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On the Study of Modified Variance Estimators in the Presence of Measurement Errors

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ABSTRACT

Often in sample survey, the estimators of population parameters do based on the assumption that the collected observations used for estimation are free from any error but mostly this assumption is violated and collected information is contaminated with measurement error due to many reasons like use of faulty instruments for data collections, inexperienced interviewer etc. This study considered the estimation problem of finite population variance under simple random sampling (SRS) scheme when the information of the study and auxiliary variables are characterized with measurement errors. The approximate expressions of biases and mean square errors (MSEs) were derived and calculated for the suggested estimators up to the first order using Taylor expansion. The efficiency comparisons of the suggested estimators were made with the existing estimators. A numerical study was also conducted to support the performance of the suggested estimators using sample size of n= 30. It was observed that modified estimators performed better than the existing related estimators considered.

Keywords: Measurement error, Estimator, Population Variance, Efficiency

1.0 INTRODUCTION

Estimation of variance is very important in the area of sample survey. It provides the information on the accuracy of the estimators and allows drawing valid conclusion about the true value of the population parameter. Measurement error is a serious problem in survey sampling which arises when the answer provided by the respondents departs from the true value.

The estimation of variance for finite population has large significance in many fields of life like Agriculture, Biological Sciences, Medical and Industry. For examples, blood pressure, pulse rate, body temperature and sugar level of blood are very basic diagnosis where treatment prescribed is to control their variation. Many authors including Das and Tripathi (1978), Isaki (1983), Bahl and Tuteja (1991), Singh *et al.* (2011), Yadav and Kadilar (2013), Singh *et al*. (2014), Muili *et al*. (2019), Qureshi *et al.* (2019), Yadav *et al*. (2019) estimated the variance of population using auxiliary information. It is a common assumption of most of statistical analysis that the collected observations are free from any error. However, Cochran (1963, 1968) and Biemer *et al.* (1991) suggested that mostly this assumption is violated and collected information is contaminated with measurement error due to many reasons. Measurement errors are the discrepancy between the observed value and true value. Therefore, it is essential to study the measurement errors. Several authors like Srivastava and Jhajji (1980), Fuller (1995), Singh and Karpe (2009), Diana and Giordan (2012), Sharma and Singh (2013), Misra *et al*. (2016), Khalil *et al*. (2018), Khalil *et al*. (2019), Zahid and Shabbir (2019), Singh *et a*l. (2020), have worked on the development of estimators for estimating different population parameters.

This paper considered the estimation problem of finite population variance using the information of the auxiliary variable in presence of measurement error under simple random sampling (SRS) design. Here, some estimators for population variance have been suggested for the study variable *Y*. The proposed estimators were based on modification of Audu *et al.* (2016) estimators.

2.0 LITERATURE REVIW

2.1 SOME EXIXSTING VARIANCE ESTIMATORS IN THE PRESENCE OF MEASUREMENT ERRORS

Assuming that the observed population values of the study and auxiliary characters Y and X respectively are $(Y_i, X_i) \in \mathbb{R}^+ > 0)$ and the true values are $(Y_i^*, X_i^*) \in \mathbb{R}^+ > 0)$, then the measurement or observational errors on Y and X are defined respectively as

$U_i = Y_i - Y_i^*$ and $V_i = X_i - X_i^*$, such that U and V follow normal distribution with zero mean, S_u^2 and S_v^2variances, $Cov(U,V) = Cov(U,X) = Cov(U,Y) = Cov(V,X) = Cov(V,Y) = 0$ and $Y_i^* = Y_i + U_i$ and $X_i^* = X_i + V_i$.

Similarly, let the values of observed sample units be $(y_i, x_i) \in \mathbb{R}^+ > 0)$ and the true values are $(y_i^*, x_i^*) \in \mathbb{R}^+ > 0)$, then the measurement or observational errors on y and x are defined respectively as $u_i = y_i - y_i^*$ and $v_i = x_i - x_i^*$ such that $y_i^* = y_i + u_i$ and $x_i^* = x_i + v_i$.

$s_x^{*2} = (s_x^2 + s_v^2)$, $s_y^{*2} = (s_y^2 + s_u^2)$,

The usual variance estimator under measurement error is defined as

$$t_1 = s_y^{*2}$$

where

$$s_y^{*2} = \frac{1}{n-1}\sum_{i=1}^{n}\left(y_i^* - \bar{y}^*\right), \ \bar{y}^* = \frac{1}{n}\sum_{i=1}^{n} y_i^* \qquad (2.1.0)$$

The MSE/Variance of the estimator is given as:

$$Var(t_1) = \frac{1}{n} S_y^4 \left(\mu_{2y} + \mu_{2u}\frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) \qquad (2.1.1)$$

where

$$\psi_{rs} = \frac{\lambda_{rs}}{\lambda_{20}^{r/2}\lambda_{02}^{s/2}}, \ \mu_{2y} = \psi_{40} - 3, \ \lambda_{rs} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)^r \left(X_i - \bar{X}\right)^s, \ S_y^2 = \lambda_{20},$$

$$\Psi_{rs} = \frac{\upsilon_{rs}}{\upsilon_{20}^{r/2}\upsilon_{02}^{s/2}}, \ \mu_{2u} = \Psi_{40} - 3, \ \upsilon_{rs} = \frac{1}{N-1}\sum_{i=1}^{N} U_i^r V_i^s, \ S_u^2 = \upsilon_{20}$$

