on an item in a study by O'Connor and Norton (2016) involving $(x - 3)(x - 5) = 0$, students show lack of knowledge of algebraic processes. Students' work on the item showed that, most of the students try to find the value for x by treating the equation as a linear equation, instead of simply applying the null factor law to obtain the roots of the equation, on the item, the students proceeded with the solution as $0 = x^2 - 3x - 5x + 15$ and arrived at $x = \sqrt{15} + 7$ as solution to the problem. Moreover, students make mistakes in the transformation and multiplication of algebraic forms, they also show lack of understanding by assuming that $(a + b)^2$ is equivalent to $a^2 + b^2$ (Fachrudin et al., 2014). Additionally, O'Connor and Norton (2016) reported that, in solving quadratic equations, students face challenges with quadratic equations involving fractions. For example, in their instrument for the study, students' responses on an item involving quadratic equation with fraction, only 28% of the students correctly solved the item. While 56% answered the item incorrectly and 16% did not attempt the item. In another item of the study with quadratic equation involving $\frac{1}{2}$, out of 24 respondents 21of them express it as 0.5 indicating their preference to work with decimals than with fractions. Another area in which students have problem with quadratic equation is in in transforming word problems into quadratic equations (Usman 2018; Thomas & Mahmud 2021).

It can be seen from the literature that, majority of the studies conducted on identifying students' problems in solving quadratic equation were outside Nigeria. Therefore, this study is an elaboration of what we know on students' problems in solving quadratic equations in Nigeria.

Methodology

This study is qualitative research. Specifically, a case study. A case study is an in-depth exploration of a particular system (e.g., activity, event, process or individuals) based on extensive study of data collected (Creswell, 2012). The population for the study comprises of all the senior secondary schools within Bauchi metropolis. A purposive sampling was used to select a senior secondary school within the metropolis for the conduct of the study. Volunteer sampling was used to select the sample for the study. The sample of the study consists of 17 male and 17 female senior secondary school III students from the selected school. A Quadratic Equation Diagnostic Test (QEDT) was used as the instrument for data collection. The instrument consists of 6 items, in which: question one explored students' ability to extract information from a graph; item two explored students' ability to compute table of values and plot quadratic graph; item three are symbolic quadratic equations; items four, five and six are quadratic word problems. Data collected were analysed based on Newman's Mathematical Error Analysis Model (1977, 1983).

According to Newman Mathematical Error Analysis Model (1977,1983), for a person to successfully solve a mathematical word problem, that person has to go through a number of hurdles and any problem with the hurdles leads to an error. These errors according to Newman's model include:

A. Reading Error

B. Comprehension Error

C. Transformation Error

D. Process Skills Error and

E. Encoding Error

Reading error occurs when a person lacks basic reading skill to appropriately read the mathematical word problem to be solved; Comprehension Error occurs when a person failed to understand what the question want him to do and there he cannot proceed with the solution or he make the wrong

process; Transformation Error is an error when a person transform a word problem into mathematical expression wrongly; Process Skill Error is an error when a person makes an arithmetical error as a results of making a wrong process; Encoding Error occur when a person wrongly present the final answer after a successful work on the problem, it can be an in ability of the person make a meaningful conclusion at the end of a problem solving.

The analysis will be guided by the Miles Huberman and Saldana's Interactive Data Analysis Model (1994, 2014). The model provides three steps for qualitative content analysis after data collection. The steps help simplify the handling of qualitative data because of its bulkiness. The three steps are:

A. Data Reduction

B. Data Display

C. Conclusion/Verifying

Data reduction refers to the process of selecting, focusing, simplifying, observing and transforming the data that appears in written-up field notes or transcriptions. As we see it, data reduction occurs continuously throughout the life of any qualitatively oriented project; Data Display: Data display is the second major flow of analysis. Generically, a display is an organized, compressed assembly of information that permits conclusion drawing and action. Looking at displays help us to understand what is happening and to do something, either analyze further or take an action based on that understanding; Conclusion/Verifying is the third phase of analysis which is conclusion drawing and verification. The types and number of errors committed by the students were summarized and presented using frequency and percentage.

