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Investigation of Compressive Strength Property of Hybrid Polypropylene - Nylon Fibre Reinforced Concrete (HPNFRC) Based on Scheffe's (6,3) Model

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ABSTRACT

In the continuous bit to demonstrate that compressive strength of concrete can increase more when two or more fibres are combined in a mixture design using Scheffe's higher degree order, a concrete mixture containing Hybrid Polypropylene – Nylon Fibre has been presented in this work to illustrate this phenomenon. This research work is therefore aimed at using Scheffe's Third Degree Mathematical Model for six component mixtures, Scheffe's (6,3) to obtain the optimized compressive strength of Hybrid Polypropylene - Nylon Fibre Reinforced Concrete (HPNFRC). Through the use of Scheffe's simplex mathematical model, the compressive strengths of HPNFRC based on Scheffe's third model were evaluated for 112 different mix ratios. As in the case of previous HPNFRC work based on Scheffe's second degree model by Nwachukwu and others (2022j), the mix proportion of Polypropylene – Nylon was in 50% - 50% ratio. Control experiments based on 112 mix ratios were also carried out, and the compressive strengths determined. Through the use of the Student's t-test statistics, the adequacy of the model when validated showed that there is a good correlation between the Lab results and the model results. The maximum compressive strength of HPNFRC based on third degree model is 71.36 MPa while the minimum value is 37.25MPa. The maximum value is slightly higher than the value obtained by Nwachukwu and others (2022j) as 60.05 MPa based on second degree model. However, the maximum values from both models are higher than the minimum required value specified by the American Concrete Institute (ACI), as 20 MPa and also the minimum required value specified by ASTM C 469 and ASTM C 39, as 30.75 for good and high performance concrete respectively. Therefore, the HPNFRC compressive strength values can adequately sustain construction of ground –level application, basement foundation as well as supporting both commercial and industrial construction works as high performance concrete at the best possible economic, aesthetic and safety advantages.

Keywords: HPNFRC, Scheffe's (6,3) Optimization Model, Compressive Strength, Mixture Design

1.INTRODUCTION

HPNFRC is typical example of Hybrid Fibre Reinforced Concrete (HFRC) which is the use of two or more fibres in a single concrete mixture matrix with the aim of improving its overall properties such as compressive strength, flexural strength, etc. The process of combination of HFRC is called hybridization. However, the fibres can be combined through the optimization process which is seen as less laborious. Optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as workability, strength and durability. According to Shacklock (1974), the objective of mix design is to determine the most appropriate proportions in which to use the constituent materials to meet the crucial needs of construction work On the account of the widely varying properties of the constituent materials, the conditions that prevail at the site of work, the exposure condition, and the conditions that are demanded for a particular work for which the mix is designed, the design of concrete mix according to (Shetty, 2006) has not being a simple task .Again, concrete mix design according to Jackson and Dhir (1996) has been defined as the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. Thus, the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, batching, mixing and placing the concrete as well as the site supervision. In the context of the above guidelines, there are methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988). These methods are empirical methods and are seen as more complex and time consuming as they involve several trial mixes and deep statistical calculations before the desired strength of the concrete can be achieved. Thus, optimization of the concrete mixture design remains both the fastest and best method and option as well as the most efficient way of selecting concrete proportions for better efficiency and performance of concrete when compared with the above mentioned empirical methods. A typical example of optimization model is the Scheffe's Mathematical Model, which can be in the form of Scheffe's Second Degree model or Scheffe's Third Degree model. Thus, in this present study, Scheffe's Third Degree Model for six components mixtures (namely water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and nylon fibre) are presented.

Concrete has been defined by Oyenuga (2008) as a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete which is classified as the most widely used construction material has been undergoing changes both as a material and due to technological advancement.. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Concrete, according to Neville (1990), plays a crucial role in all building structures owing to its numerous advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. But despite the numerous role that concrete plays, it has got its drawbacks, especially the plain type. According to Shetty (2006), plain concrete possesses a very low tensile strength, limited ductility and little resistance to cracking. Consequently, there have been continuous search for the upgrading of the concrete properties which include using conventional reinforced steel bars to improve the tensile properties of concrete members. Although this method provides tensile strength to the concrete members, it however, does not increase the inherent tensile strength of concrete itself. Sequel to further researches and recent developments in concrete technology, it has been established that the addition of fibres (either as glass fibre, polypropylene fibre, nylon fibre, steel fibre , plastic fibre, asbestos (mineral fibre), or carbon fibres , etc.) to concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). FRC is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed fibres. Hybrid Fibre Reinforced Concrete (HFRC) is the use of two or more fibres in a single concrete mixture matrix with the objective of improving its overall properties. Hybrid – Polypropylene - Nylon Fibre Reinforced Concrete (HPNFRC) is a hybrid concrete mixture where the conventionally steel reinforcement in concrete production is replaced (wholly or partially) with polypropylene fibre and nylon fibre. Before now, works on optimization of compressive strength of PFRC and NFRC as well as work on HPNFRC based on Scheffe's Second Degree model have been carried out.

