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Modeling of Magnetohydrodynamic (MHD) Waves Propagation in the Solar Corona

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ABSTRACT:

Modeling the propagation of magnetohydrodynamic waves in the outermost layer of the sun's atmosphere is to better understand the sun's complex and dynamic behaviour. Investigating the dynamic behaviour of the solar corona, an ideal magnetohydrodynamics equations were linearized under certain conditions to obtain a dispersion relation. In solving this dispersion relation numerically, three wave modes were obtained which are; Alfven waves, fast magnetosonic waves and the slow magnetosonic waves. Further analysis also gave some fundamental speeds of the solar corona, acoustic speed which was found to be 151km/s while the Alfven speed varies from 89.05km/s – 890.50km/s depending on the values of the magnetic field strength . The result confirms that the Alfven waves depends only on the Alfven speed and the angle between the wave vector and the magnetic field while the fast and slow magnetosonic waves depends on the acoustic speed, Alfven speed and the propagation angle. This result also depicts that as the propagation angle increases the phase velocity of the fast magnetosonic waves also increases, while the phase velocity of the Alfven waves and the phase velocity of the slow magnetosonic waves decreases, confirming that the Alfven waves is an intermediate waveform between the fast magnetosonic waves and the slow magnetosonic waves.

Keywords: Sun, solar Corona, Magnetohydrodynamic waves,

1. INTRODUCTION

The sun is at the center of the solar system and radiates energy generated by nuclear reaction outwardly into the earth and other planets (Gudiksen 2004, Tomczyk et al 2016), the sun is structured in a way that it is separated by layers into inner and outer layers. The inner layer consist of the core, radiative and convective zones, while the outer layer consist of the photosphere, chromosphere, transition region and the corona (Priest 2012, Bittencourt 2013, Wiegelmann et al 2014, Roberts 2019).

The solar corona is one of the outer layers of the sun which has been identified as the outermost layer of the sun's atmosphere, (Gabriel 1988, Nakariakov 2020), having a temperature of the order of one to two million Kelvin (Hayashi 2005, Asara et al 2022). The corona lies above the chromosphere and extend millions of kilometers into the outer space and can only be observed during a total eclipse or with the use of a coronagraphs (Sakurai 2017, Mehta 2022).

Solar corona is identified as a high temperature plasma containing dynamical and extremely complex magnetic structures (Mehta 2022) and the plasma is defined as the fourth state of matter along side with solid, liquid and gases (Sturrock 1994, Bittencourt 2013, Jovanovic 2021) consisting mainly of hydrogen which is approximately 90%, helium 10% and trace amount of other elements 0.1%, which are held together by gravity (Laing 1996, Priest 2012).

Magnetohydrodynamic (MHD) waves in the solar corona are essential for understanding various activities in the solar atmosphere. The solar corona region is highly dynamic and complex, where magnetic fields plays a significant role in shaping its behaviour (Solanki et al 2006, Wedemeyer-Bohn et al 2009, Jiang et al 2021, Chen et al 2022). The propagation of MHD waves in the solar corona has been recognized as an important signature for many dynamical processes (Roberts 2000, Wu et al 2005), related literature have confirmed that there are several types of MHD waves that propagates in the solar corona, which includes Alfven waves, fast and slow magnetosonic waves (Kudoh and Shibata 1999, Arregui 2015, Morton et al 2015) and investigating the behaviours of this MHD waves propagation in the solar corona is the aim of this research.