Isaki (1983) ratio type estimator for population variance in the presence of measurement error is as follow:

$$t_2 = s_y^{*2}\left(\frac{S_x^2}{s_x^{*2}}\right) \qquad (2.1.2)$$

$$Bias(t_1) = \frac{1}{n} S_y^2 \left(1 + \mu_{2x} + \mu_{2v}\frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 - \frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} \right) \qquad (2.1.3)$$

$$MSE(\eta_1) = \gamma S_y^4 \left(\begin{array}{l} \left(\mu_{2y} + \mu_{2u}\frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + \left(\mu_{2x} + \mu_{2v}\frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) \\ -2\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 2 \end{array} \right) \qquad (2.1.4)$$

where

$$\mu_{22}(X,Y) = \frac{1}{N}\sum_{i=1}^{N}\left(X_i - \bar{X}\right)^2\left(Y_i - \bar{Y}\right)^2, \ S_x^2 = \lambda_{02}, \ S_v^2 = \upsilon_{02}$$

Singh *et al.* (2011**)** estimators in the presence of measurement error is given as

$$t_3 = s_y^{*2}\exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}}\right) \qquad (2.1.5)$$

$$Bias(t_3) = \frac{1}{2n} S_y^2 \left(\frac{3}{4}\left(\mu_{2x} + \mu_{2v}\frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - \frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 1 \right) \qquad (2.1.6)$$

$$MSE(t_3) = \gamma S_y^4 \left(\begin{array}{l} \left(\left(\mu_{2y} + \mu_{2u} \frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + \frac{1}{4}\left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) \right) \\ -\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 1 \end{array} \right) \tag{2.1.7}$$

Tariq *et al*. (2021) suggested arithmetic, geometric and harmonic-based estimators of population variance in the presence of measurement error as

$$t_4^{AM} = \frac{t_1 + t_2}{2} = \frac{s_y^{*2}}{2\left(1 + S_x^2 / s_x^{*2}\right)} \tag{2.1.8}$$

$$t_4^{GM} = \left(t_1 t_2\right)^{1/2} = s_y^{*2} \left(\frac{S_x^2}{s_x^{*2}} \right)^{1/2} \tag{2.1.9}$$

$$t_4^{HM} = \frac{2}{\left(\frac{1}{t_1} + \frac{1}{t_2} \right)} = \frac{2 s_y^{*2}}{\left(1 + \frac{s_x^{*2}}{S_x^2} \right)} \tag{2.10}$$

$$t_5^{AM} = \frac{t_1 + t_3}{2} = \frac{s_y^{*2}}{2} \left(1 + \exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}} \right) \right) \tag{2.11}$$

$$t_5^{GM} = \left(t_1 t_3\right)^{1/2} = s_y^{*2} \left(\exp\left(\frac{S_x^2 + s_x^{*2}}{S_x^2 - s_x^{*2}} \right) \right)^{1/2} \tag{2.12}$$

$$t_5^{HM} = \frac{2}{\left(\frac{1}{t_1} + \frac{1}{t_3} \right)} = \frac{2 s_y^{*2}}{\left(1 + 1/\exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}} \right) \right)} \tag{2.13}$$

$$t_6^{AM} = \frac{t_2 + t_3}{2} = \frac{s_y^{*2}}{2} \left(\frac{S_x^2}{s_x^{*2}} + \exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}} \right) \right) \tag{2.14}$$

$$t_6^{GM} = \left(t_2 t_3\right)^{1/2} = s_y^{*2} \left(\frac{S_x^2}{s_x^{*2}} \exp\left(\frac{S_x^2 + s_x^{*2}}{S_x^2 - s_x^{*2}} \right) \right)^{1/2} \tag{2.15}$$

$$t_6^{HM} = \frac{2}{\left(\frac{1}{t_2} + \frac{1}{t_3} \right)} = \frac{2 s_y^{*2}}{\left(1/\left(S_x^2 / s_x^{*2} \right) + 1/\exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}} \right) \right)} \tag{2.16}$$

$$t_7^{AM} = \frac{t_1 + t_2 + t_3}{3} = \frac{s_y^{*2}}{3} \left(1 + \frac{S_x^2}{s_x^{*2}} + \exp\left(\frac{S_x^2 - s_x^{*2}}{S_x^2 + s_x^{*2}} \right) \right) \tag{2.17}$$

$$t_7^{GM} = \left(t_1 t_2 t_3\right)^{1/3} = s_y^{*2} \left(\frac{S_x^2}{s_x^{*2}} \exp\left(\frac{S_x^2 + s_x^{*2}}{S_x^2 - s_x^{*2}} \right) \right)^{1/3} \tag{2.18}$$

$$t_7^{HM} = \frac{3}{\left(\frac{1}{t_1}+\frac{1}{t_2}+\frac{1}{t_3}\right)} = \frac{3s_y^{*2}}{\left(1+1/\left(S_x^2/s_x^{*2}\right)+1/\exp\left(\frac{S_x^2-s_x^{*2}}{S_x^2+s_x^{*2}}\right)\right)} \quad (2.19)$$

