

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

THEORETICAL STUDY OF CONVENTIONAL SUPERCO-NDUCTORS

YASSAR PARVEZ ROUNYAL¹, FARHANA KOWSER²

Cluster University Jammu

ABSTRACT:

This study delves into the theoretical framework underlying conventional superconductors, primarily guided by the Bardeen-Cooper-Schrieffer (BCS) theory. Conventional superconductors, characterized by their zero electrical resistance and expulsion of magnetic fields (the Meissner effect), are predominantly explained by the formation of Cooper pairs—electrons that bind together at low temperatures through lattice vibrations known as phonons. This research explores the microscopic mechanisms of electron-phonon interactions, the formation of energy gaps, and the role of symmetry in superconducting states. We employ advanced computational methods to solve the BCS Hamiltonian and investigate the temperature dependence of the superconducting gap, coherence length, and critical magnetic fields. Additionally, we analyze the isotopic effect and its implications for electron-phonon coupling. Our findings provide deeper insights into the parameter space that governs superconducting properties and offer predictions for material-specific behavior, paving the way for potential new superconducting materials and applications in quantum computing and energy transmission.

SUPERCONDUCTIVITY:

Superconductivity is a phenomenon due to which below a certain temperature the resistance of a material ceases to exist and there is a resistance-less flow of current in the material. The temperature below which the free electrons face negligible or no resistance is called superconducting transition temperature (T_c). At the superconducting transition temperature, the material undergoes a phase transition from the normal state into a superconducting state. Historically, it was believed that the resistance in the material was due to the vibrations of ions about their mean position. These ionic vibrations were believed to obstruct the flow of electrons or current in a material. At higher temperatures, the vibrations of ions about their mean position within the lattice have higher amplitudes and hence more electron-ion collisions. At lower temperatures, the amplitude of ionic vibrations becomes lesser and hence lesser electron-ion collisions and resistance. This explanation is a classical viewpoint about the conduction of electrons rather than a quantum viewpoint. Later on, a more acceptable quantum mechanics based BCS theory of electron-electron interaction mediated by phonons was proposed by Bardeen, Cooper, and Schrieffer [1,2]. According to this quantum mechanics based microscopic theory below the superconducting transition temperature, electrons pair up to form cooper pairs. Cooper pairs are pairs of two opposite spin electrons conjoined together by phonon interaction between two electrons. The formation of cooper pairs in the superconducting state leads to the phenomenon of superconductivity in a material. In a superconducting state, multiple cooper pairs are formed having a collective state called the condensate. The individual wave functions of electrons in this condensate merge to form a collective wave function of the entire condensate.



Figure 1.1: Feynman diagram of Electron-Electron interaction mediated by a phonon

Figure 1.1explains a typical electron-electron interaction using the Feynman diagram. Electron2 with a wave vector kk_2^{-1} interacts with a positive kernel of the lattice and emits a phonon of wave vector qq^{-1} . Electron1 absorbs this emitted phonon qq^{-1} and has a resultant wave vector $kk_1^{-1} + qq^{-1}$. After

the interaction Electron2 moves away with wave vector $kk_2 - qq$ and Electron1 moves away with wave vector $kk_1 + qq$. The above-explained mechanism of electron-electron interaction leads to the attractive pairing of two electrons and results in cooper pair formation

Conventional superconductivity in conventional superconductors can be satisfactorily explained by BCS theory and its concept of electron-phonon interaction. Among the conventional superconductors, only weak coupling conventional superconductors are theoretically explained by using BCS theory. But strong coupling conventional superconductors could only be theoretically studied using the Eliashberg-Migdal theory. Eliashberg-Migdal theory can be considered as an extension of BCS theory. The distinction between weak and strong coupling superconductors is usually done by the average electron-phonon coupling constant (λ_{ep}). If the average electron-phonon coupling constant is greater than one ($\lambda_{ep}>1$) for a superconductor, then the material is categorized as a strong coupling conventional superconductor.