2. Methodology

2.1 Governing Equations

In studying the behaviours of the plasma and the propagation of magnetohydrodynamic waves in the solar corona, the fluid dynamics equations and electromagnetism equations (Maxwell's equations) will be used in their ideal forms as stated below

Continuity equation

$$\frac{\partial\rho}{\partial t}+\rho(\vec{\nabla}.\vec{V})=0 \qquad (2.1)$$

Momentum equation

$$\rho\frac{\partial\vec{V}}{\partial t}+\rho(\vec{V}.\vec{\nabla})\vec{V}=\frac{1}{\mu_0}\vec{B}X\left(\vec{\nabla}\,X\,\vec{B}\right)-\vec{\nabla}P \qquad (2.2)$$

Magnetic induction equation

$$\frac{\partial\vec{B}}{\partial t}=\vec{\nabla}\,X\,(\vec{V}\,X\,\vec{B}) \qquad (2.3)$$

No magnetic monopoles

$$\vec{\nabla}.\vec{B}=0 \qquad (2.4)$$

Energy equation

$$\frac{\partial P}{\partial t}-\frac{\gamma P\partial\rho}{\rho\;\partial t}=0 \qquad (2.5)$$

2.2 Equilibrium state

For static equilibrium, where there is no motion $\overrightarrow{V_0}=0$ and $\frac{\partial}{\partial t}=0$. Putting these in the ideal MHD equations yields,

$$\vec{\nabla}P_0-\frac{1}{\mu_0}\overrightarrow{B_0}X\left(\vec{\nabla}\,X\,\overrightarrow{B_0}\right)=0 \qquad (2.6)$$

Applying triple vector product to Equ. (2.6) yields,

$$-\vec{\nabla}P_T+\frac{1}{\mu_0}\left(\overrightarrow{B_0}.\nabla\right)\overrightarrow{B_0}=0 \qquad (2.7)$$

where the term P_T of Equ. (2.7) is the total pressure consisting of two terms, the thermodynamic (gas) pressure P_0, and the magnetic pressure $\frac{\overrightarrow{B_0^2}}{2\mu_0}$, and the other term is the magnetic tension. Now, the plasma- β is given as the ratio of the plasma pressure to the magnetic pressure. That is

$$\beta=\frac{2\mu_0P_0}{\overrightarrow{B_0^2}} \qquad (2.8)$$

2.3 Perturbation and Linearization

Now we consider a little deviation of the physical variables of the medium from their equilibrium values and write them as a sum of the equilibrium and perturbation values and ignoring terms of higher order

$$\rho=\rho_0+\rho_1\;,\vec{B}=\overrightarrow{B_0}+\overrightarrow{B_1},P=P_0+P_1\;,\vec{V}=\overrightarrow{V_1} \qquad (2.9)$$

where the symbols "0" and "1" denotes the stable and perturbed values. In quasi-linear approximation the perturbed values are taken to be insignificant in comparison with the equilibrium values ie, $\rho_1\ll\rho_0$, $\vec{B}_1\ll\vec{B}_0$, $P_1\ll P_0$, $\vec{V}_1$ is also assumed to be small (Pekunlu et al 2001).

Putting Equ. (2.9) into Eqs. (2.1) – (2.5) and ignoring the products of scalars and vectors give the perturbed continuity, momentum, induction, magnetic monopole and energy equations as

$$\frac{\partial\rho_1}{\partial t}+\rho_0(\nabla.\overrightarrow{V_1})=0 \qquad (2.10)$$

$$\rho_0\frac{\partial\vec{V}_1}{\partial t}+\vec{\nabla}P_1=\frac{1}{\mu_0}\vec{B}_0\,X\,(\vec{\nabla}\,X\,\vec{B}_1) \qquad (2.11)$$

$$\frac{\partial\vec{B}_1}{\partial t}=\vec{\nabla}\,X\,(\vec{V}_1\,X\,\vec{B}_0) \qquad (2.12)$$

$$\vec{\nabla}.\overrightarrow{B_1}=0 \qquad (2.13)$$

$$\frac{\partial P_1}{\partial t}-\frac{\gamma P_0}{\rho_0}\frac{\partial\rho_1}{\partial t}=0 \qquad (2.14)$$