The biases and MSEs of the above estimators are given below;

$$Bias(t_4^{AM}) = \frac{1}{2n}S_y^2\left(1+\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2-\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.20)$$

$$Bias(t_4^{GM}) = \frac{1}{8n}S_y^2\left(4+3\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{4\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.21)$$

$$Bias(t_4^{HM}) = \frac{1}{4n}S_y^2\left(2+\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{2\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.22)$$

$$Bias(t_5^{AM}) = \frac{1}{8n}S_y^2\left(4+3\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{4\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.23)$$

$$Bias(t_5^{GM}) = \frac{1}{4n}S_y^2\left(1+\frac{5}{8}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.24)$$

$$Bias(t_5^{HM}) = \frac{1}{4n}S_y^2\left(1+\frac{3}{4}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.25)$$

$$Bias(t_6^{AM}) = \frac{1}{8n}S_y^2\left(3+\frac{11}{2}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{3\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.26)$$

$$Bias(t_6^{GM}) = \frac{1}{4n}S_y^2\left(3+\frac{21}{8}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{3\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.27)$$

$$Bias(t_6^{HM}) = \frac{1}{4n}S_y^2\left(3+\frac{5}{2}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{3\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.28)$$

$$Bias(t_7^{AM}) = \frac{1}{2n}S_y^2\left(1+\frac{11}{12}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}\right) \quad (2.29)$$

$$Bias(t_7^{GM}) = \frac{1}{2n} S_y^2 \left(1 + \frac{11}{4} \left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - \frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} \right) \quad (2.30)$$

$$Bias(t_7^{HM}) = \frac{1}{2n} S_y^2 \left(1 + \frac{1}{3} \left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - \frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} \right) \quad (2.31)$$

$$MSE(t_4^{AM}) = MSE(t_4^{GM}) = MSE(t_4^{HM}) = \frac{S_y^4}{n} \left(\left(\mu_{2y} + \mu_{2u} \frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + 4\left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - 4\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 4 \right) \quad (2.32)$$

$$MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) = \frac{S_y^4}{4n} \left(4\left(\mu_{2y} + \mu_{2u} \frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + \frac{1}{4}\left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - 2\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 2 \right) \quad (2.33)$$

$$MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) = \frac{S_y^4}{4n} \left(4\left(\mu_{2y} + \mu_{2u} \frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + \frac{9}{4}\left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - 6\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 6 \right) \quad (2.34)$$

$$MSE(t_7^{AM}) = MSE(t_7^{GM}) = MSE(t_7^{HM}) = \frac{S_y^4}{n} \left(\left(\mu_{2y} + \mu_{2u} \frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2 \right) + \frac{1}{4}\left(\mu_{2x} + \mu_{2v} \frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2 \right) - \frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 1 \right) \quad (2.35)$$

3.0 MATERIALS AND METHODS

The work of Audu *et. al.* (2016) can be extended to capture measurement errors thus:

$$T_1 = \varpi_1 s_y^{*2} \left(\frac{S_x^2}{s_x^{*2}} \right) \quad (3.1.0)$$

$$T_2=\varpi_2 s_y^{*2}\left(\frac{S_x^2-C_x}{s_x^{*2}+C_x}\right) \quad (3.1.1)$$

$$T_3=\varpi_3 s_y^{*2}\left(\frac{S_x^2-\beta_2(x)}{s_x^{*2}+\beta_2(x)}\right) \quad (3.1.2)$$

$$T_4=\varpi_4 s_y^{*2}\left(\frac{S_x^2\beta_2(x)-C_x}{s_x^{*2}\beta_2(x)+C_x}\right) \quad (3.1.3)$$

$$T_5=\varpi_5 s_y^{*2}\left(\frac{S_x^2C_x-\beta_2(x)}{s_x^{*2}C_x+\beta_2(x)}\right) \quad (3.1.4)$$

$$T_6=\varpi_6 s_y^{*2}\left(\frac{S_x^2+\beta_2(x)}{s_x^{*2}-\beta_2(x)}\right) \quad (3.1.5)$$

$\varpi_i,\ i=1,2,3,4,5,6$ is unknown weight which will be determined to minimized

$MSE(T_d)$, $s_x^{*2}=(s_x^2+s_v^2)$ and $s_y^{*2}=(s_y^2+s_u^2)$, s_u^2 and s_v^2 are sample variances based on measurement errors on Y and X respectively

3.1 Properties of the Proposed Estimators $T_i,\ i=1,2,3,4,5,6$

In this section the theoretical biases and means square error (MSE) of newly extended estimators are derived up to second degree approximation using Taylor series expansion

