



## Heat transfer analysis through semi-spherical fin with Nano-fluid flow

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### ABSTRACT :

The objective of the investigation is to analyze the thermal performance of nanofluid flow in different base fluids (water and kerosene) over a fully wet, semi-spherical porous fin, taking into account the impacts of an internal heat source and radiation. Optimal thermal distribution performance was investigated in two distinct base fluids. A Darcy law model is used to create the heat equation. The non-dimensional governing equation solved using the Maple programmer and the Runge Kutta Fehlberg technique's 4th and 5th orders. The computational results obtained under various constraints using two base fluids are graphed and thoroughly examined. It should be observed that convection, radiation and wet parameters reduce thermal performance, whilst internal ambient parameters and heat generation increase it. Water performs better in terms of thermal distribution than kerosene base fluid when titanium alloy is present

**Keywords:** Heat transfer, semi spherical fin, Nano fluid

### 1. Introduction :

Due to the numerous applications of nano-fluids in heat exchangers, electrical cooling systems, solar collector, and biomedicine, to minimize the size of the body, an extended surface known as a fin is attached; this improves heat transfer rate ( $q$ ). Applications like aircraft demand lighter fins with excellent heat transfer capabilities, but high thermal conductivity metals can be costly. Consequently, enhancing ( $q$ ) can be obtained by utilizing compact, cost-effective fins with high ( $q$ ). In general, increasing the temperature difference, improving heat transfer coefficient ( $h_c$ ), and expanding the heat transfer area will enhance the ( $q$ ) transmission. Technological constraints limit the ability to improve the temperature difference, and managing ( $h_c$ ) is more challenging. Therefore, increasing the surface heat transfer area is the most significant way to enhance the heat exchange rate. Employing fins is a highly effective method for greatly expanding the surface heat transfer area of a system Nabati et al., have studied the heat transport characteristics of a moving wet fin using numerical methods[1]. Hatami and Ganji (2014) have studied the interaction of heat and mass transfer in a porous circular fin when fully saturated with liquid [2]. Ghasemi et al. (2014) conducted a thermal analysis of a convective fin, examining the effects of temperature-dependent thermal conductivity ( $K$ ) and heat generation on fin heat transfer [3]. Sobamowo (2017) have studied effect thermal conductivity on fin efficiency by using finite difference method [4]. Gireesha et al. (2020) have studied effect of thermal boundary condition on the effectiveness of fin [5]. Torabi et al. (2012) have studied moving fin with constant velocity by using Differential Transform Method (DTM)[7].

Darvishi et al. (2016) have conducted a numerical study the hyperbolic annular fin with temperature-dependent thermal conductivity[8]. Sowmya et al.,(2020) have studied longitudinal porous fin analyzing the impacts of radiation and convection under wet conditions, [9]. Xu et al. (2014) [9] and Xu (2017) [10] have studied the heat transfer dynamics of thermal non-equilibrium in a solar collector filled in metal foam and in micro heat exchangers containing porous media, respectively. Some of the recent studies by Dogonchi and Ganji (2016), Ma et al.,(2018) Ray et al.(2018) and Turkyilmazoglu.et.al (2018).

### 2. Problem Description :

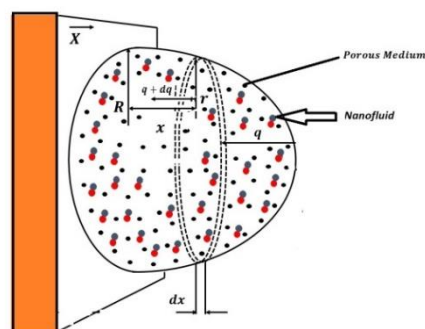


Figure 1: Semi-spherical porous fin[15]

Titanium alloy nanoparticle is immersed in water and kerosene forming base fluids as shown above Figure. 1 and thermo physical properties are shown in Table 1. Fin having a radius  $R$ . There is a dependence of temperature and fin surface heat transfer is through radiative-convective heat transfer and fin generates internal heat. Atmospheric (Ambient) air temperature ( $T_a$ ) serves as a thermal radiation absorber, considering that the exposed surfaces possess emissivity. Surface emissivity ( $\epsilon$ ) is dependent on temperature, but  $h$  is kept be constant.