The major objective of engineering design is to ensure that the structure being designed will not reach a Serviceability Limit State (SLS), which is connected with deflection, cracking, vibration etc, and Ultimate Limit State (ULS), which is generally connected with collapse (Ettu, 2001). subsequently, the concrete's compressive strength is one of the most important properties of concrete that require very close examination because of the crucial role it plays in the strength of structural members . By definition, compressive strength of concrete is the strength of hardened concrete measured by the compression test. It is also a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine (UTM). Conclusively, the compressive strength of the concrete cube test provides an idea about all the characteristics of concrete under investigation.

This present study examines the application of Scheffe's Third Degree Mathematical Model in the optimization of the compressive strength of HPNFRC. Before now, a lot of researchers have done related works on polypropylene fibre, nylon fibre as well as other fibres, but none has been able to address the subject matter sufficiently. For instance, on Polypropylene Fibre Reinforced Concrete (PFRC) and HFRC, MK-Yew and others (2011) investigated the Strength Properties of Hybrid Nylon- Steel and Polypropylene –Steel Fiber-Reinforced High Strength Concrete. Bayasi and Zeng (1993) and Patel and others (2012) have investigated the properties of PFRC. Similarly, Kumbhar and others (2014) investigated the compressive strength of Hybrid Fibre Concrete. In his contribution, Richardson (2014) also investigated the compressive strength of concrete with polypropylene fibre addition. On NFRC and other fibres, Ganesh Kumar and others (2019) have carried out a study on waste nylon fibre in concrete. Samrose and Mutsuddy (2019) have investigated the durability of NFRC. Hossain and others (2012) have also investigated the effect of NF in concrete rehabilitation. Ali and others (2018) have carried out a study on NFRC through partial replacement of cement with metakaolin. Song and others (2005) also investigated the strength properties of NFRC and PFRC respectively. Varma and Raji (2019) have presented an experimental investigation to quantity the improved mechanical properties of Hybrid - Polypropylene-Steel Fibre-Reinforced Concrete. Qian and Stroeven (2000) investigated the optimization of fibre size , fibre content, and fly ash content in hybrid polypropylene- steel fibre concrete based on general mechanical properties. As this work is aimed at converting disposable waste like polypropylene and nylon into useful construction material, the work of Ishaya and others (2016) is a guide. Recent works on optimization show that many researchers have used Scheffe's optimization method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were also based on the use of Scheffe' mathematical model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) . Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and Opara (2023) in the their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Finally, Nwachukwu and others (2024c) applied Scheffe's (5,2) model to evaluate the compressive strength of Plastic Fibre Reinforced Concrete [PLFRC**]**. From the works reviewed so far, it can be envisaged that the subject matter has not been fully addressed as it is obvious that no work has been done on the use of Scheffe's Third Degree Model to optimize the compressive strength of HPNFRC. Henceforth, the need for this present research work.

2. METHODOLOGY

2.1 MATERIALS FOR HPNFRC MIXTURES

In this recent work, the component materials under investigation in line with Scheffe's (6,3) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates, Polypropylene and Nylon Fibres. Usually potable water is obtained from clean water source and applied in accordance with ASTM C1602/C1602M-22 (2022). The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate, whose size ranges from 0.05 - 4.5mm was procured from the local river. Crushed granite (as a coarse aggregate) of 20mm size was obtained from a local stone market and was downgraded to 4.75mm. Both fine and coarse aggregates were procured and prepared in accordance with ASTM C33/C33M-18 (2018). The same size and nature of polypropylene fibre and nylon fibre used previously by Nwachukwu and others (2022c), Nwachukwu and others (2022d) respectively and Nwachukwu and others (2022j) are the same as the one being used in this present work based on Scheffe's third degree model.

2.2 THEORITICAL BACKGROUND ON HPNFRC SCHEFFE'S (6,3) MATHEMATICAL MODEL

A simplex lattice can be defined as a structural representation of lines joining the atoms of a mixture, whereas these atoms are constituent components of the mixture. For HPNFRC, the six constituent elements are Water/ cement ratio, Cement, Fine Aggregate, Coarse Aggregate, Polypropylene Fibre and Nylon Fibre. Subsequently, a simplex of six-component mixture is a five -dimensional solid. According to Obam (2009), mixture components are usually subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \ ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad (1)$$

where $X_i \geq 0$ and i = 1, 2, 3… q, and q = the number of mixtures.