Results

The study explored the errors committed by students in solving quadratic equations. Summary of the findings of the study is presented in table 1. On the table, the first column is the 6 items presented to the students for data collection. The second column is the number and percentage of students that solved the problems correctly without any error. The third Column (A) present the number and percentage of Reading Errors committed. The fourth Column (B) is the number and percentage of Comprehension Errors committed. The fifth Column (C) is the number and percentage of Transformation Errors committed. The sixth Column (D) is the number and percentage of Process Skill Errors committed and finally, and the seventh Column (E) is the number and percentage of Encoding Errors committed.

Table 1 Types, Frequency and Percentage of Errors Committed by the Participants in Solving Quadratic Equation.

QUESTION	CORRECT	A	B	C	D	E
1	0(0.0%)	0(0.0%)	0(0.0%)	21 (61.8%)	9 (26.5%)	4 (11.8%)
2	3(8.8%)	0(0.0%)	0 (0.0%)	20 (58.8%)	11 (32.4%)	0 (0.0%)
3	2(5.9%)	0(0.0%)	0(0.0%)	0(0.0%)	32(94.1%)	0(0.0%)
4	1(2.9%)	0(0.0%)	6(17.7%)	19(55.9%)	7(20.6%)	1(2.9%)
5	1(2.9%)	0(0.0%)	3(8.8%)	27(61.8%)	2(5.9%)	1(2.9%)
6	1(2.9%)	0(0.0%)	6(17.7%)	16(44.1%)	6(11.8%)	5(14.7%)
TOTAL	8(3.9%)	0(0%)	15(7.4%)	103(50.5%)	67(32.8%)	11(5.4%)

From Table 1, 34 students responded to the 6 questions in the test. This means a total of 204 (6 x 34) responses were obtained. Out of the 204 responses, only 8 (3.9%) approximately 4% were error free. This means 96% contain one form of error or the other. The highest number of errors committed is in the Transformation Error, where 105 (50.5%) Transformation Errors were committed, followed by Process Skill Error accounting for 67 (32.8%). Then Comprehension Error accounting for 15 (7.4%). Then Encoding Error which is 11 (5.4%) and there was no Reading Error committed.

Question one, was on the use of graph to answer questions on quadratic equations. The responses showed that none of the participants were able to answer the question without committing any error, 21 (61.8%) of the participants commit Transformation Error. The second most committed error was the Process Skill Error, where 9 (26.5%) Process Skill Errors were committed by the participants. On Encoding Error, 4 (11.8%) Encoding Errors were committed.

Question two was on computing a table and plotting quadratic graph. It was aimed at exploring students' ability to compute table of values and plot the quadratic graph. Only 3 (8.8%) participants were able to solve the problem successfully without committing any error. Twenty (58.8%) committed Transformation Error. Eleven (32.4%) committed Process Skill Error and no participant committed Reading, Comprehension and Encoding Error.

Question three include 3 symbolic quadratic equations, it is aimed at exploring students' ability to solve each of the three equations symbolically using any method. Only two 2 (approximately 6%) of the participants solve the three questions without committing any error and the remaining 32 (94.1%) of the participants commit Process Skill Error. None of the participant commit Reading, Comprehension, Transformation and Encoding Error

Question four, five and six aimed at exploring students' ability in solving quadratic word problems, but each question is testing a different concept. In question four, only 1 (approximately 3%) participant answered the question without any error. Six (17.7%) participants committed Comprehension

Error. Nineteen (55.9%) participants committed Transformation Error. Seven (20.6%) participants committed Process Skill Error. and 1 (approximately 3%) committed Encoding Error. No participant committed Reading Error.