2.4 Dispersion Relation

Assuming the equilibrium magnetic field $\overrightarrow{B_0}$ be in x-z plane

$$\overrightarrow{B_0} = B_0 \sin\theta\, \widehat{e_x} + B_0 \cos\theta\, \widehat{e_z} \qquad (2.15)$$

where θ is the angle between the magnetic field and the z-axis. Now considering a plane waves, propagating along $\widehat{e_z}$, so that all perturbed quantities are proportional to $\exp(ik.z - i\omega t)$ (2.16)

Putting Eqs.(2.15) and (2.16) into Eqs. (2.10 – 2.14) and using the fact that $\frac{\partial}{\partial t} = -i\omega$ and $\nabla = ik$ yields the normal mode equations as;

$$\omega\rho_1 - k\rho_0 V_{1z} = 0 \qquad (2.17)$$

$$\omega\rho_0 V_{1x} + \frac{kB_0 \cos\theta B_{1x}}{\mu_0} = 0 \qquad (2.18)$$

$$\omega\rho_0 V_{1y} + \frac{kB_0 \cos\theta B_{1y}}{\mu_0} = 0 \qquad (2.19)$$

$$\omega\rho_0 V_{1z} - kP_1 - \frac{k\,B_0 \sin\theta B_{1x}}{\mu_0} = 0 \qquad (2.20)$$

$$\omega B_{1x} + kB_0 \cos\theta\, V_{1x} - kB_0 \sin\theta\, V_{1z} = 0 \qquad (2.21)$$

$$\omega B_{1y} + kB_0 \cos\theta\, V_{1y} = 0 \qquad (2.22)$$

$$\omega B_{1z} = 0 \qquad (2.23)$$

$$P_1 = C_s^2 \rho_1 \qquad (2.24)$$

The acoustic speed is given by $C_s = \sqrt{\frac{\gamma P_0}{\rho_0}}$ (2.25)

2.4.1 Alfven Waves

Considering Equ (2.19) and Equ (2.22) containing the variables variables V_{1y} and B_{1y} give the Alfven waves and also the phase velocity

$$\omega = \pm k V_A \cos\theta \qquad (2.26)$$

And the corresponding phase velocity V_p^A is given by

$$V_p^A = \frac{\omega}{k} = V_A \cos\theta \qquad (2.27)$$

$V_A = \frac{B_0}{\sqrt{\mu_0 \rho_0}}$ is the Alfven speed (2.28)

2.4.2 Magneto-acoustic Waves

Taking into account the variables B_{1x}, B_{1z} and P_1, eliminating them from Equations (2.17), (2.18), (2.20), (2.21), (2.23) and (2.24) give the dispersion relation of the magnetosonic waves and its solution gives the fast magnetosonic waves that is the positive sign while the negative sign is the slow magnetosonic waves.

$$\omega^4 - A\omega^2 + B = 0 \qquad (2.29)$$

where

$$A = \mathrm{k}^2(\mathrm{V}_\mathrm{A}^2 + C_s^2) \qquad (2.30)$$

$$B = k^4 \mathrm{V}_\mathrm{A}^2 C_s^2 \mathrm{Cos}^2\theta \qquad (2.31)$$

$$\omega = k\sqrt{\frac{(\mathrm{V}_\mathrm{A}^2 + C_s^2) \pm \sqrt{(\mathrm{V}_\mathrm{A}^2 + C_s^2)^2 - 4\mathrm{V}_\mathrm{A}^2 C_s^2 \mathrm{Cos}^2\theta}}{2}} \qquad (2.32)$$

3. Results

This section focuses on the presentation of results obtained from a detailed analysis of the data. The results are presented both in graphical and tabular format.