Let $e_0=\dfrac{s_y^{*2}-S_y^2}{S_y^2},\ e_1=\dfrac{s_x^{*2}-S_x^2}{S_x^2}$ be sampling error terms with expected values defined as

$$\left.\begin{aligned}&E(e_0)=E(e_1)=0, E(e_0^2)=\frac{1}{n}\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)\\&E(e_1^2)=\frac{1}{n}\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right), E(e_0e_1)=\frac{1}{n}\left(\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}-1\right)\end{aligned}\right\} \quad (3.1.6)$$

Expressing $T_i,\ i=1,2,3,4,5,6$ in terms of e_0 and e_1

$$T_i=\left.\begin{matrix}\varpi_1S_y^2(1+e_0)(1+e_1)^{-1} & i=1\\ \varpi_iS_y^2(1+e_0)(1+m_ie_1)^{-1} & i=2,3,4,5,6\end{matrix}\right\} \quad (3.1.7)$$

where

$$m_2=\frac{S_x^2}{S_x^2+C_x}, m_3=\frac{S_x^2}{S_x^2+\beta_2(x)}, m_4\frac{S_x^2\beta_2(x)}{S_x^2\beta_2(x)+C_x}, m_{5=}\frac{S_x^2C_x}{S_x^2C_x+\beta_2(x)}, m_{6=}\frac{S_x^2}{S_x^2-\beta_2(x)},$$

Simplify eq. (3.17) up to first order approximation, we have

$$T_i-S_y^2=\left.\begin{matrix}S_y^2\left((\varpi_1-1)+\varpi_1\left(e_0-e_1+e_1^2-e_0e_1\right)\right) & i=1\\ S_y^2\left((\varpi_i-1)+\varpi_i\left(e_0-m_ie_1+m_i^2e_1^2-m_ie_0e_1\right)\right) & i=2,3,4,5,6\end{matrix}\right\} \quad (3.1.8)$$

Taking expectation of eq. (3.18), the $Bias\left(T_i\right),\ i=1,2,3,4,5,6$ are obtained as

$$Bias\left(T_i\right)=\left.\begin{array}{ll} S_y^2\left(\left(\varpi_1-1\right)+\varpi_1\frac{1}{n}\left(\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)-\left(\frac{\mu_{22}\left(X,Y\right)}{S_x^2S_y^2}-1\right)\right)\right) & i=1 \\ S_y^2\left(\left(\varpi_i-1\right)+\varpi_i\frac{1}{n}\left(m_i^2\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)-m_i\left(\frac{\mu_{22}\left(X,Y\right)}{S_x^2S_y^2}-1\right)\right)\right) & i=2,...,6 \end{array}\right\} \quad (3.1.9)$$

Square eq. (3.18) and simplify up to first order approximation, we obtained

$$\left(T_i-S_y^2\right)^2=\left.\begin{array}{ll} S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(e_0^2+e_1^2-2e_0e_1\right)+2\varpi_1\left(\varpi_1-1\right)\left(e_1^2-e_0e_1\right)\right) & i=1 \\ S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(e_0^2+m_i^2e_1^2-2m_ie_0e_1\right)+2\varpi_1\left(\varpi_1-1\right)\left(m_i^2e_1^2-m_ie_0e_1\right)\right) & i=2,...,6 \end{array}\right\} \quad (3.20)$$

Take expectation of the result of (3.20), the $MSE\left(T_i\right),\ i=1,2,3,4,5,6$ are obtained as

$$MSE\left(T_i\right)=\left.\begin{array}{ll} S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(Q_1+Q_2-2Q_3\right)+2\varpi_1\left(\varpi_1-1\right)\left(Q_2-Q_3\right)\right) & i=1 \\ S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(Q_1+m_i^2Q_2-2m_iQ_3\right)+2\varpi_1\left(\varpi_1-1\right)\left(m_i^2Q_2-m_iQ_3\right)\right) & i=2,...,6 \end{array}\right\} \quad (3.21)$$

where

$$Q_1=\frac{1}{n}\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right),\ Q_2=\frac{1}{n}\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right),Q_3=\frac{1}{n}\left(\frac{\mu_{22}\left(X,Y\right)}{S_x^2S_y^2}-1\right)$$

Differentiate (3.21) with respect to ϖ, equate to zero.

$$\frac{\partial MSE\left(T_i\right)}{\partial\varpi}=\left.\begin{array}{l} \frac{\partial\left(S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(Q_1+Q_2-2Q_3\right)+2\varpi_1\left(\varpi_1-1\right)\left(Q_2-Q_3\right)\right)\right)}{\partial\varpi} \\ \frac{\partial\left(S_y^4\left(\left(\varpi_1-1\right)^2+\varpi_1^2\left(Q_1+m_i^2Q_2-2m_iQ_3\right)+2\varpi_1\left(\varpi_1-1\right)\left(m_i^2Q_2-m_iQ_3\right)\right)\right)}{\partial\varpi} \end{array}\right\} \quad (3.22)$$

$$\left.\begin{array}{ll} S_y^4\left(2\left(\varpi_1-1\right)+2\varpi_1\left(Q_1+Q_2-2Q_3\right)+2\left(2\varpi_1-1\right)\left(Q_2-Q_3\right)\right)=0 & i=1 \\ S_y^4\left(2\left(\varpi_i-1\right)+2\varpi_i\left(Q_1+m_i^2Q_2-2m_iQ_3\right)+2\left(2\varpi_i-1\right)\left(m_i^2Q_2-m_iQ_3\right)\right)=0 & i=2,...,6 \end{array}\right\} \quad (3.23)$$

