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Awavelet Methodologies for Solving Reaction-Diffusion Complications in Science

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ABSTRACT

The wavelet method is a comparatively recent innovation in applied mathematics that has garnered considerable interest in the field of engineering research. This interest is largely due to the method's ability to analyze a wide range of dynamic phenomena through the use of waves. The wavelet method has proven to be a versatile tool in various contexts, ranging from providing solutions to differential equations to capturing nonlinearity in data distribution. In the domain of scientific phenomena, which are typically modeled through linear or nonlinear differential equations, wavelets have emerged as an effective solution. Studies have demonstrated the wavelet method's efficiency and potency in solving a broad array of linear and nonlinear reaction-diffusion equations. This paper discusses potential future directions and scope for developing wavelet algorithms to solve reaction-diffusion equations.

Keywords and phrases: wavelet method; nonlinear partial differential equations; reaction-diffusion system; reaction-diffusion equations.

Introduction

Nonlinear partial differential equations of the reaction-diffusion type emerge in numerous applications across various fields, such as physical sciences, developmental biology, ecology, physiology, and finance, among others. These systems typically comprise multiple parabolic partial differential equations that are coupled together. In the context of population biology, the reaction term represents growth, while the diffusion term accounts for migration. Reaction-diffusion systems also serve as models for pattern formation in morphogenesis, predator-prey and other ecological systems, conduction in nerves, epidemics, carbon monoxide poisoning, and oscillating chemical reactions, and many others.

Reaction-diffusion systems are typically composed of pairwise coupled parabolic partial differential equations, and they often play a significant role in various fields, such as morphogenesis, predator-prey and ecological systems, nerve conduction, epidemics, carbon monoxide poisoning, oscillating chemical reactions, pulse splitting and shedding, reactions and competitions in excitable systems, and stability issues. The most basic form of reaction-diffusion equations (RDEs) is given by

$$V_t = \frac{\partial V}{\partial t} = D\frac{\partial^2 V}{\partial x^2} + f(V),$$

where D is the diffusion coefficient, $U = U(x, t)$ is the vector of dependent variables, and $f(U)$ is a nonlinear vector-valued value of u (the response term). Any interaction between U's components generates the reaction term. The parameter u could be a vector of competition, symbiosis, or predator-prey interactions. Diffusion terms may be applied to either molecular diffusion or "random" individual movement within a population [1]. A reaction-diffusion-convection system identical to the system provided can be developed by

$$\frac{\partial U}{\partial t} = U_t = f(U) + D\frac{\partial^2 U}{\partial x^2} + C\frac{\partial U}{\partial x}.$$

The convection coefficient is represented by C. It is well known that the numerical handling of the reaction terms affects the numerical outcomes of reaction-diffusion systems.

Derivation of reaction-diffusion equations (RDEs)

The functioning of diffusion models is characterized by the migration of numerous individuals within a specific environment or medium. These particles typically occupy a particular area.

Open set of *Rn* (the *n*th dimensional space with the Cartesian coordinate system), which we refer to as *X*, with $n = 1.$ Due to the diverse nature of the environment, the diffusion coefficient $D(x)$ is typically not a constant. However, we can assume that $D(x) = D$ when the region is roughly homogeneous, which simplifies the previous equation to

$$\frac{\partial q}{\partial t} = D\Delta q + f(t, x, q),$$

$$\Delta q = \operatorname{div}(\nabla q) = \sum_{i=1}^{n} \frac{\partial^2 q}{\partial x_i}$$

where is the Laplacian operator.

Importance of reaction-diffusion problems in engineering

Civil engineering

Recently, the investigation of the aggregate alkali reaction in fluid leaching procedures has garnered significant interest in the assessment of concrete dams in civil engineering. The numerical results obtained here are aimed at enhancing comprehension of the model [2].

Chemical engineering

The solution of reaction-diffusion systems presents a formidable challenge due to the time-dependent, highly nonlinear, and coupled nature of the equations. Therefore, the development of efficient solution techniques that can characterize a wide range of parameter combinations is essential. Thus the ultimate goal is the design, optimization, or real-time control of the system of interest.

Reaction-diffusion equations are commonly encountered in both chemical and biological contexts. These equations can give rise to a wide variety of structures, including pattern formation and traveling waves. In groundwater aquifers, reaction-diffusion equations govern the kinetic absorption and growth of biofilm-forming microbes, as well as their transport. The equations may also contain advective terms [3]. Similarly, in hydrology, these types of equations are used to model the transport and fate of absorbing contaminants and microbe-nutrient systems in groundwater. Hariharan et al. [4] introduced the Haar wavelet method for modeling methylene blue adsorption onto plant leaf powder using film-pore diffusion.