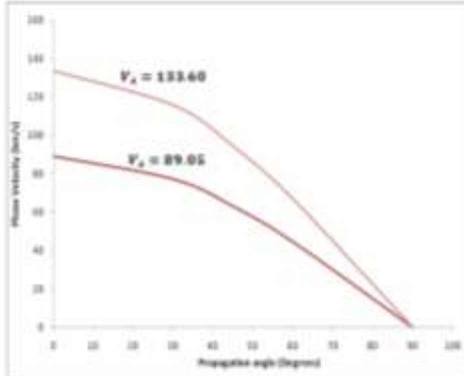

Fig 3.1: Phase Velocity against Propagation Angle of Alfven Waves for High Plasma Beta

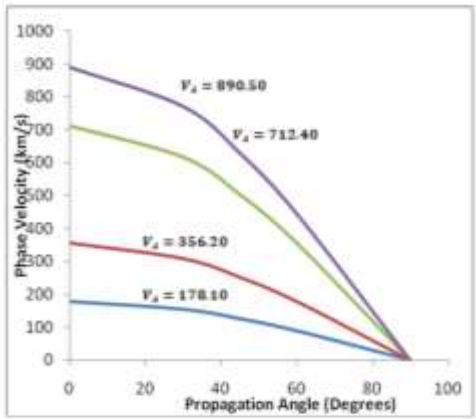

Fig 3.2: Phase Velocity against Propagation Angle of Alfven Waves for Low Plasma Beta

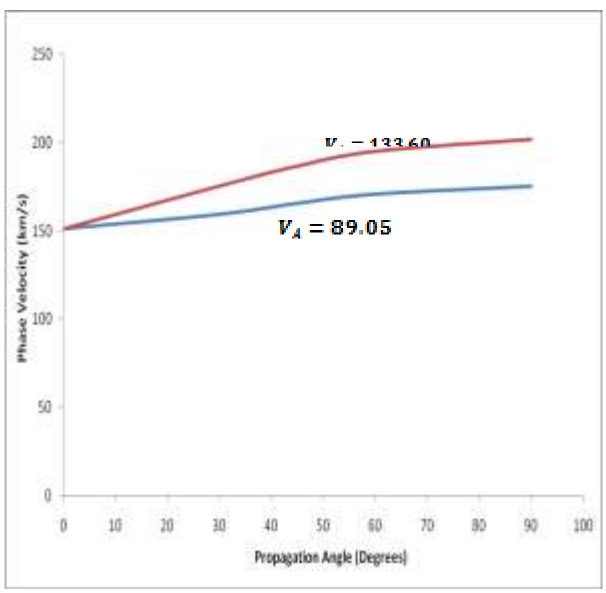

Fig 3.3: Phase Velocity against Propagation Angle of Fast Magnetoacoustic Waves for High Plasma Beta

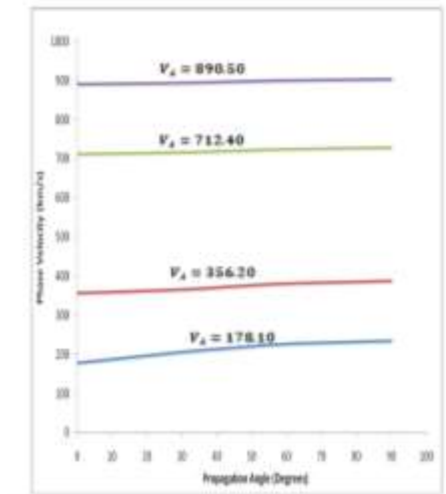

Fig 3.4: Phase Velocity against Propagation Angle of Fast Magnetoacoustic Waves for Low Plasma Beta