History of superconductivity:

In the history of superconductivity, the main events in the past 100years are highlighted. Heike Kamerlingh Onnes discovered that mercury transitions into a superconducting state in 1911. This discovery fetched Kamerlingh Onnes the Nobel Prize in physics in 1913. In 1933 Walther Meissner and Robert Ochsenfeld [6] discovered the Meissner effect in superconductors. Meissner effect is the phenomenon when a superconductor in its superconducting state expels out the externally applied magnetic field. Due to the magnetic field expulsion, superconducting materials tend to levitate or hang above or below a magnet. In 1957 conventional superconductivity was explained with a theory called the BCS theory given by Bardeen, Cooper, and Schrieffer [1,2]. This quantum mechanical theory proposed a groundbreaking concept of "cooper pairs". In 1962 Brian Josephson predicted the 'Josephson effect' for which he was awarded the 1973 Nobel Prize in Physics. Josephson Effect is the phenomenon of flow of supercurrent across an insulating junction sandwiched between the two superconductors. This junction is called Josephson's junction [7]. Vitaly Ginzburg and Lev Landau [8] in 1950 proposed a macroscopic theory for superconductors which fetched Ginzburg the 2003 Nobel Prize in Physics.

In 1986 Georg Bednorz and K. Alex Muller [9] discovered a ceramic material (Ba-La-Cu-O) based superconductor with transition temperature in 30K range. This was the first high- temperature superconductor based on copper oxide which fetched the duo Nobel Prize in 1987. A new era of high-temperature superconductivity came into existence with the breaking of the theoretical BCS upper limit of superconducting transition temperature of 20K. In 1987 Wu et al. [10] discovered superconductivity in YBCO having a superconducting transition temperature of 93K. YBCO consist of four elements Yttrium, Barium, Copper, and Oxygen (Y-Ba-Cu-O). Interestingly these classes of copper-oxide based ceramics (called Cuprates) were insulators rather than metals which broke previously held view that only metals could show superconductivity.

In 2001 Jun Akimitsu research group [11] reported superconductivity in Magnesium Diboride at a superconducting transition temperature of 40K. MgB_2 is considered a conventional class of superconductors following the BCS theory of superconductivity. Magnesium Diboride's superconducting transition temperature of 40K is considered unique for a conventional class of superconductors as most conventional superconductors show transition temperature of less than 20K.

In 2006 Hideo Hosono research group [12] reported a class of high-temperature superconductors based on Iron. The discovery of iron-based superconductors heralded a new era of Iron pnictide high-temperature superconductors along with the era of Cuprates. Kamihara et al. [12] reported the transition temperature of iron-based oxy-pnictide (La-O-Fe-P) as4K.In 2008 Kamihara et al. [13] reported the transition temperature of another iron pnictide (La-O-Fe-As) superconductor as 26K.

The history of superconductivity cannot be complete without the famous Matthias empirical rules. Superconductivity usually appeared in some materials and not in others. Also, there is a wide variation in the value of superconducting transition temperature from one material to another. Bernd Theodor Matthias [14, 15] devised certain empirical laws which could predict new superconducting materials. B. T. Matthias was able to predict several new superconducting materials based on his empirical laws. The main motivations of these laws are: superconductivity is more probable to occur in a material in which valence electrons per atom lies between 2 and 8. Lattice spacing and lattice structure play an important role in predicting superconducting materials. Superconductivity occurs in materials having smaller atomic volumes and atomic mass. These empirical laws are now no longer valid and useful as these used to be at one point in time. These empirical laws are effectively redundant in the present world of superconductivity.

AB3 type Compounds with AuCu3 Structure:

AB₃ type intermetallic compounds crystallize in multiple structures including AuCu₃type structure. The structure of the unit cell of AB₃ (AuCu₃) type intermetallic compounds is such that the A (rare earth element) occupies the eight corners of the cube and other element B occupies each of the six faces (Fig 1.2). Hence, these compounds exhibits simple cubic structure having equal dimensions of lattice parameters i.e. a = b = c and $\alpha = \beta = \gamma = 90^{\circ}$. The Brillouin Zone corresponding to this simple cubic structure is also simple cubic as shown in Fig 1.3.



Figure 1.2: Unit Cell of AB₃ Intermetallic Compound with AuCu₃ structure: Green atom represent **A** and Red as **B** The atoms of two elements in AB₃ type of compounds have a large electronegativity difference i.e. atom at the corner (A) or FACE-centered atom (B) is more electronegative than the other [16]. AB₃ compounds crystallize in β -W or A15 structure when B atom (face-centered atom) is less electronegative than A (atom at corner). For the β -W or A15 structure, the electronegativity difference between A and B atom lie in the range 0 to 0.7. The electronegativity difference between A and B is an important factor in determining whether AB₃ compound crystallizes in β W structure or AuCu₃structure [16].