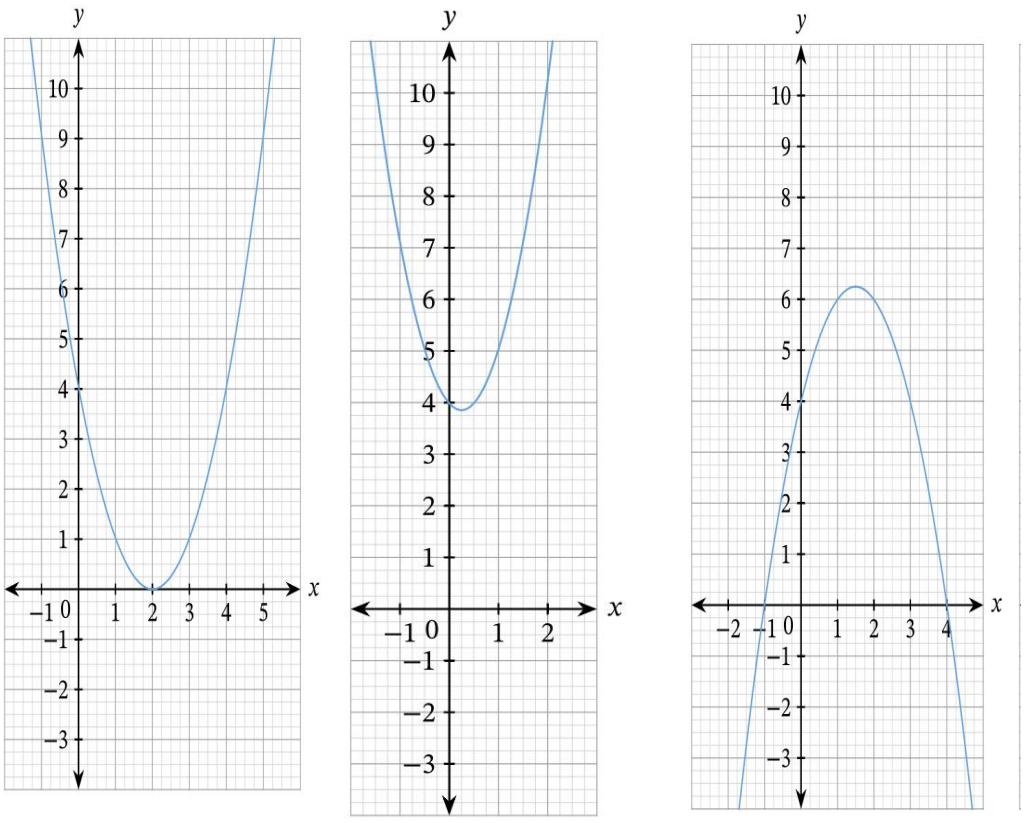
In question five. Only 1 (approximately 3%) participant answered the question without any error. Three (8.8%) participants committed Comprehension Error. Twenty-seven (61.8%) committed Transformation Error. Only 2 (approximately 6%) committed Process Skill Error and Only 1 (approximately 3%) participant committed Encoding Error. No 0 (0.0%) participant committed Reading Error.

One (approximately 3%) participant in question six answered the question without any error. Six (17.7%) committed Comprehension Error. Sixteen (44.1%) committed Transformation Error. Six (17.7%) committed Process Skill Error. Five (14.7%) committed Encoding Error. None of the participant committed Reading Error,

Discussion of findings

Question one of the instruments for this study aimed at exploring students' ability in extracting information from graph and formulating quadratic equations from a given graph. The responses showed that, most of the participants 61.8% were unable to transform the information on the graph into mathematical form, which led to Transformation Error, these include students' failure to find the x and y intercepts, vertices, axis of symmetry and maximum or minimum of a graph (see figure 1). Although about 26.5% were able to find the roots of the equation from the graph, they still could not use the roots to get the originating equation. This is a Process skill Error (see figure 2). The results also revealed that, 11.8% of the participants committed Encoding Errors (see students work in figure 1 and 2). No reading and comprehension errors were found in the question.

1. Use the graphs below to answer the following questions



A B C

i. What are the roots of equations represented in graph A, B and C above?
ii. What are the vertices of graph A, B and C above?
iii. Find the minimum or maximum (as the case may be) value of the curves?
iv. What is the y-intercept of graph A, B and C?
v. Find the equation of graph A, B and C?
vi. Why is the graph of A and that of B looking up, while that of C looking down?
vii. What types of roots are the equations of A, B and C having and why?

viii. What is the axis of symmetry in graph A, B and C?