The preceding energy balance equation can be written as.

$$q - (q + dq) + q^* A - m C_p (T - T_a) - 2\pi r h (1 - \psi) (T - T_a) dx - 2\pi r h_D i_{fg} (1 - \psi) (\omega - \omega_a) dx - 2\pi r \epsilon \sigma f (T^4 - T_a^4) dx \quad (2.1)$$

Where  $q^*$  internal heat generation,  $A$  is cross sectional area of the fin,  $m$  fluid flow of mass,  $C_p$  is specific heat,  $\psi$  is porous term,  $(1 - \psi)$  represents the fraction of the solid area of the fin decreased compared to the pore area needed for convection surface.,  $f$  is the shape factor,  $\epsilon$  is emissivity radiation discharge inside the ambient fluid,  $T$  fin temperature,  $T_a$  is the ambient (air) temperature of the fin.

The varying internal heat generation rate as a function of temperature, given by.

$$q^* = q_0 (1 + \lambda (T - T_a)) \quad (2.2)$$

Mass flow rate through porous material

$$m = \rho v 2\pi r dx \quad (2.3)$$

Darcy's model for Passage velocity given

$$v = \frac{g K \rho \beta (T - T_a)}{\mu_f} \quad (2.4)$$

Substituting equation (2.2)-(2.4) into the equation (2.1) resulted in following equation

$$q - (q + dq) + q_0 (1 + \lambda (T - T_a)) A - \frac{2\pi r (\rho C_p)_{nf} (\rho \beta)_{nf} g K (T - T_a)^2}{\mu_{nf}} - 2\pi r h (1 - \psi) (T - T_a) dx - 2\pi r h_D i_{fg} (1 - \psi) (\omega - \omega_a) dx - 2\pi r \epsilon \sigma f (T^4 - T_a^4) dx \quad (2.5)$$

In addition, Fourier law of conduction is

$$q = -k_{nf} A \frac{dT}{dx} \quad (2.6)$$

Here,  $A$  denotes the cross sectional area of the fin.  $A = \pi r^2$ . Heat transmission is observed to occur at  $x$  and The dependence between two variables  $r$  and  $X$  can be expressed as follows (figure 1).

$$r^2 = R^2 - X^2 \quad (2.7)$$

Substitute equations (2.6)-(2.7) to the equation (2.5)

$$-\frac{d}{dx} \left( -k_{nf} \pi (R^2 - x^2) \frac{dT}{dx} \right) + q_0 (1 + \lambda (T - T_a)) \pi (R^2 - x^2) - \frac{2\pi \sqrt{R^2 - x^2} (\rho C_p)_{nf} (\rho \beta)_{nf} g K (T - T_a)^2}{\mu_{nf}} - 2\pi \sqrt{R^2 - x^2} h (1 - \psi) (T - T_a) - 2\pi \sqrt{R^2 - x^2} h_D i_{fg} (1 - \psi) (\omega - \omega_a) - 2\pi \sqrt{R^2 - x^2} \epsilon \sigma f (T^4 - T_a^4) \quad (2.8)$$

Thermal power is the function (h)

$$h = \left[ \frac{T - T_a}{T_b - T_a} \right]^n = h_p C_p Le^{\frac{2}{3}}$$

(2.9)

Here,  $h_a$  shown as heat transfer coefficient at  $T_a$  and  $n$  is the power law index.