2.2.1. POSSIBLE DESIGN POINTS FOR HPNFRC SCHEFFE'S (6,3) MIXTURES DESIGN

It is a well-known fact that the (q, m) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen mathematical equation to represent the response surface over the entire simplex region (Aggarwal, 2002). The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each component proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is Scheffe's polynomial degree. For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is

$x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1, 0, 0), (0, 1, 0), (0, 0, 1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. Generally, the formula for determining the number of coefficients/terms/points required for a given lattice is always given by:

$$k = \frac{(q+m-1)!}{(q-1)!.\ m!} \quad \text{Or} \quad {}^{q+m-1}C_m \qquad \textbf{2(a-b)}$$

Where k = number of coefficients/ terms / design points, q = number of components = 6 in this study and m = number of degree of polynomial = 3 in this present work. Using either of Eqn. (2), $k_{(6,3)}$ = 56. Thus, 56 possible design points/coefficients for HPNFRC Scheffe's (6,3) lattice can be as follows:

A_1 (1,0,0,0,0,0); A_2 (0,1,0,0,0,0); A_3 (0,0,1,0,0,0); A_4 (0,0,0,1,0,0), A_5 (0,0,0,0,1,0); A_6 (0, 0,0,0, 0, 1); A_{112} (0.67,0.33, 0, 0,0, 0); A_{122} (0.67, 0, 0.33,0,0,0); A_{113} (0.67, 0, 0, 0.33,0,0); A_{133} (0.67, 0, 0, 0,0.33, 0); A_{114} (0.67, 0, 0, 0, 0, 0.33); A_{144} (0,0.67,0.33,0,0,0); A_{115} (0, 0.67, 0, 0.33, 0,0); A_{155}, (0, 0.67,0, 0,0.33, 0); A_{116} (0,0.67,0,0,0,0.33); A_{166} (0,0,0.67,0.33,0,0); A_{223} (0,0,0.67,0,0.33,0); A_{233} (0,0,0.67,0,0,0.33);A_{224} (0,0,0,0.67, 0.33,0);A_{244}(0,0,0,0.67,0,0.33);A_{225}(0,0,0,0,0.67,0.33);A_{255}(0.50,0.50,0,0,0,0);A_{226}(0.50,0,0.50,0,0,0) ;A_{266}(0.50,0,0,0.50,0,0);A_{334}(0.50,0,0,0,0.50,0);A_{344}(0.50,0,0,0,0,0.50);A_{335}(0,0.50,0.50,0,0,0) A_{355}(0,0.50,0,0.50,0,0);A_{336}(0,0.50,0,0,0.50,0);A_{366}(0,0.50,0,0,0,0.50);A_{445}(0,0,0.50,0.50,0,0); A_{455}(0,0,0.50,0,0.50,0);A_{446}(0,0,0.50,0,0,0.50);A_{466}(0,0,0,0.50,0.50,0);A_{556}(0,0,0,0.50,0,0.50); A_{566}(0,0,0,0,0.50,0.50);A_{123}(0.80,0.20,0,0,0,0);A_{124}(0.80,0,0.20,0,0,0);A_{125}(0.80,0,0,0.20,0,0);A_{126}(0.80,0,0,0,0.20,0);A_{134}(0.80,0,0,0,0,0.20);A_{135}(0,0.80, 0.20,0,0,0);A_{136}(0,0.80,0,0.20,0,0);A_{145}(0,0.80,0,0,0.20,0);A_{146}(0,0.80,0,0,0,0.20);A_{156}(0,0,0.80,0.20,0,0);A_{234}(0,0,0.80,0,0.20,0);A_{235}(0,0,0.80,0,0,0.20); A_{236}(0,0,0,0.80,0.20,0);A_{245}(0,0,0,0.80,0,0.20);A_{246}(0,0,0,0,0.80,0.20);A_{256}(0.60,0.40,0,0,0,0);A_{345}(0.60,0,0.40,0,0,0);A_{346}(0.60,0,0,0.40,0,0); A_{356}(0.60,0,0,0,0.40,0); A_{456}(0.60,0,0,0,0,0.40) **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable X_1, X_2, X_3, X_4 … X_q is given in the form of Eqn.(4): N= b_0 + $\sum b$i xi + $\sum b$ij$x$j + $\sum b$i $jxjxk$ + + $\sum b$i ${}^{j}{}_{2}$ +… $i_n x i_2 x i_n$ **(4)**

where ($1 \le i \le q$, $1 \le i \le j \le k \le q$, $1 \le i_1 \le i_2 \le \ldots \le i_n \le q$ respectively) , b = constant coefficients and N is the response which represents the property (compressive strength) under investigation. As this research work is based on the Scheffe's (6, 3) simplex, the actual form of Eqn. (4) for six component mixture, degree three has been developed by Nwachukwu and others (2023a).