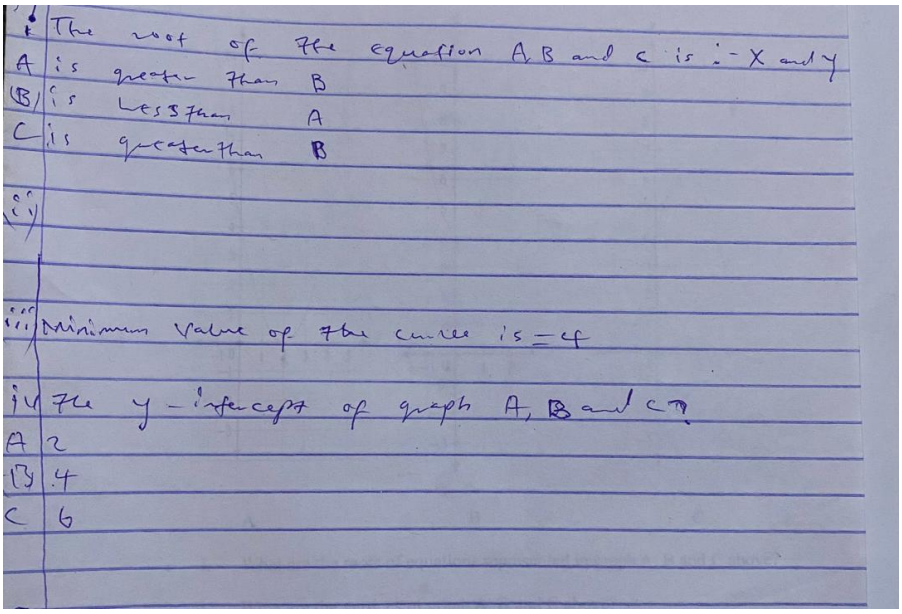


Figure 1: A Student's Work with Difficulty in Extracting Information from a Graph.