Substituting equation (2.9) into the equation (2.8), we have

$$\begin{aligned} & k_{nf} (R^2 - x^2) \frac{d^2 T}{dx^2} - 2xk_{nf} \frac{dT}{dx} + q_0 (1 + \lambda(T - T_a))(R^2 - x^2) \\ & - \frac{2\sqrt{R^2 - x^2} (\rho C_p)_{nf} (\rho\beta)_{nf} gK (T - T_a)^2}{\mu_{nf}} - 2\sqrt{R^2 - x^2} h_a (1 - \psi) \frac{(T - T_a)^{n+1}}{(T_b - T_a)^n} \\ & - 2\sqrt{R^2 - x^2} \frac{h_a}{C_p Le^{\frac{2}{3}}} \frac{(T - T_a)^{n+1}}{(T_b - T_a)^n} i_{fg} (1 - \psi) b_2 - 2\sqrt{R^2 - x^2} \varepsilon \sigma f (T^4 - T_a^4) \end{aligned} \quad (2.10)$$

Boundary conditions at a Temperature=Constant:

$$T(0) = T_b \quad \text{at } x = 0; \quad \frac{dT}{dx} = 0 \quad \text{at } x = R \quad (2.11)$$

Non-dimensional of the parameters:

$$\begin{aligned} X = \frac{x}{R}, \quad \theta = \frac{T}{T_b}, \quad \theta_a = \frac{T_a}{T_b}, \quad Ng = \frac{q_0 R^2}{T_b k_f}, \quad B = \lambda T_b, \quad Nc = \frac{2(\rho C_p)_f (\rho\beta)_f gKT_b}{k_f \mu_f}, \\ m_0 = \frac{2Rh_a (1 - \psi)}{k_f}, \quad m_1 = \frac{2Rh_a i_{fg} b_2 (1 - \psi)}{C_p Le^{\frac{2}{3}} k_f}, \quad Nr = \frac{2R\varepsilon \sigma f T_b^3}{k_f}, \\ (\omega - \omega_a) = b_2 (T - T_a), \quad m_2 = m_0 + m_1. \end{aligned} \quad (2.12)$$

Substituting equation (2.12) into the equation (2.10), we get

$$\begin{aligned} & \frac{d^2 \theta}{dX^2} - \frac{2X}{1 - X^2} \frac{d\theta}{dX} + Ng (1 + B(\theta - \theta_a)) \frac{k_f}{k_{nf}} - \frac{Nc}{\sqrt{1 - X^2}} (\theta - \theta_a)^2 \\ & - \frac{(\rho C_p)_{nf} (\rho\beta)_{nf} \mu_f k_f}{(\rho C_p)_f (\rho\beta)_f \mu_{nf} k_{nf}} - \frac{m_2}{\sqrt{1 - X^2}} \frac{(\theta - \theta_a)^{n+1} k_f}{(1 - \theta_a)^n k_{nf}} - \frac{Nr}{\sqrt{1 - X^2}} (\theta^4 - \theta_a^4) \frac{k_f}{k_{nf}} \end{aligned} \quad (2.13)$$

and the following (2.11) will becomes

$$\theta(X) = 1 \quad \text{at } X = 0; \quad \frac{d\theta}{dX} = 0 \quad \text{at } X = 1 \quad (2.14)$$

Where,  $Ng$  is the generation parameter,  $B$  non-dimensional internal heat generation parameter,  $Nc$  is convective parameter,  $m_2$  is wet porous parameter,  $n$  is power law index,  $Nr$  is radiative parameter,  $\theta_a$  is the ambient non-dimensional temperature and  $\theta$  is the non-dimensional temperature. The non-dimensional governing equation solved using the Maple programmer and the Runge Kutta Fehlberg technique's 4<sup>th</sup> and 5<sup>th</sup> orders.