Solve for ϖ_i, the expressions for ϖ_i are obtained as

$$\varpi_i=\left\{\begin{array}{ll} \frac{1+Q_2-Q_3}{1+\left(Q_1+3Q_2-4Q_3\right)} & i=1 \\ \frac{1+m_i^2Q_2-m_iQ_3}{1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)} & i=2,3,4,5,6 \end{array}\right. \quad (3.24)$$

By substituting (3.25) in (3.21), the minimum MSEs of $T_i, \quad i=1,2,3,4,5,6$ are obtained as

$$MSE_{\min}\left(T_i\right)=\begin{cases} S_y^4\left(1-\dfrac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right) & i=1 \\ S_y^4\left(1-\dfrac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right) & i=2,3,4,5,6 \end{cases} \tag{3.25}$$

3.2 Theoretical Efficiency Comparisons

The efficiencies of the modified estimators are compared theoretically with some related existing estimators and the conditions for which the proposed estimators performed better than the existing related estimators have been established.

The usual variance estimator t_1 under measurement error against $T_i,\ i=1,2,3,4,5,6$

$$Var(t_1)=\frac{1}{n}S_y^4\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right) \tag{3.26}$$

$$Var(t_1)=S_y^4Q_1 \tag{3.27}$$

$$MSE_{\min}\left(T_i\right)=\begin{cases} S_y^4\left(1-\dfrac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right) & i=1 \\ S_y^4\left(1-\dfrac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right) & i=2,3,4,5,6 \end{cases} \tag{3.28}$$

$$\left.\begin{aligned} &Var\left(t_1\right)-MSE\left(T_1\right)>0 && i=1 \\ &Var\left(t_1\right)-MSE\left(T_i\right)>0 && i=2,3,4,5,6 \end{aligned}\right\} \tag{3.29}$$

$$\left.\begin{aligned} &S_y^4Q_1-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 && i=1 \\ &S_y^4Q_1-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 && i=2,3,4,5,6 \end{aligned}\right\} \tag{3.30}$$

$$\left.\begin{aligned} &Q_1+\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}>1 && i=1 \\ &Q_1+\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}>1 && i=2,3,4,5,6 \end{aligned}\right\} \tag{3.31}$$

Isaki (1983) estimator t_2 under measurement error VS $T_i,\ i=1,2,3,4,5,6$

$$MSE(t_2)=\frac{1}{n}S_y^4\left(\begin{array}{l}\left(\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)+\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)\right)\\ -2\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}+2\end{array}\right) \quad (3.32)$$

$$MSE(t_2)=S_y^4\left(Q_1+Q_2-2Q_3+2\right) \quad (3.33)$$

$$\left.\begin{array}{ll}MSE(t_2)-MSE(T_1)>0 & i=1\\ MSE(t_2)-MSE(T_i)>0 & i=2,3,4,5,6\end{array}\right\} \quad (3.34)$$

$$\left.\begin{array}{ll}S_y^4\left(Q_1+Q_2-2Q_3+2\right)-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 & i=1\\ S_y^4\left(Q_1+Q_2-2Q_3+2\right)-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 & i=2,3,4,5,6\end{array}\right\} \quad (3.35)$$

$$\left.\begin{array}{ll}\left(\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)-\left(Q_1+Q_2-2Q_3\right)<1 & i=1\\ \left(\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)-\left(Q_1+Q_2-2Q_3\right)<1 & i=2,3,4,5,6\end{array}\right\} \quad (3.36)$$

Singh *et al.* (2011) estimator t_3 under measurement error VS $T_i,\ i=1,2,3,4,5,6$

$$MSE(t_3)=\gamma S_y^4\left(\begin{array}{l}\left(\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)+\frac{1}{4}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)\right)\\ -\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}+1\end{array}\right) \quad (3.37)$$

$$MSE(t_3)=S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right) \quad (3.38)$$

$$\left.\begin{array}{ll}MSE(t_3)-MSE(T_1)>0 & i=1\\ MSE(t_3)-MSE(T_i)>0 & i=2,3,4,5,6\end{array}\right\} \quad (3.39)$$

$$\left.\begin{array}{l} S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right)-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1 \\ S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right)-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{array}\right\} \tag{3.40}$$

$$\left.\begin{array}{l} \left(Q_1+\frac{1}{4}Q_2-Q_3\right)+\left(\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1 \\ \left(Q_1+\frac{1}{4}Q_2-Q_3\right)+\left(\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{array}\right\} \tag{3.41}$$