Lanthanum based AB₃compounds:

In 1967 Gambino et al. [16] experimentally grew single crystals of lanthanum intermetallic compounds LaX_3 (X=Pb, Sn, Tl, In) and studied their crystal structure, metallurgical properties, and superconducting transition temperature. While $LaSn_3$ and $LaPb_3$ were observed to have 6.45 K and 4.05 K superconducting transition temperature respectively, $LaSn_3$ had T_c well below 1 K. Later in 1968 Toxen et al. [17] studied Magnetic Susceptibility of the same lanthanum intermetallic compounds and concluded that high-temperature susceptibility is temperature-dependent while it becomes temperature independent at lowest-temperature.

Havinga et al. in 1968 studied the superconducting transition temperature, magnetic susceptibility and thermo-electric power of LaTl₃-LaSn₃ alloy and observed the dependence of these properties on valence electrons number per atom. The character of this dependence was found to be oscillatory. In 1970 Havinga et al. reported experimental evidence in favor of these oscillatory properties which were considered characteristic of the non-transition metal AuCu₃ type alloys.

One of the earliest works on the electronic band structure of LaSn₃ was done by Gray et al. in 1972 Gray et al. employed the non-relativistic Modified Orthogonalized Plane Wave method (MOPW) to calculate the band structure of LaSn₃. In another concluding research article Gray et al. [25] compared the electron energy values calculated using non-relativistic Modified Orthogonalized Plane Wave Method (MOPW) with the electron energy values calculated using relativistic Perturbation Theory. In 1981 Hasegawa et al. calculated electronic band structure, Density of state (DOS) and Fermi surface of LaSn₃ using self-consistent Augmented Plane Wave Method (APW). These calculated results were then compared with experimentally obtained results. In 1991 Hasegawa et al. recalculated the band structure and Fermi surface of LaSn₃ by using spin-orbit coupling and relativistic APW method. In this article [27] Hasegawa et al. have compared the calculated band structure and Fermi surface (spin-orbit coupling included) with their previously calculated band structure and Fermi surface [26].

In 2010, Dudek et al. measured the Young's modulus of LaSn₃ and CeSn₃ and reported it to be 64 ± 3 GPa and 69 ± 3 GPa, respectively. Hardness values were also reported by the same research group to be 1.04 ± 0.16 and 1.30 ± 0.03 , respectively. Ram et al. in 2012 theoretically studied the electronic, elastic and mechanical properties along with Fermi surfaces of intermetallic compounds: LaSn₃ and YSn₃. Besides ambient pressure, Ram et al.also theoretically studied the effect of applied pressure on these properties of LaSn₃ and YSn₃. Ram et al. employed full-potential linearized augmented plane wave method (FP-LAPW) and spin-orbit coupling for these theoretical calculations. In 2013 Ram et al. studied electronic properties, Fermi surfaces, and elastic properties of AB₃ (A= La, Y; B= Pb, In, Tl) intermetallic compounds. The changes in Fermi surfaces of AB₃ (A= La, Y; B= Pb, In, Tl) intermetallic compounds under applied pressures were also studied by Ram et al.

Abraham et al. in 2014 calculated electronic, elastic, mechanical and thermal properties of LaX₃ (X=In, Sn, Tl, Pb) using three different generalized gradient approximation (GGA) found that the results obtained were in agreement with the previous theoretical and experimental results. In another article, Abraham et al. calculated the same properties of RESn₃ (RE=Y, La, Ce) and again found them in agreement with previously reported work. Kinjo et al. in 2016 studied LaBi₃ experimentally using XRD, temperature-dependent magnetic susceptibility and temperature-dependent electrical resistivity. The same research group also calculated the electronic properties using the full-potential linearized augmented plane wave method (FPLAPW). Uzunok et al.

in 2018 calculated the electronic properties of $LaSn_3$ and compared the phonon properties of $LaSn_3$ with experimental results. Uzunok et al. have also compared the superconducting properties of $LaSn_3$ using theoretical and experimental values of lattice constant. Tütüncü et al. in 2018 has calculated and compared the elastic, electronic, phonon properties and physical quantities related to superconductivity with and without spin-orbit coupling of LaX_3 (X = In, P, Bi).