The thermo physical expression of nanofluid is expressed mathematically as  $\rho_{nf} = (\phi\rho_s + (1-\phi)\rho_f)$

$$(\rho C_p)_{nf} = (\phi(\rho C_p)_s + (1-\phi)(\rho C_p)_f)$$

$$(\rho\beta)_{nf} = [(1-\phi)(\rho\beta)_f + \phi(\rho\beta)_s]$$

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

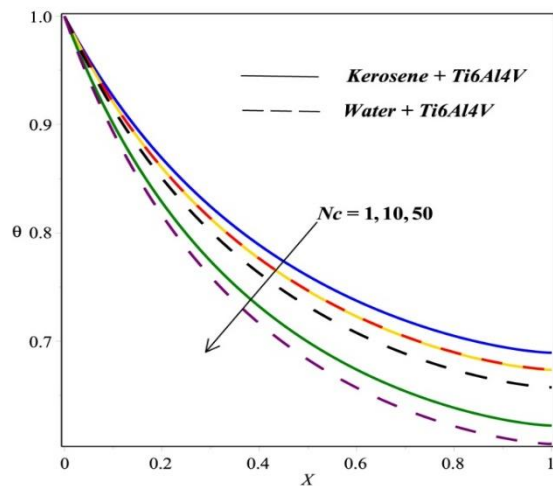
$$k_{bf} = \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} k_f$$

Here  $\phi$  represents solid volume fractions and  $s,nf$ , indicates solid nanoparticles of Ti6Al4V viscous fluid, nanofluid respectively. The thermo physical properties represent in below table 1

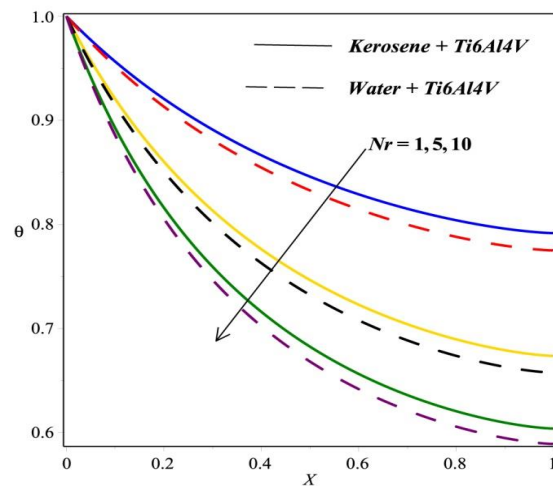
**Table.1: Thermo-physical Properties of the fluid and Nano-fluid**

Material	Kerosene	Water	Ti6Al4V
P (kg-m <sup>3</sup> )	780	997.1	4420
C <sub>p</sub> (J-kg <sup>-1</sup> K <sup>-1</sup> )	2090	4179	0.56
K(W-m <sup>-1</sup> ,K <sup>-1</sup> )	0.149	0.613	7.2
$\beta$ ( $\times 10^{-5}$ ) K <sup>-1</sup>	99	21	0.86

### 3. Results and Discussions :



**Fig (2) Impact of Convection parameter on temperature filed**



**Fig (3) Impact of Radiative parameter on temperature filed**

The varying  $N_c$ =radiative parameter,  $N_g$ =heat generation factor,  $N_g$ =internal, ambient temperature  $\theta_a$  wet porous parameter  $m_2$ , were graphically showed and by altering each parameter by keeping others constant. Figure 2 shows the variation in  $N_c$  on heat transfer. An increase in the convective parameter ( $N_c$ ) leads to a significant decrease in the temperature distribution. This is due to variation in Rayleigh (Ra) and Darcy (Da) number and direct impact of buoyance of the system. This is due of higher (Q) from fin surfaces to environment. The variation of radiative factor ( $N_r$ ) on the (Q) for nanofluid shown in figure 3. Increase in radiative factors reduces the fin (Q). This is because of enhancement of radiation effect.