Tariq *et al*. (2021) estimator t_4^{AM}, t_4^{GM}, t_4^{HM} under measurement error VS $T_i,\ i=1,2,3,4,5,6$

$$MSE(t_4^{AM})=MSE(t_4^{GM})=MSE(t_4^{HM})=\frac{S_y^4}{n}\left(\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)\right.$$
$$\left.+4\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-4\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}+4\right) \tag{3.42}$$

$$MSE(t_4^{AM})=MSE(t_4^{GM})=MSE(t_4^{HM})=S_y^4\left(Q_1+4Q_2-4Q_3+4\right) \tag{3.43}$$

$$\left.\begin{array}{ll} MSE(t_4^{AM})=MSE(t_4^{GM})=MSE(t_4^{HM})-MSE\left(T_1\right)>0 & i=1 \\ MSE(t_4^{AM})=MSE(t_4^{GM})=MSE(t_4^{HM})-MSE\left(T_i\right)>0 & i=2,3,4,5,6 \end{array}\right\} \tag{3.44}$$

$$\left.\begin{array}{l} S_y^4\left(Q_1+4Q_2-4Q_3+4\right)-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1 \\ S_y^4\left(Q_1+4Q_2-4Q_3+4\right)-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{array}\right\} \tag{3.45}$$

$$\left.\begin{array}{l} \left(Q_1+4Q_2-4Q_3\right)+\left(\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>-3 \quad i=1 \\ \left(Q_1+4Q_2-4Q_3\right)+\left(\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>-3 \quad i=2,3,4,5,6 \end{array}\right\} \tag{3.46}$$

i. Tariq *et al*. (2021) estimator t_5^{AM}, t_5^{GM}, t_5^{HM} under measurement error VS $T_i,\ i=1,2,3,4,5,6$

$$MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) = \frac{S_y^4}{4n}\left(4\left(\mu_{2y} + \mu_{2u}\frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2\right)\right.$$
$$\left. + \frac{1}{4}\left(\mu_{2x} + \mu_{2v}\frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2\right) - 2\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 2\right) \quad (3.47)$$

$$MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) = S_y^4\left(Q_1 + \frac{1}{16}Q_2 - \frac{1}{2}Q_3 + \frac{1}{2}\right) \quad (3.48)$$

$$\left.\begin{aligned} &MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) - MSE(T_1) > 0 && i = 1 \\ &MSE(t_5^{AM}) = MSE(t_5^{GM}) = MSE(t_5^{HM}) - MSE(T_i) > 0 && i = 2,3,4,5,6 \end{aligned}\right\} \quad (3.49)$$

$$\left.\begin{aligned} &S_y^4\left(Q_1 + \frac{1}{16}Q_2 - \frac{1}{2}Q_3 + \frac{1}{2}\right) - S_y^4\left(1 - \frac{\left(1 + (Q_2 - Q_3)\right)^2}{\left(1 + (Q_1 + 3Q_2 - 4Q_3)\right)}\right) > 0 \quad i = 1 \\ &S_y^4\left(Q_1 + \frac{1}{16}Q_2 - \frac{1}{2}Q_3 + \frac{1}{2}\right) - S_y^4\left(1 - \frac{\left(1 + (m_i^2 Q_2 - m_i Q_3)\right)^2}{\left(1 + (Q_1 + 3m_i^2 Q_2 - 4m_i Q_3)\right)}\right) > 0 \quad i = 2,3,4,5,6 \end{aligned}\right\} \quad (3.50)$$

$$\left.\begin{aligned} &\left(Q_1 + \frac{1}{16}Q_2 - \frac{1}{2}Q_3\right) + \left(\frac{\left(1 + (Q_2 - Q_3)\right)^2}{\left(1 + (Q_1 + 3Q_2 - 4Q_3)\right)}\right) > \frac{1}{2} \quad i = 1 \\ &\left(Q_1 + \frac{1}{16}Q_2 - \frac{1}{2}Q_3\right) + \left(\frac{\left(1 + (m_i^2 Q_2 - m_i Q_3)\right)^2}{\left(1 + (Q_1 + 3m_i^2 Q_2 - 4m_i Q_3)\right)}\right) > \frac{1}{2} \quad i = 2,3,4,5,6 \end{aligned}\right\} \quad (3.51)$$

Tariq *et al*. (2021) estimator t_6^{AM}, t_6^{GM}, t_6^{HM} under measurement error VS $T_i,\ i = 1,2,3,4,5,6$

$$MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) = \frac{S_y^4}{4n}\left(4\left(\mu_{2y} + \mu_{2u}\frac{S_u^4}{S_y^4} + 2\left(1 + \frac{S_u^2}{S_y^2}\right)^2\right)\right.$$
$$\left. + \frac{9}{4}\left(\mu_{2x} + \mu_{2v}\frac{S_v^4}{S_x^4} + 2\left(1 + \frac{S_v^2}{S_x^2}\right)^2\right) - 6\frac{\mu_{22}(X,Y)}{S_x^2 S_y^2} + 6\right) \quad (3.52)$$

$$MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) = S_y^4\left(Q_1 + \frac{9}{16}Q_2 - \frac{3}{2}Q_3 + \frac{3}{2}\right) \quad (3.53)$$