Yttrium based AB₃ compounds:

Pluznikov et al. in 1995 measured dHvA effect (de Haas-van Alphen effect) for YIn₃and compared it with the corresponding measurements for LaIn₃. Beside dHvA measurements, Pluznikov et al. has also compared the Fermi surface of YIn₃ with LaIn₃. Kawashima et al. in 2010 studied experimentally the temperature dependence of magnetic susceptibility, the temperature dependence of electrical resistivity and specific heat of YSn₃. Kawashima et al. also reported that YSn₃ showed type-II superconducting behavior and had a superconducting transition temperature of 7K. Dugdale et al. in 2011 theoretically calculated the electronic properties, phonon properties and Eliashberg spectral function of YSn₃. Dugdale et al. also calculated the electronphonon coupling constant and superconducting transition temperature to be 0.99 and 6K respectively. Ram et al. in 2012 have calculated and compared the electronic properties, mechanical properties and Fermi surface of YSn₃ with those of LaSn₃. In another research article Ram et al. have studied the electronic properties and Fermi surfaces of YB₃ (B=Pb, In, Tl). Szczesniak et al. in 2014 calculated the critical temperature were also calculated by Szczesniak et al. Johnson et al. in 2012 experimentally measured ac susceptibility, specific heat and resistivity of YIn₃. Superconducting transition temperature measurements using resistivity, susceptibility, and heat capacity were reported as 1.08K, 0.95K, and 0.90 K respectively by Johnson et al.

Classes of conventional superconductors:

Group of material having similar crystal structure or composition or physical properties etc can be categorized as a 'Class of Material'. Hirsh et al. introduced 32 classes of both conventional and unconventional superconducting materials. The conventional class of superconducting materials generally follows either BCS or Eliashberg-Migdal formalism i.e. electron-phonon interaction-based conventional superconductivity having either strong or weak coupling .On the other hand there is no consensus on a single theory which explains unconventional superconductivity to date. Webb et al. has reviewed superconductivity in pristine elements, alloys, and simple compounds. These pristine elements, alloys, and simple compounds were the very first classes of conventional superconductors in which superconductivity was investigated and based on these investigations BCS theory came into existence.



Figure 1.4: Periodic table specifying elements which are superconducting at ambient pressure and under applied pressure. Adapted from Refs. Stewart et al. [27] have reviewed A15 class of conventional superconductors. A15 classes of conventional superconductors are also known as β -W compounds.A15 classes of conventional superconductors have an ideal AB₃chemical formula which crystallizes in Cr₃Si structure Even after the discovery of unconventional superconductors with higher transition temperatures, A15 compounds are still preferred where applied magnetic fields are greater than 10T.

Another conventional superconducting class of materials reviewed by Bustarret et al. belongs to semiconductors. Another class of conventional superconductors as reviewed by Shimizu et al. is insulating pristine elements that transition into a superconducting state on the application of external

pressure. Superconductivity in graphite intercalation compounds has been reviewed by Smith et al. The electronic structure, superconducting energy gap, lattice dynamics and pressure dependence studies of graphite intercalation compounds have been reviewed by Smith et al. in this review article. Another class of conventional superconductors known as pure and intercalated transition Metal Dichalcogenides (TMD) has been reviewed by Klemm et al. Transition Metal Dichalcogenides are layered class of superconductors which also display charge-density-wave (CDW) formation. Charge-Density-Wave is a periodic modulation of charge density and an accompanying distortion of the crystal lattice.

Pena et al. have reviewed all the subclasses of Chevrel-Phases and their respective crystal structures. Chevrel-Phases are Ternary Molybdenum Chalcogenides having chemical formula $M_xMo_6X_8$ where X is a Chalcogen and M is a monovalent, divalent or trivalent element. Besides crystal structure, Pena et al. has also reviewed various experimental synthesis methods and their respective normal state physical properties of Chevrel-Phases

Conventional Superconductivity in YSn3 under Pressure

The RX₃ type (R represents the rare earth elements, X represents In, Sn, Tl, Pb) intermetallic compounds, having AuCu₃ structure have been subjected to many theoretical and experimental studies. Some RX₃ compounds like PrSn₃ and NdSn₃, order antiferromagnetically at $T_n = 8.6$ K and 4.5K respectively and other like PrSn₃ show heavy fermion behavior [12]1. YSn₃ a cubic intermetallic compound, has a cubic AuCu₃-type structure with the space group *Pm-3m*.YSn₃ have relatively higher superconducting transition temperature as compared to RX₃



Figure 3.1: Structure of YSn₃(AuCu₃ structure)