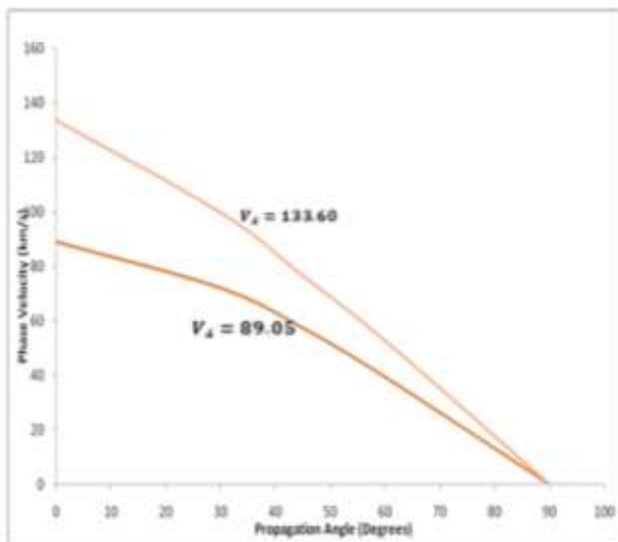

Fig 3.5: Phase Velocity against Propagation Angle of Slow Magnetoacoustic Waves for High Plasma Beta

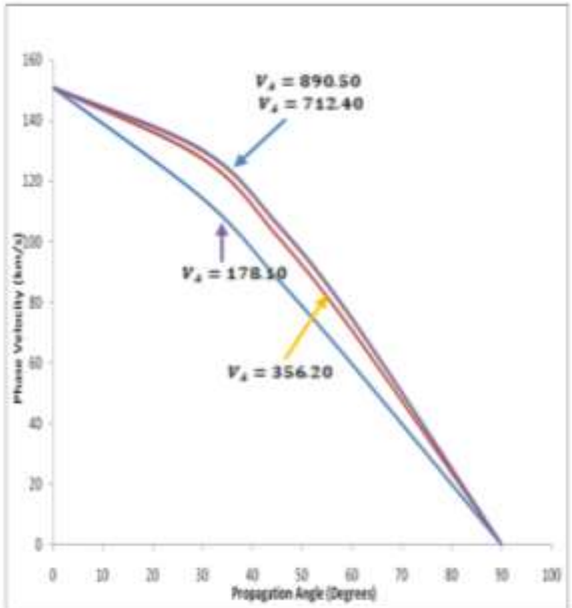

Fig 3.6: Phase Velocity against Propagation Angle of Slow Magnetoacoustic Waves for Low Plasma Beta

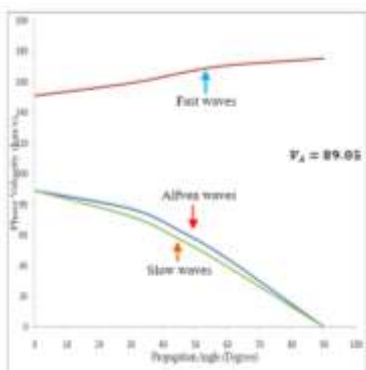

Fig 3.7: Phase Velocity against Propagation Angle of Alfven, Fast and Slow Waves for High Plasma Beta

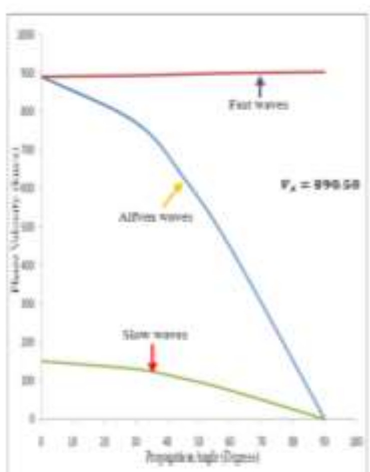

Fig 3.8: Phase Velocity against Propagation Angle of Alfven, Fast and Slow Waves for Low Plasma Beta

Table 3.2 Phase Velocities of Fast Magnetoacoustic Waves for Various Values of Alfven Speed

Alfven Speed, V_A (km/s)	Acoustic Speed, C_s (km/s)	Propagation Angle, θ (Degrees)	Phase Velocity, V_p^f (km/s)
89.05	151	0	151
		30	159.3
		45	165.6
		60	170.8
		90	175.3
133.60	151	0	151.1
		30	175.2
		45	186.5
		60	194.8
		90	201.6
151.40	151	0	151.4
		30	185.2
		45	197.5
		60	206.5
		90	213.8
178.10	151	0	178.0
		30	203.5
		45	216.3
		60	225.8
		90	233.5
356.20	151	0	356.2
		30	265.3
		45	373.2
		60	380.4
		90	386.9
712.40	151	0	712.4
		30	716.5
		45	720.5
		60	724.4
		90	728.2
890.50	151	0	890.5
		30	893.8
		45	897.0
		60	900.1
		90	903.2