$$\left.\begin{aligned} &MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) - MSE(T_1) > 0 && i = 1 \\ &MSE(t_6^{AM}) = MSE(t_6^{GM}) = MSE(t_6^{HM}) - MSE(T_i) > 0 && i = 2,3,4,5,6 \end{aligned}\right\} \quad (3.54)$$

$$\left.\begin{aligned} &S_y^4\left(Q_1+\frac{9}{16}Q_2-\frac{3}{2}Q_3+\frac{3}{2}\right)-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1\\ &S_y^4\left(Q_1+\frac{9}{16}Q_2-\frac{3}{2}Q_3+\frac{3}{2}\right)-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{aligned}\right\} \tag{3.55}$$

$$\left.\begin{aligned} &\left(Q_1+\frac{9}{16}Q_2-\frac{3}{2}Q_3\right)+\left(\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>-\frac{1}{2} \quad i=1\\ &\left(Q_1+\frac{9}{16}Q_2-\frac{3}{2}Q_3\right)+\left(\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>-\frac{1}{2} \quad i=2,3,4,5,6 \end{aligned}\right\} \tag{3.56}$$

Tariq *et al*. (2021) estimator $t_7^{AM}, t_7^{GM}, t_7^{HM}$ under measurement error VS $T_i,\ i=1,2,3,4,5,6$

$$MSE(t_7^{AM})=MSE(t_7^{GM})=MSE(t_7^{HM})=\frac{S_y^4}{n}\left(\left(\mu_{2y}+\mu_{2u}\frac{S_u^4}{S_y^4}+2\left(1+\frac{S_u^2}{S_y^2}\right)^2\right)+\frac{1}{4}\left(\mu_{2x}+\mu_{2v}\frac{S_v^4}{S_x^4}+2\left(1+\frac{S_v^2}{S_x^2}\right)^2\right)-\frac{\mu_{22}(X,Y)}{S_x^2S_y^2}+1\right) \tag{3.57}$$

$$MSE(t_7^{AM})=MSE(t_7^{GM})=MSE(t_7^{HM})=S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right) \tag{3.58}$$

$$\left.\begin{aligned} &MSE(t_7^{AM})=MSE(t_7^{GM})=MSE(t_7^{HM})-MSE\left(T_1\right)>0 \quad i=1\\ &MSE(t_7^{AM})=MSE(t_7^{GM})=MSE(t_7^{HM})-MSE\left(T_i\right)>0 \quad i=2,3,4,5,6 \end{aligned}\right\} \tag{3.58}$$

$$\left.\begin{aligned} &S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right)-S_y^4\left(1-\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1\\ &S_y^4\left(Q_1+\frac{1}{4}Q_2-Q_3+1\right)-S_y^4\left(1-\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{aligned}\right\} \tag{3.60}$$

$$\left.\begin{aligned} &\left(Q_1+\frac{1}{4}Q_2-Q_3\right)+\left(\frac{\left(1+\left(Q_2-Q_3\right)\right)^2}{\left(1+\left(Q_1+3Q_2-4Q_3\right)\right)}\right)>0 \quad i=1\\ &\left(Q_1+\frac{1}{4}Q_2-Q_3\right)+\left(\frac{\left(1+\left(m_i^2Q_2-m_iQ_3\right)\right)^2}{\left(1+\left(Q_1+3m_i^2Q_2-4m_iQ_3\right)\right)}\right)>0 \quad i=2,3,4,5,6 \end{aligned}\right\} \tag{3.61}$$

4.0 RESULTS AND DISCUSSION

In this section, efficiency of the modified estimators over some related existing estimators were examined using real life data to support the theoretical comparisons in section three.

Data 1: Data set of Statistics Department on Student's CGPA (Y) and Student's AGE (X)

$N = 94,\ \bar{Y} = 2.48,\ \bar{X} = 22.93,\ \bar{U} = 0.06,\ \bar{V} = 0.43,\ S_y^2 = 0.082,\ S_x^2 = 11.12,\ S_u^2 = 0.02,\ S_v^2 = 8.74,\ \mu_{2y} = 1.287,\ \mu_{2X} = -0.477,\ \mu_{2U} = 5.37,\ \mu_{2V} = 3.02,\ \beta_{2(y)} = 3.287,\ \beta_{2(x)} = 2.523,\ \beta_{2(u)} = 8.37,\ \beta_{2(v)} = 6.02,\ CX = \frac{\sqrt{Sx^2}}{\bar{X}} = 0.15,\ \mu_{22(yx)} = 0.777043,\ \mu_{22}(U,V) = 0.216756,\ \rho_{xy} = 0.71$, for when n = 30.

Data 2: Data set of Science Laboratory Technology Department on Student's CGPA (Y) and Student's AGE (X)

$N = 87,\ \bar{Y} = 2.76,\ \bar{X} = 24.33,\ \bar{U} = 0.08,\ \bar{V} = -0.21,\ S_y^2 = 0.104,\ S_x^2 = 7.27,\ S_u^2 = 0.09,\ S_v^2 = 6.46,\ \mu_{2y} = 0.146,\ \mu_{2x} = -0.18,\ \mu_{2U} = 6.14,\ \mu_{2V} = 2.65,\ \beta_{2(y)} = 3.146,\ \beta_{2(x)} = 2.12,\ \beta_{2(u)} = 9.14,\ \beta_{2(v)} = 5.65,\ CV(X) = 0.11,\ \mu_{22(yx)} = 0.46,\ \mu_{22}(U,V) = 0.2287,\ \rho_{xy} = 0.65$, for when n = 30.