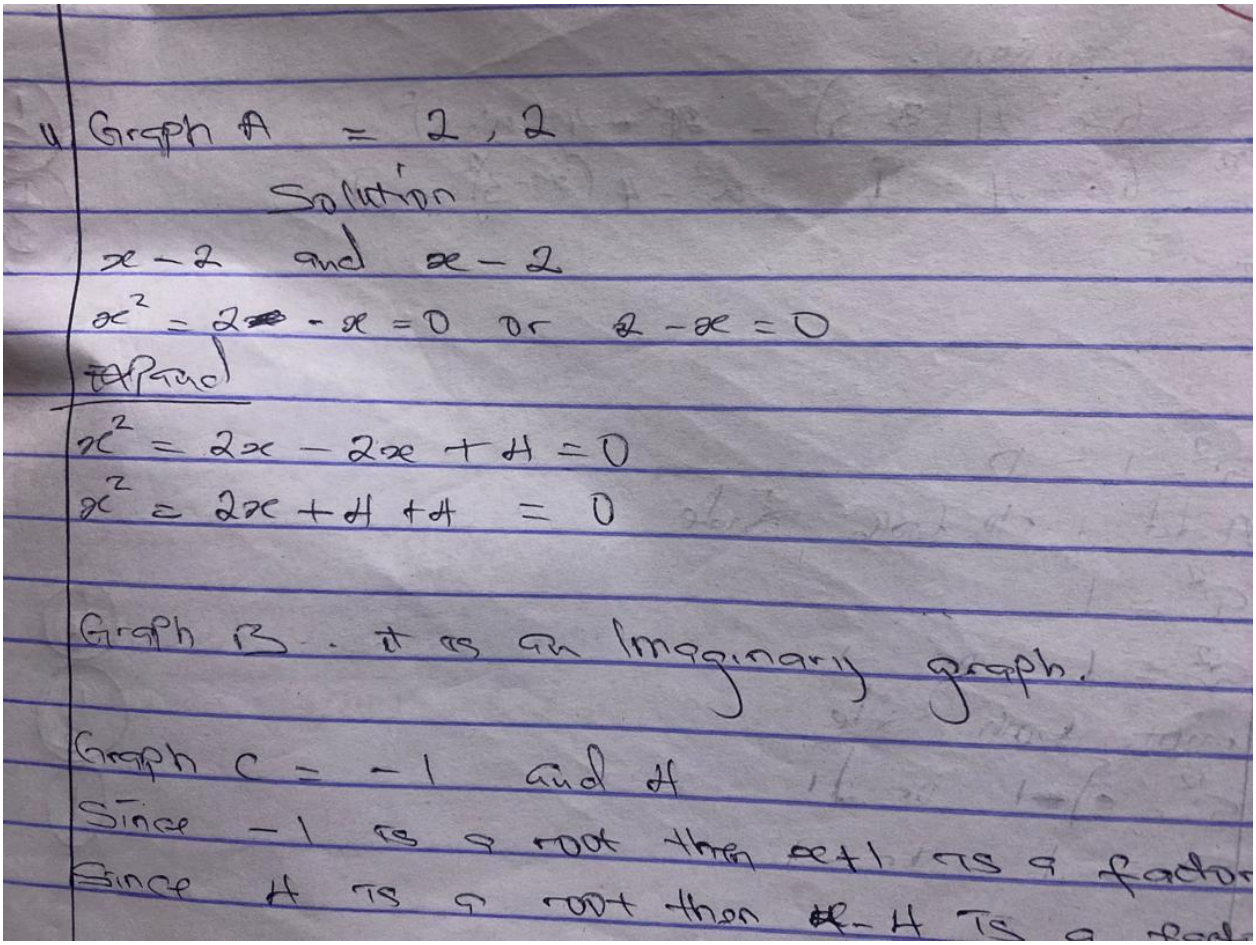


Figure 2: A Participant work with correct root but wrong transformation.

Responses from question two, which was aimed at exploring students' ability to compute and plot quadratic graph revealed that, only 8.8% of the respondents were able to appropriately plot the graph for the table. Majority 58.8% of the participants committed Transformation Error, as a result they

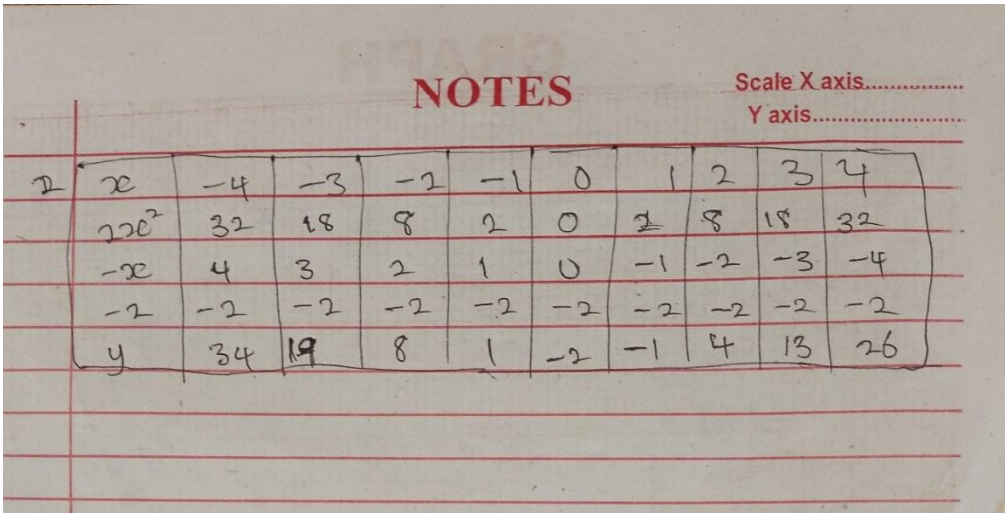
were unable to plot the graph correctly. While 32.4% of the participants were unable to compute table of values for a quadratic equation as a result of Process Skill Error. See figure 3 and 4.

Question 2: Copy and complete the table below for the relation $y = 2x^2 - x - 2$ for

$-4 \leq x \leq 4$

X	*-4*	*-3*	*-2*	*-1*	*0*	*1*	*2*	*3*	*4*
Y		*19*			*-2*				*26*

Using the scale of 2cm to 1unit on the x-axis and 2cm to 5unit on the y axis, draw the graph of the equation.



	x	−4	−3	−2	−1	0	1	2	3	4
	$2x^2$	32	18	8	2	0	2	8	18	32
	$-x$	4	3	2	1	0	−1	−2	−3	−4
	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
	y	34	19	8	1	−2	−1	4	13	26

Figure 3: A Student's Work on Completing Quadratic Table.

Figure 3 shows a table whose graph (as plotted by the participant) is in figure 4. It can be seen that, the participant committed a Transformation Error. The participant was able to construct the table of values correctly. However, she was unable to transform it into a correct quadratic graph.