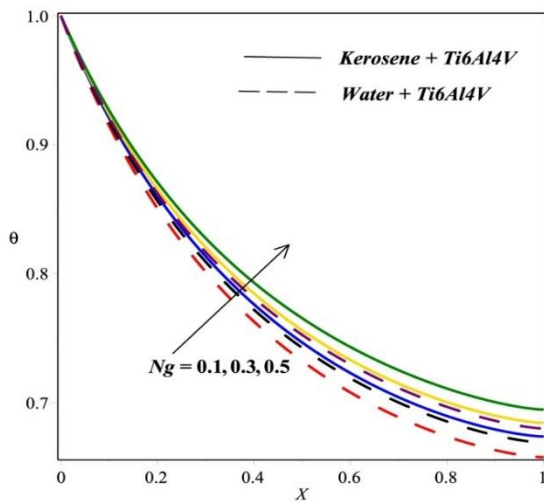


Fig (4) Impact of Internal heat on temperature filed

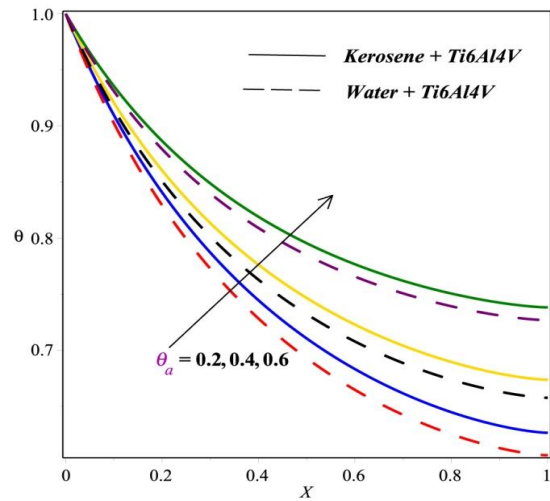


Fig (5) Impact of ambient temperature on temperature filed

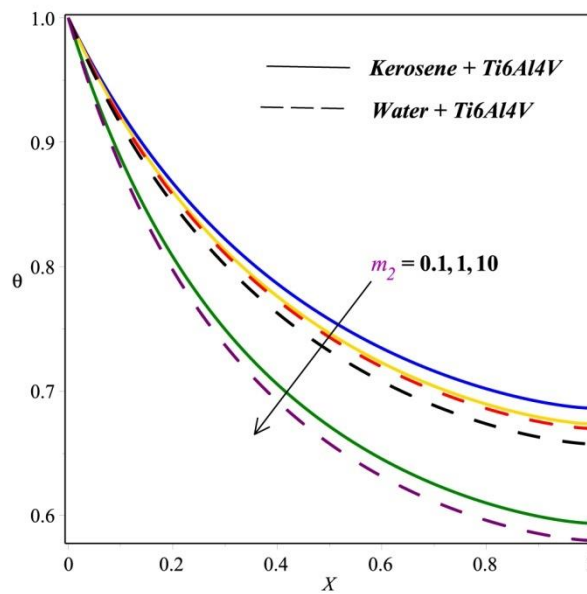


Fig (6) Impact of Wet porous parameter on temperature filed

Figure 4 shows the variation of  $N_g$  on thermal characteristics. Thermal profile increases as heat generation parameter increases. This is because of to development of internal heat inside the fin, improves the temperature rise of the surface area of fin. This resulting in decrease in thermal distribution. Figure 5 variation of ambient temperature  $\theta_a$  on the heat transfer of fin. As the value of  $\theta_a$  improves the fin temperature. This due to heat transfer of fin decrease as surrounding temperature fluid rises. The variation of wet porous factor  $m_2$  on temperature distribution of the fin as shown in Figure 6. The fin's moist characteristics helps remove additional heat from the atmosphere, while its permeability permits fluid to pass through. Therefore, the wet porous nature of the fin aids in temperature distribution and benefits the fin's surface.. As a result, it is clearly established that as this parameter increases, the thermal profile of the fin decreases, leading to an improvement in the fin's cooling effect

#### 4. Conclusion :

Thermal properties of natural convection and radiation effects have been investigated using a semispherical permeable fin which is completely saturated in a nanofluid. The temperature field is greatly affected by natural convection, radiation, and the moist environment around the porous fin, with improvements attributed to the wet porous parameter, internal heat generation, and generation number. Additionally, increases in the power law index, ambient temperature, and solid volume fraction enhance thermal behaviour, especially in nanofluids compared to regular nanofluids.

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