The Biases, MSEs and PREs of the considered estimators were computed using their properties as shown in the tables below.

Table 4.1: Biases, MSEs and PREs of Modified and Some Existing Estimators using Data from Department of Statistics, KPT for $N = 94,\ n = 30$

Estimators	**Bias**	**MSE**	**PRE**	**Rank**
sample variance t_1	0	0.00105	100	
Isaki (1983) t_2	0.02255	0.000609	172.4138	17th
Singh et al.(2011) t_3	0.0085	0.000765	137.2549	16th
Tariq et al. (2021)				
t_4^{AM}	0.01127	0.000338	310.6509	14th
t_4^{GM}	0.0085	0.000338	310.6509	11th
t_4^{HM}	0.00574	0.000338	310.6509	1th
t_5^{AM}	0.0085	0.0009794	107.2085	19th
t_5^{GM}	0.00356	0.0009794	107.2085	19th
t_5^{HM}	0.00425	0.0009794	107.2085	19th
t_6^{AM}	0.01538	0.0005618	186.8993	14th
t_6^{GM}	0.01483	0.0005618	186.8993	14th
t_6^{HM}	0.01414	0.0005618	186.8993	14th
t_7^{AM}	0.01035	0.0003102	338.4913	8th
t_7^{GM}	0.02715	0.0003102	338.4913	8th
t_7^{HM}	0.00272	0.0003102	338.4913	8th
Modified Estimators				
T_1	-0.0149	0.0003087	340.1801	6th
T_2	-0.01492	0.0003087	340.1801	5th
T_3	-0.01488	0.0002806	374.1981	3rd
T_4	-0.01491	0.0003087	340.1801	4th
T_5	-0.01277	0.000252	416.6667	2nd
T_6	-0.01329	4.687E-05	2240.239	1st

COMMENT: Table 4.1 shows the biases, MSEs and Percentage Relative Efficiency (PREs) of modified estimators $T_i,\ i=1,2,3,4,5,6$ and that of some existing related. The above result revealed that the modified estimators $T_i,\ i=1,2,3,5,6$ have minimum MSE and high PRE compared to other existing related estimators in the study. Therefore, the modified estimator T_6 is the most efficient among other estimators considered data 1.

Table 4.2: Biases, MSEs and PRE of Modified and Some Existing Related Estimators using Data from Department of SLT, KPT for *N=87, n=30*

Estimators	Bias	MSE	PRE	Rank
sample variance t_1	0	0.01266	100	
Isaki (1983) t_2	0.10189	0.0237	53.46727	20th
Singh et al.(2011) t_3	0.03872	0.0156	81.01758	19th
Tariq et al. (2021)				
t_4^{AM}	0.05095	0.006606	191.644	17th
t_4^{GM}	0.03872	0.006606	191.644	17th
t_4^{HM}	0.02649	0.006606	191.644	17th
t_5^{AM}	0.03872	0.00608	208.2408	8th
t_5^{GM}	0.0163	0.00608	208.2408	8th
t_5^{HM}	0.01936	0.00608	208.2408	8th
t_6^{AM}	0.06878	0.005706	221.8717	14th
t_6^{GM}	0.06725	0.005706	221.8717	14th
t_6^{HM}	0.06419	0.005706	221.8717	14th
t_7^{AM}	0.04687	0.006117	206.9777	11th
t_7^{GM}	0.12705	0.006117	206.9777	11th
t_7^{HM}	0.01518	0.006117	206.9777	11th
Modified Estimators				
T_1	-0.02484	0.00258	490.2105	2nd
T_2	-0.02579	0.00268	472.1074	4th
T_3	-0.03765	0.00392	323.3689	5th
T_4	-0.02529	0.00263	481.3979	3rd
T_5	-0.05386	0.0056	226.0443	6th
T_6	0.00737	0.00077	1652.465	1ts

COMMENT: Table 4.2 shows the biases, MSEs and Percentage Relative Efficiency (PREs) of modified estimators $T_i,\ i=1,2,3,4,5,6$ and that of some existing related. The above result revealed that the modified estimators $T_i,\ i=1,2,3,5,6$ have minimum MSE and high PRE compared to other existing related estimators in the study. Therefore, the modified estimator T_6 is the most efficient among other estimators considered data 2.

4.1 CONCLUSION

From the results of empirical study, it is inferred that the modified estimators suggested in this study demonstrated high relative efficiency over existing related estimators, the suggested modified estimator T_6 is the most efficient estimator among all the estimators considered in the study. Hence, the proposed modified estimators ($T_{i,\ i=1,2,3,4,5,\ 6}$) are hereby recommended for usage by the researchers in sampling theory using measurement errors.

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