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Light Attenuation as a Function Distance: A Factional Order Correction

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ABSTRACT

This paper aims to explore the possibility of applying fractional calculus to bridge the classical models with complicated physical processes. It investigates the application of fractional calculus in the Beer-Lambert law, which deals with how light interacts with matter. This exploration delves beyond the mere application of fractional calculus, uncovering intriguing connections between it and established models in light-matter interactions domain. In the realm of light, fractional calculus provides a superior description of light absorption in intricate scenarios, surpassing the limitations of the Beer-Lambert Law. This paves the way for breakthroughs in solar cell technology and material design with tailored light absorption properties.

Keywords: Fractional Calculus, Beer Lambert Law, Light Intensity, Photon

INTRODUCTION

Classical physics has distinguished itself as possessing a high level of accuracy in describing various natural occurrences[1]. However, its drawbacks arise when the system under study possesses memory effects and nonlinear characteristics, slowing progress in many scientific fields. To meet this challenge, following the preceding outline and approach, this paper investigates fractional calculus, which generalizes regular calculus with the ability to deal with derivatives and integrals of non-integer order[2]. Our attention is based on applying fractional calculus to improve equation such as the Beer-Lambert Law The Beer-Lambert Law is a fundamental law applied to spectroscopy that describes the relationship between absorbance and a material's concentration; however, this law has limitations because it cannot explain nonlinear systems[3]. Fractional calculus provides a solution to the problem by admitting fractional derivatives up to 0.1, which helps in coming up with a better understanding of these laws in complex cases where other forms of calculus cannot be applied. This investigation focuses on commonly known law to improve the scope and efficiency of its usage in various practical fields.

LITERATURE REVIEW

The potential of fractional calculus as a tool to improve and enhance recognized laws, such as the Beer-Lambert Law, was investigated in this paper, which is a mathematical framework containing non-integer derivatives and integrals. Fractional calculus offers a deeper understanding of intricate behaviors that elude traditional calculus, such as those described by the Generalized Fractional Derivative (GFD) equation[4]:

$$\frac{d^{\alpha}f(t)}{dt^{\alpha}} = \lim_{\varepsilon \to 0} \frac{f(t+\varepsilon)-f(t)}{\varepsilon}$$

Here, α ($0 < \alpha \leq 1$) is the fractional order that characterizes "memory effects" that are pivotal for the description of the temporal evolution of processes. The Beer-Lambert Law is useful in spectroscopy and, though it has some drawbacks identifying highly complex and non-linear absorption-based phenomena[5]. Fractional calculus can be used to overcome these limitations since it provides different concentration-dependent absorption characteristics effectively[6]. This opens the possibility to the development of subsequent studies to improve these laws with the help of reasonable fractional calculus and clarify the theoretical fundamentals of applying it in various branches of science and research including medicine, material science, and the construction of optimal optical devices.

METHODOLOGY

This paper's research approach addresses a significant issue. We explore the relation of light intensity to photocurrent. A very controlled experiment involves the use of a photocell enclosed in a box to eliminate extraneous light. We change the distance between the photocell and steady-voltage incandescent lamp using a meter ruler. An accurate, highly sensitive microammeter records the photocurrent arising from the photocell at each distance.

This approach guarantees the acquisition of a good dataset for photocurrent as a function of distance. We then analyze this data through the lens of the Beer-Lambert Law, which describes the weakening of light intensity with distance traveled through a material, as expressed by the equation:

$I=I_o e^{-\mu_o x}$

Then to get more refined results, we incorporated fractional calculus into Beer-Lambert Law that offered a more nuanced description of light-matter interactions compared to the integer-order derivatives used in the traditional law.

RESULT AND DISCUSSION:

Our analysis of the data begins with a close examination of the data based on the Beer-Lambert Law experiment.

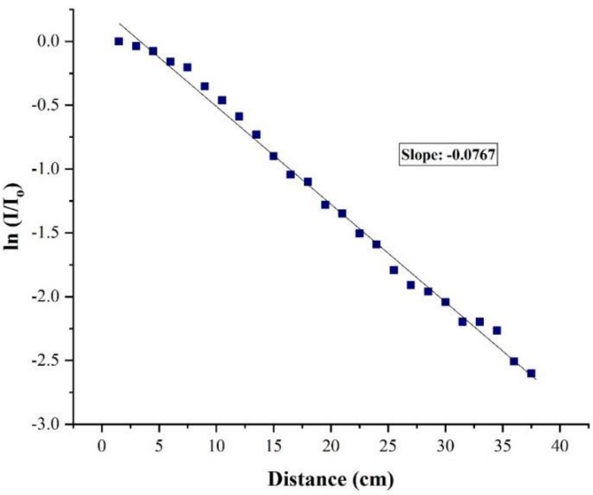

Figure 1 : Beer-Lambert Law Analysis in (I/I_o) vs. Distance

Figure 1 depicts a vital aspect of the experiment: the relationship between the natural logarithm of normalized intensity $\ln(I/I_o)$ and the distance traveled by light. This transformed data, unlike a raw intensity plot, reveals a linear connection, validating the Beer-Lambert Law. The slope of this line offers the effective absorption coefficient (μ_o) of the sample. Interestingly, it provides an $x_{1/2}$value of 9 cm based on this calculation, which deviates from the 13 cm distance which is the experimental value

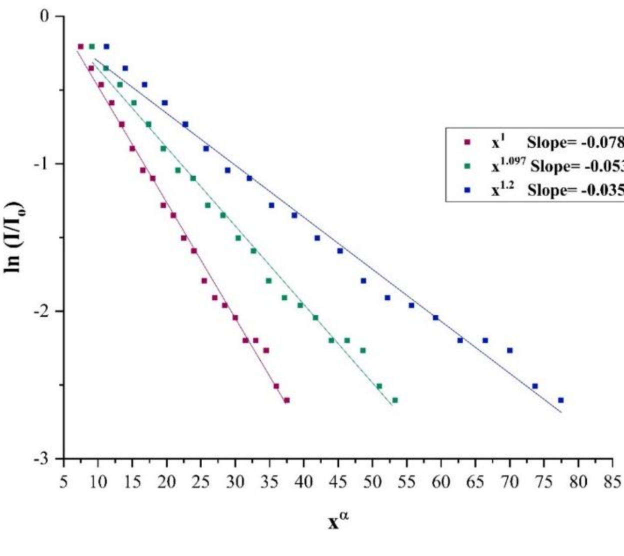

Figure 2 : Refining Light Attenuation Modeling with Fractional Calculus

The most profound discovery is found when alpha is approximately equal to 1. 097. Here, the obtained value of $x_{1/2}$ which is 13. 01 cm agrees quite well with the experimental value of 13 cm. This implies that instead of using the Beer-Lambert Law, fractional calculus with a certain fractional order (alpha) can give a better account of the light loss in this experiment. Figure 2 represents this convergence between the fractional calculus model and the experimental data.

CONCLUSION:

This research journey ventured beyond simply applying fractional calculus; it uncovered fascinating connections with existing models in two domains: light-matter interactions and circuit behavior. In the field of light, fractional calculus offered a more detailed explanation of light absorption in various circumstances, which could give better results than Beer-Lambert Law. This opens the path for the improvements of the solar cell and the material having specific light absorbing characteristics.

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