Table 3.3 Phase Velocities of Slow Magnetoacoustic Waves for Various Values of Alfven Speed

Alfven Speed, V_A (km/s)	Acoustic Speed, C_s (km/s)	Propagation Angle, θ (Degrees)	Phase Velocity, V_p^S (km/s)
89.05	151	0	89.1
		30	72.1
		45	57.4
		60	39.4

		90	0
133.60	151	0	133.6
		30	99.7
		45	76.5
		60	32.7
		90	0
151.40	151	0	151.0
		30	106.9
		45	81.8
		60	55.3
		90	0
178.10	151	0	151.0
		30	114.4
		45	87.9
		60	59.6
		90	0
356.20	151	0	151.0
		30	127.5
		45	101.9
		60	70.7
		90	0
712.40	151	0	151.1
		30	130.0
		45	105.6
		60	74.2
		90	0
890.50	151	0	151.0
		30	130.3
		45	106.0
		60	74.7
		90	0

4. Discussion

Magnetohydrodynamic wave modes are propagating disturbances found in electrically conducting fluids permeated by magnetic fields whose magnetic tension provides a restoring force moving across field lines. The following MHD wave modes were identified

4.1 Alfven Waves

The behaviour of the Alfven waves indicates that as the propagation angle increases the phase velocity decreases. For a parallel propagation (that is) the phase velocity equals the Alfven speed and for perpendicular propagation (that is) the phase velocity vanishes and this clearly points out that the phase velocity of the Alfven waves depends on the propagation angle. From the figures and tables it can be observed that as the propagation angle increases the Alfven waves vanishes, and also the values of the phase velocity of the Alfven wave does not exceed the Alfven speed. Since this waves carries zero perturbation in density and pressure, they are incompressible, and also transverse in nature (Jovanovic 2021, Mehta 2022).

4.2 Fast Magnetosonic waves

The propagation of the fast magnetosonic waves in the solar corona as it is indicated in the tables and figures shows that as the propagation angle increases, the phase velocity also increases. For parallel propagation ( ), it is observed that for Alfven speeds of = 89.05, = 133.60 and =151.40 respectively their phase velocities is approximately equal to the acoustic speed, while for Alfven speeds of =178.10, =356.20, =712.40 and =890.50 respectively their phase velocities is approximately equal to their Alfven speeds. Since they perturb the perturb density, they are compressive and longitudinal in nature (Jovanovic 2021, Mehta 2022).

4.3 Slow Maggnetosonic Waves

For parallel propagation ( ), it is observed that for Alfven speeds =151.40, =178.10, =356.20, =712.40 and =890.50 respectively their phase velocities is approximately equal to the acoustic speed, this indicates that, as the propagation angle increases the phase velocity decreases and it also points out that for a perpendicular propagation (that is) the phase velocity vanishes. Since they perturb the perturb density, they are compressive and longitudinal in nature (Jovanovic 2021, Mehta 2022).

5. Conclusion

Aimed at investigating the behaviours of this MHD waves propagation in the solar corona, a mathematical model describing the types of MHD waves that propagates in the solar corona. The Alfven waves which has a dispersion relation that relates the wave frequency to the wave number k has been observed to depend only on the Alfven speed and the angle between the wave vector and magnetic field. This study confirms that the Alfven speed is smaller in magnitude to the value of speed of light.

The fast and slow magnetosonic waves has a dispersion relation of the fourth order polynomial, and they depends on the acoustic speed, Alfven speed and angle of propagation.

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