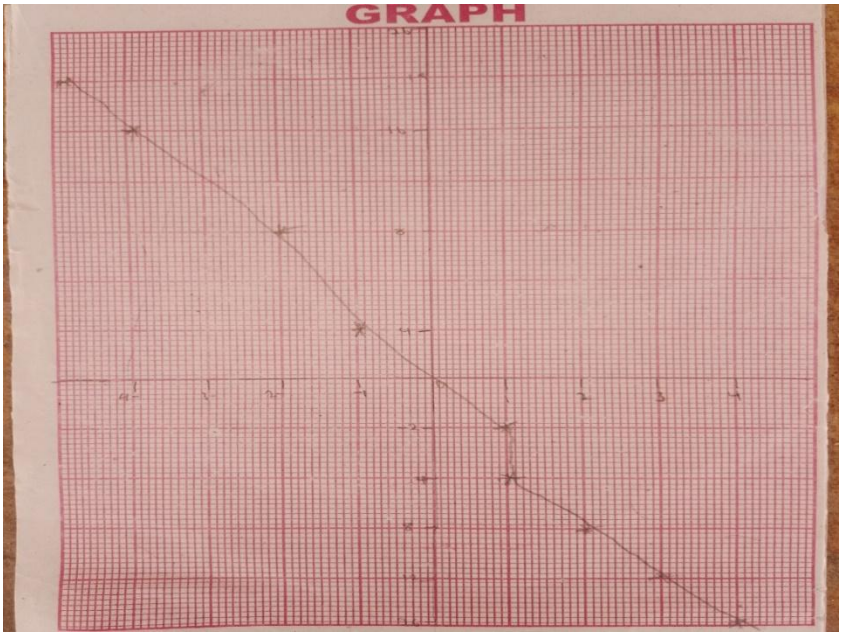


Figure 4: A Student's Work on Plotting Quadratic Graph with Transformation Error.

Since the participant showed ability to compute table of values. There's a need explore why she could not plot the graph. Therefore, she was called for an interview and the following conversation took place with the participant.

Researcher: Please read question 2

Student: Read the question fluently without any problem

Researcher: What did the question want you to do?

Student: to complete the table and plot the graph

Researcher: shows the correct working to the student. can you see the difference with your graph?

Student: yes, sir I don't know how to plot graph

These responses on question one and two clearly indicated that, the students have weaknesses in graphical solution of quadratic equation. The findings are in line with the WAEC chief examiners' annual report for over a decade (2010-2021).

Question three was aimed at measuring the students' ability to solve symbolic quadratic equations. The responses showed that, the participants have challenges in solving symbolic quadratic equations. This could be due to their lack of conceptual understanding of quadratic equation as the participants showed inability to determine which method is appropriate to apply to a particular problem. This has led majority of the participants 94.1% to commit a Process Skill Error. In figure 5 for instance, the participant applied a wrong method in the solution, where he tried to factorise the equation while it cannot be factorised. This error is due to the participants' lack of conceptual understanding of using the delta to determine which method is appropriate and which is not to solve the equation.

Question 3: Solve the following equations using any method

(a) $x^2 - 6x - 4 = 0$

(b) $a^2 - 1 = 0$

(c) $3b^2 = 5b$

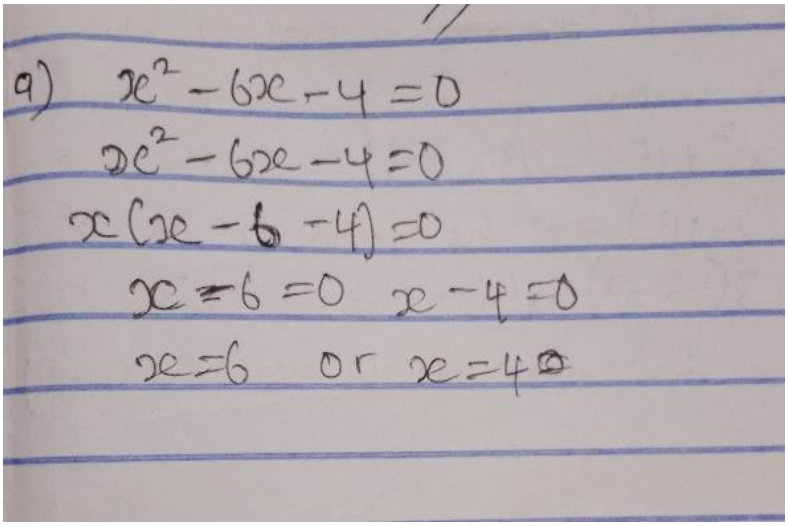


Figure 5: A student's Work with Process Skill Error.