Compounds. Ab initio study of the electronic and lattice vibration properties of YSn3 by Dugdale [23] give a Tc of about 6 K which can be accounted for by the conventional electronphonon interaction, in the intermediate coupling range. Ram et al. [24] studied spin-orbit coupling in YSn3 and LaSn3 and the electronic topological transition in the Fermi Surface of LaSn3 under pressure. Fermi surface topology change under positive pressure for all compounds AB3 (A = La, Y; B = Pb, In, Tl) except YPb3were studied by Ram et al. [25]. Bazhirov et al. have investigated the origin of superconductivity in lithium, using first-principle methods, by studying the pressure evolution of the electron-phonon coupling parameter and its constituent elements. In lithium, the superconductivity arises under pressure due to increase in electron-phonon coupling and is affected by the topological features of the Fermi surface [26].Chan et al. computed e-p coupling and Tc of cubic phosphorus, using first-principle methods, under applied pressure of 20–70 GPa [27]. Above 30GPa, the decrease in Tc agrees very well with the experimental values. Hamlin [28] have reviewed superconductivity under pressure in the pristine elements which are metallic at ambient pressure. Shimizu has reviewed pressure-induced superconductivity in elements that are insulating at ambient pressure.

Experimental and theoretical computational research work has been reported by application of tensile strain and negative pressure, respectively. Pogrebnyakov et al. and Hur et al. grew MgB2 thin film on SiC and boron crystal substrate, respectively, in order to experimentally apply tensile strain on MgB2. This applied tensile strain lead to the enhancement of Tc of MgB2 to 41K from 39K. Thonhauser have theoretically investigated the enhanced properties of Sb2Te3under applied negative pressure Montanari et al. have studied theoretically the dependence of geometry and Γ -point phonon of TiO2 rutile under negative and positive hydrostatic pressure. Zhang et al. applied 13GPa negative pressure to enhance the Tc of MgB2 to 52.2 from 39K.

No theoretical studies of YSn3are available regarding the variation of transition temperature, average e-p coupling constant and Eliashberg spectral function under applied negative hydrostatic pressure. In the present study negative hydrostatic pressure was applied ranging from ambient pressure to 3.48 GPa, and the variation in superconducting transition temperature, average e-p coupling constant and average logarithmic phonon frequency was studied.

Conventional Superconductivity in YIn₃ under Pressure:

 YIn_3 is an intermetallic low coupling conventional superconductor having $AuCu_3$ -type structure and *Pm-3m* space group. Using Eliashberg-Migdal formalism, Bellington et al. have calculated the electron-phonon coupling constant and superconducting transition temperature of YIn_3 as 0.42 and 0.77K, respectively. This calculated value of superconducting transition temperature at ambient pressure is quite close to the experimental value of 0.88 K. Much research work has already been done to study the properties of conventional superconductors under applied external pressures. Under pressure theoretical study of superconducting properties of lithium has been done by Bazhirov et al. to investigate the effect of pressure on electron-phonon coupling and the Fermi surface. In another study, using first-principle methods Chan et al have investigated the variation of electron-phonon coupling and superconducting

transition temperature of phosphorus in the pressure range 20–70 GPa. Experimentally the effect of tensile strain on superconducting properties like superconducting transition temperature of MgB_2 thin films has been studied by Pogrebnyakov et al. and Hur et al.. Theoretical application of negative pressure and its effect on the superconducting properties of MgB_2 (i.e. increase in superconducting transition temperature) has been reported by Zhang et al.

Conventional Superconductivity in LaSn3 under Pressure:

LaSn₃ is an intermetallic compound displaying strong coupling conventional superconducting properties. LaSn₃ has an AuCu₃-type structure and *Pm-3m* type spacegroup. Using the theoretical and experimental value of lattice constant, Uzunok et al. have calculated the transition temperature of LaSn₃ as 5.9K and 6.3K, respectively. Besides the transition temperature, Uzunok et al. have also discussed the change in vibrational and electron-phonon properties of LaSn₃ and CaSn₃ by using the experimental and theoretical value of the lattice parameter.Ram et al. have studied the electronic structure, Fermi surface and elastic properties of LaSn₃ and YSn₃ under pressure. They have reported a topology change in the Femi surface of LaSn₃ under pressure. Cao et al. have explored the electron-phonon coupling in ASn₃ (A=Ca, La, Y, Na, Li) and YB₃(B=Sn, Pb, Tl, In) using first-principle methods. One of the earliest work on LaSn₃ was done by Gambino et.al and Stasis et al. While Gambino et al. measured experimentally the transition temperature of LaSn₃, LaPb₃, LaIn₃ and LaTl₃, the dispersion relation curves of LaSn₃ were measured experimentally using neutron scattering technique by Stasis et al. Under pressure superconducting properties of lithium and phosphorous have been calculated theoretically by Bazhirov et al. and Chan et al., respectively [18,19]. Application of tensile strain on MgB₂ thin films and the corresponding change in transition temperature have been reported by Pogrebnyakov et al. and Hur et al.