Mechanical engineering

A formal presentation of a simplified kinematic description for a spiral formed in a two-component reaction-diffusion medium through the application of a free-boundary approach is provided [5]. The potential energy created by an external force due to deformation is propagated among mass points by the principle of reaction-diffusion. The novelty of this methodology lies in the establishment of reaction-diffusion techniques to describe the potential energy of deformation and extrapolate the internal forces of a deformed object. The reaction-diffusion model is developed to facilitate the natural propagation of the energy generated by the external force [6].

Biological engineering

The transport of oxygen from the blood plasma to the tissues in the skeletal muscle or brain is a topic of great importance in modern times. By using reaction-diffusion equations (RDEs), it is possible to answer various questions such as the factors that affect the oxygen supply to tissue cell respiration, what happens when oxygen is inhaled at low concentrations, and the influence of axial and radial diffusion of oxygen in blood and oxygen diffusivity in tissue. Recent research has shown that the classical diffusion equation is insufficient to model many real-world situations, where particles spread faster than predicted and exhibit significant asymmetry, which is referred to as anomalous diffusion [7]. An example of such a situation is the modeling of gliomas growth and their mechanical impact on the surrounding brain tissue. This requires a strongly coupled system of nonlinear partial differential equations to represent the tumor growth and a piecewise linearly elastic material to represent the background tissue [8].

In the past, various researchers have made significant contributions to the field of reaction-diffusion models in population genetics [9].

Reaction-diffusion modeling

The significant advancements in nonlinear analysis and the theory of dynamical systems over the past 30 years have enabled a comprehensive analysis of many reaction-diffusion models. These advancements encompass developments in bifurcation theory, the formulation of reaction-diffusion models as dynamical systems, the creation of mathematical theories of persistence or permanence in dynamical systems, and the systematic incorporation of ideas based on monotonicity into the theory of dynamical systems [10].

To classify reaction-diffusion models as dynamical systems, it is necessary to define suitable state spaces of functions and determine how the models operate on them. Although we may not be able to solve reaction-diffusion models explicitly, as is also the case with many nonlinear systems of

ordinary differential equations, we can use methods from the theory of dynamical systems to determine when a model predicts persistence and when it predicts extinction, as well as to describe some features of its dynamics. However, there are new technical challenges that arise when formulating reaction-diffusion models as dynamical systems [11].

In recent years, a potent analytical approach for nonlinear problems, known as the homotopy analysis method (HAM), has been introduced by [12]. This method offers a practical means of controlling the convergence of approximation series and adjusting convergence regions as needed. FitzHugh-Nagumo (FN) equation has been tackled using the HAM. An analytical and approximate method for the Nagumo telegraph reaction-diffusion equation is proposed in [13], while [14] provided explicit solutions for Fisher's equation for a specific wave speed. The sinc-collocation method is employed in [15] to solve Fisher's equation. The finite difference method is applied in [11] to solve reaction-diffusion equations in modeling amperometric biosensors in batch and flow injection analysis. A variational iteration method is developed based on He's method for solving the Huxley equation and the generalized Burgers-Huxley equation. The article [13] carried out multi-resolution simulations of reaction-diffusion systems with strong degeneracy.

A significant new tool that allows for the natural treatment of quantities with several length scales is the ripple. In this review composition, we demonstrate the usage of ripples to break response-prolixity equations, which are difficult to solve with other numerical techniques due to their widely changing length scales. The Daubechies ripples are applied in most studies. Since these ripples are orthogonal, there is a need to solve the differential equation using the Haar wavelet approach. The strategy with much fewer degrees of freedom and a shorter CPU time produces better outcomes than classical methods.

When solving differential, integral, and integro-differential equations, the Haar wavelet technique shows a number of beneficial characteristics:

(i) Fast transformation times with extremely high accuracy and the ability to develop algorithms quickly in comparison to other established techniques.

(ii) The low number of significant wavelet coefficients and the sparsity of the transform matrices contribute to the low cost and simplicity of the computation.

The method's ease of use in solving boundary value problems stems from its automatic handling of boundary conditions.