The above solution on question 3(a) shows that the student committed a Process Skill Error, he failed in determining what is the appropriate method to use in order to solve the problem. The participant supposed to use completing the square or quadratic formula to solve the problem. The following conversation took place with the participant to find out if he is aware of the other methods of solving quadratic equation. This is an indication that students are more familiar with factorization method than the other methods. This is in line with findings by O'Connor and Norton (2016).

Researcher: Please read question 3

Student: Read the question fluently without any problem

Researcher: (shows the correct working to the student) can you see where your mistake is?

Student: I use a different method

Researcher: can you mention the methods of solving quadratic equation?

Student: factorization, graph, formula and completing the square.

From the conversation the student knows all the methods of solving quadratic equation, but lack conceptual understanding that there's a rationale for applying specific method to specific problem. This students' problem in solving symbolic quadratic equation was also reported by Didis and Erbas (2015).

Question four, five and six are word problems that lead to quadratic equations. A lot of the participants 44.4% indicated difficulty in understanding what the question wants them to do, which led them to commit Comprehension Error (see figure 6). As a result, they were unable to transform the word problems into correct quadratic equations (see figure 7) and consequently led to Transformation Error. Another error committed by the students in solving quadratic word problems is Process Skill Error (see figure 8). The least error committed in solving quadratic word problems was the Encoding Error.

Question 6: The product of two consecutive positive odd numbers is 195. By constructing a quadratic equation and solving it, find the two numbers.

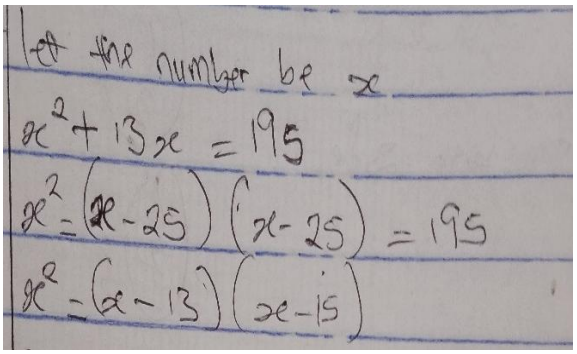


Figure 6: A Student's Work with Comprehension Error.

Figure 6 is a participant's work on question 6, the participant committed a Comprehension Error because he omitted the term 'consecutive positive odd numbers' which is $x + 2$ and consequently she transformed the question as $x^2 + 13x = 195$. The right working supposed to be: Let the number be x and $x + 2$, their product is $x(x + 2)$ to arrive at $x^2 + 2x = 195$. The following discussion took place with the participant to find out his level of comprehension.

Researcher: Please read question 6

Student: Read the question fluently without any problem

Researcher: What is the meaning of the word consecutive?

Student: Hmm, I don't know sir.

Researcher: shows the correct working to the student can you see where your mistake is?

Student: yes, sir I didn't include x and multiply it by $(x + 2)$

From the above conversation it can be seen that, the participant did not understand what the question wants and that led to not understanding a vital information (consecutive odd numbers).

A surprising instance was found on question four where, a participant displayed a complete lack of algebraic knowledge (see figure 7). The participant tried to solve the problem using only numbers. This is an indication that the participant is still operating at the arithmetic level.

Question 4: A woman is 4 times older than her child, 5 years ago, the product of their ages was 175. Find their present age.

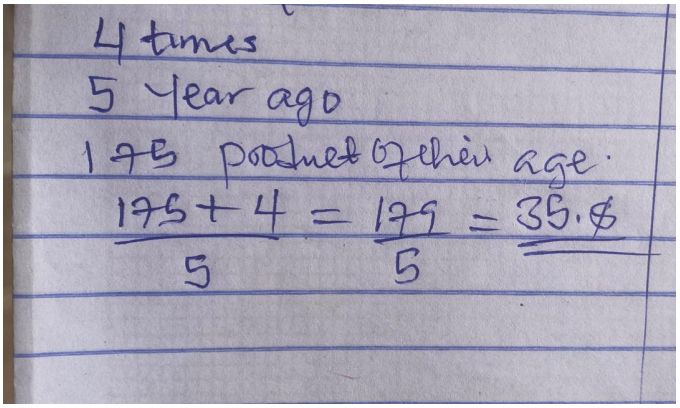


Figure 7: A Student's Work with Transformation Error.