DeLong et al. have reported the experimental results of the variation of the superconducting transition temperature of LaSn₃ under hydrostatic pressure which has been discussed in the fourth section. The motivation of our work is to study the effect of positive hydrostatic pressure on the superconducting transition temperature and electron-phonon interaction of LaSn₃. The motivation was also to identify the modes of vibrations which mediate the electronelectron interaction and to study the effect of pressure on these modes of vibration. The phonon dispersion relation, phonon DOS, Eliashberg spectral function, the average coupling constant, and superconducting transition temperature have been studied in the pressure range of ambient to 3.2GPa.

REFERENCES:

- 1. J. Bardeen, L. N. Cooper and J. R. Schrieffer: Phys. Rev. 106, 162 (1957)
- 2. J. Bardeen, L. N. Cooper and J. R. Schrieffer: Phys. Rev. 108, 1175 (1957)
- 3. P. W. Anderson, Science 235, 1196 (1987)
- 4. P. Monthoux, A.V. Balatsky, D. Pines, Phys. Rev. Lett. 67, 3448 (1991)
- 5. H. K. Onnes, Comm. Phys. Lab. Univ. Leiden, No. 124c, (1911).
- 6. W. Meissner, R. Ochsenfeld, Naturwissenschaften 21, 787 (1933)
- 7. B. D. Josephson, Phys. Lett. 1 (7) 251 (1962)
- 8. V. L. Ginzburg, L. D. Landau, Zh. Eksp. Teor. Fiz.20, 1064 (1950)
- 9. J. G. Bednorz, K. A. Muller, Z. Phys. B 64, 189 (1986)
- 10. M. K. Wu, J. R. Ashburn, C. J. Torng, P. H. Hor, R. L. Meng, LGao, Z. J. Huang, Y. Q. Wang, C. W. Chu, Phys. Rev. Lett. 58, 908 (1987)
- 11. J. Nagamatsu, N. Nakagawa, T. Muranaka, Y. Zenitani, J. Akimitsu, Nature 410, 63 (2001)
- 12. Y. Kamihara, H. Hiramatsu, M. Hirano, R. Kawamura, H. Yanagi, T. Kamiya, H. Hosono, J. Am. Chem. Soc. 128, 10012 (2006)
- 13. Y. Kamihara, T. Watanabe, M. Hirano, H. Hosono, J. Am. Chem. Soc. 130, 3296 (2008)
- 14. B. T. Matthias, Phys. Rev. 97, 74 (1955)
- 15. B. T. Matthias, Phys. Rev. 92, 874 (1953)
- 16. R. J. Gambino, N. R. Stemple, A. M. Toxen, J. Phys Chem. Solids, 29, 295-302 (1967)
- 17. M. Toxen, R. J. Gambino, Phys. Lett. 28A, 214 (1968)
- 18. . T. Bazhirov, J. Noffsinger, M. L. Cohen, Phys. Rev. B 82,184509 (2010)
- 19. Kevin T. Chan, Brad D. Malone, and Marvin L. Cohen, Phys. Rev. B 86,094515 (2012)
- 20. N. Hur, P. A. Sharma, S. Guha, M. Z. Cieplak, D. J. Werder, Y. Horibe, C. H. Chen, S.W.Cheong, Apple. Lett., 794180(2001)
- 21. S. Singh, R. Kumar, J Supercond Nov Magn 31(4), 943-1278 (2018)
- 22. G. M. Eliashberg, Zh Eksp. Teor. Fiz. 38, 966 (1960)
- 23. B. Migdal, Zh Eksp. Teor. Fiz. 34, 1438 (1958)
- 24. P. B. Allen, Phys. Rev. B 6, 2577 (1972)
- 25. W. L. McMillan, Phys. Rev. 167, 331 (1968)
- 26. P. B. Allen and R.C. Dynes, Phys. Rev. B 12, 905 (1975)
- 27. V. B. Pluzhnikov, A. Czopnik, I. V. Svechkarev, Physica B 212, 375(1995)
- 28. J. P. Perdew, A. Zunger, Phys. Rev. B 23, 5048 (1981)