In real-world applications, fewer words mean faster calculations and less memory usage, whereas more terms mean better resolution precision. As a result, the analysis has taken into account a trade-off between computation speed, memory savings, and solution accuracy. With increasing values of m ($\rightleftarrows\rightleftarrows$that is, $m = 16,\ m = 32,\ m = 64,\ m = 128,\ m = 256\rightleftarrows\rightleftarrows$), the numerical solution approaches the actual solution more and more. When compared to a precise answer, the Haar wavelet approach is pretty reasonable, according to the results. These calculations show that this method has a very high accuracy:

$$H(t) = \left\{1,\ \text{for } t \in \left[\left[\frac{k}{m}, \frac{k+0.5}{m}\right]\right]\right.$$

$$= \left\{-1,\ \text{for } t \in \left[\left[\frac{k+0.5}{m}\ \frac{k+1}{m}\right]\right]\right.$$

$$= \{0,\ \text{elsewhere.} \tag{1}$$

The translation parameter is $k = 0,\ 1,\ 2,\ ...,\ m-1,$ and the wavelet level is indicated by an integer. J is the maximum resolution level. In the case of minimal values, the index i is computed using the formula $i = m + k + 1.$ We have $i = 2$ with $m = 1,\ k = 0,$ and its maximum value is i $= 2J + 1.$ The scaling function for which $h_1 = 1$ in $[0,\ 1]$ corresponds to the value $i = 1,$ is assumed. After discretizing the Haar function $h_i(t)$ and defining the collocation points $t_1 = (1 - 0.5)/2M,\quad (l = 1,\ 2,\ ...,\ 2M),$ we obtain the coefficient matrix $H(i,\ 1) = (h_i(t_1)),$ which has the dimension 2M 2M. Integration P's operational matrix, which is a 2M

$$PH_{il} = \int_0^{t_1} h(t)_i(t)\,dt, \tag{2}$$

$$QH_{il} = \int_0^{t_1} dt \int_0^{t_1} h(t)_i(t)\,dt. \tag{3}$$

The elements of the matrices H, P and Q can be evaluated according to (2) and (3).

$$H_2 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad P_2 = \frac{1}{4}\begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix}, \tag{4}$$

$$H_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \end{pmatrix}, \tag{5}$$

$$P_4 = \frac{1}{16}\begin{pmatrix} 8 & -4 & -2 & -2 \\ 4 & 0 & -2 & 2 \\ 1 & 1 & 0 & 0 \end{pmatrix}, \tag{6}$$

$$H_4^{-1} = \begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 1 & -2 & 0 \\ 1 & -1 & 0 & 2 \end{pmatrix}, \tag{7}$$

$$P_8 = \frac{1}{64}\begin{pmatrix} 32 & -16 & -8 & -8 & -4 & -4 & -4 & -4 \\ 16 & 0 & -8 & 8 & -4 & -4 & 4 & 4 \\ 4 & 4 & 0 & 0 & -4 & 4 & 0 & 0 \end{pmatrix}. \tag{8}$$

Table 1. Comparison between the exact and HWM for Example 8.3.1

X	T	U_{exat}	U_{HWM}
0.25	0.5	0.81839	0.81855
	1.0	0.98292	0.98305
	2.0	0.99988	0.99999
	5.0	1.00000	1.00000
0.50	0.5	0.77590	0.77602
	1.0	0.97815	0.97824
	2.0	0.99985	0.99996
	5.0	1.00000	1.0000
0.75	0.5	0.72582	0.72595
	1.0	0.92207	0.92221
	2.0	0.99981	0.99993
	5.0	1.00000	1.00000

$$u(x,\, t) = \frac{1}{1 + (1 + e^{x-5t})^2}.$$

This process is started with

$$u(x_1,\, t_s) = \frac{1}{1 + (1 + e^{x})^2},$$

$$u'(x_1,\, t_s) = \frac{-2e^{x}}{1 + (1 + e^{x})^3},$$

$$u''(x_1,\, t_s) = -2\left[\frac{e^{x} - 2e^{2x}}{1 + (1 + e^{x})^2}\right].$$

It is worth noting that applying the proposed scheme as above for the Fisher's equation

$$u_t = u_{xx} + au(1-u),$$

solution $u(x, t)$ can be compared with the Haar solution. The Haar wavelet scheme is given by

$$U''(x_l, t_{s+1}) = u''(x_l, t_{s+1}) + u(x_l, t_{s+1})(1 - u(x_l, t_{s+1})).$$

From the above formula, the wavelet coefficients can be successively calculated. This process is started with

$$u(x_l, t_s) = \frac{1}{1 + (1 + e^x)^2},$$

$$u'(x_l, t_s) = \frac{-2e^x}{1 + (1 + e^x)^3},$$

$$u''(x_l, t_s) = -2\left[\frac{e^x - 2e^{2x}}{1 + (1 + e^x)^2}\right].$$

Results

Our results can be compared with Wazwaz and Gorguis results [15]. Good agreement with the exact solution is observed (see Table 1).

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