Another error committed by students in solving quadratic word problems is the Encoding Error, see figure 8 for an example of Encoding Error committed by a participant.

Question 4: A woman is 4 times older than her child, 5 years ago, the product of their ages was 175. Find their present age.

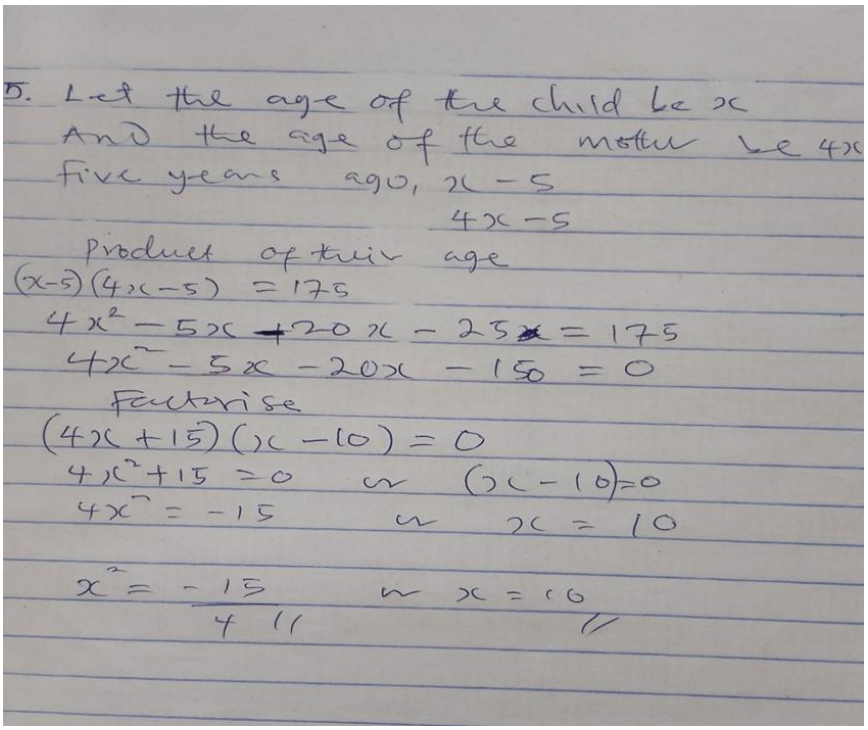


Figure 8: A Student Work with Encoding Error.

From the above solution, the participant committed an Encoding Error. The following conversation took place with the participant to find out if she really means that, $-\frac{15}{4}$ represent an age.

Researcher: Please read question 4

Student: Read the question fluently without any problem

Researcher: What did the question want you to do?

Student: to find the age of the child and the mother, five years ago

Researcher: what are the ages you got from your working?

Student: $-\frac{15}{4}$ and 10

Researcher: Are the two answers both correct?

Student: Yes

From the conversation, it shows that the participant really means that $-\frac{15}{4}$ can be an age which is an Encoding Error because it cannot be an age.

All these errors found in this study were also reported in other studies (Usman, 2018; Thomas & Mahmud, 2021). Similarly, all the causes of errors identified in this study were also reported in other studies (O'cannor & Norton, 2016; Harawaty et al., 2021).

Conclusion and Recommendation

The study investigates students' problems in solving quadratic equation. From the findings of the study, it can be concluded that, if students' problems in solving quadratic equations are not addressed, their future learning will be at stake. This is because, quadratic equation has connection with further mathematics topics and other science subjects. Therefore, its understanding or otherwise has a direct effect on students' future learning. Hence, the study recommended that: school administrators should organise seminars and workshops to enlighten teachers on the use of alternate teaching strategies with a view to improve teaching and learning of quadratic equation; researchers should investigate teaching strategies that would improve students' learning of quadratic equation.

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