2.2.2. ACTUAL AND PSEUDO COMPONENTS FOR THE HPNFRC SCHEFFE'S MODEL

There exists a transformation relationship between the pseudo components and the actual components in the Scheffe's mix design model. The established relationship is stated in Eqn.(5): Z = A * X **(5)**

Where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5) yields: $X = A^{-1} * Z$ **(6)**

2.2.3. FORMULATION OF POLYNOMIAL EQUATION FOR THE HPNFRC SCHEFFE'S (6,3)

LATTICE

The mathematical equation by Scheffe (1958), which is known as response is given in Eqn.(4). And for the Scheffe's (6,3) simplex lattice, the polynomial equation for six component mixtures in third degree capacity has been formulated based on Eqn.(4) by the work of Nwachukwu and others (2023a) as shown under:

N = ß1X1 + ß2X2 + ß3X3 + ß4X4 + ß5X5 + ß6X6 + ß12X1X2 + ß13X1X3 + ß14X1X4 + ß15X1X5 +ß16X1X6 + ß23X2X3 + ß24X2X4 + ß25X2X5 + ß26X2X6 + ß34X3X4 + ß35X3X4 + ß36X3X 6 + ß45X4X5 +ß46X4X6 + ß56X5X6 + ß123X1X2X3 + ß124X1X2X4 + ß125X1X2X5 + ß126X1X2X6 + ß134X1X3X4 + ß135X1X3X5 + ß136X1X3X6 + ß145X1X4X5 + ß146X1X4X6 + ß234X2X3X4 + ß235X2X3X5 + ß236X2X3X6 + ß245X2X4X5 + ß246X2X4X6 + ß256X2X5X6 + ß345X3X4X5 + ß346X3X4X6 + ß356X3X5X6 + ß456X4X5X6 + $\gamma_{12}X_1X_2^2 + \gamma_{13}X_1X_3^2 + \gamma_{14}X_1X_4^2 + \gamma_{15}X_1X_5^2 + \gamma_{16}X_1X_6^2 + \gamma_{23}X_2X_3^2 + \gamma_{24}X_2X_4^2 + \gamma_{25}X_2X_5^2 + \gamma_{26}X_2X_6^2 + \gamma_{34}X_3X_4^2 + \gamma_{35}X_3X_5^2 + \gamma_{36}X_3X_6^2 + \gamma_{45}X_3X_6^2 + \gamma_{46}X_4X_6^2 + \gamma_{56}X_5X_6^2$ **(7)**

2.2.4 . DETERMINATION OF THE COEFFICIENTS OF THE HPNFRC SCHEFFE'S (6, 3) MATHEMATICAL EQUATIONS

From the work of Nwachukwu and others (2023a), the coefficient has been established as follows:

$\beta_{1=}N_1$; $\beta_2 = N_2$; $\beta_3 = N_3$; $\beta_4 = N_4$; $\beta_5 = N_5$ and $\beta_6 = N_6$ **8(a-f)**

$\beta_{12} = 2.25(N_{112} + N_{122} - N_1 - N_2)$;$\beta_{13} = 2.25(N_{113} + N_{133} - N_1 - N_3)$;$\beta_{14} = 2.25(N_{114} + N_{144} - N_1 - N_4)$; **9(a-c)**

$\beta_{15} = 2.25(N_{115} + N_{155} - N_1 - N_5)$;$\beta_{16} = 2.25(N_{116} + N_{166} - N_1 - N_6)$;$\beta_{23} = 2.25(N_{223} + N_{233} - N_2 - N_3)$; **10(a-c)**

$\beta_{24} = 2.25(N_{224} + N_{244} - N_2 - N_4)$; $\beta_{25} = 2.25(N_{225} + N_{255} - N_2 - N_5)$; $\beta_{26} = 2.25(N_{226} + N_{266} - N_2 - N_6)$; **11(a-c)**

$\beta_{34} = 2.25(N_{334} + N_{344} - N_3 - N_4)$ $\beta_{35} = 2.25(N_{335} + N_{355} - N_3 - N_5)$;$\beta_{36} = 2.25(N_{336} + N_{366} - N_3 - N_6)$; **12(a-c)**

$\beta_{45} = 2.25(N_{445} + N_{455} - N_4 - N_5)$; $\beta_{46} = 2.25(N_{446} + N_{466} - N_4 - N_6)$;$\beta_{56} = 2.25(N_{556} + N_{566} - N_5 - N_6)$; **13(a-c)**

$\beta_{123} = 27N_{123} - 6.75(N_{112}+N_{122}+N_{113}+N_{133}+N_{223}+N_{233}) + 2.25(N_1+N_2+N_3)$ **(14)**

$\beta_{124} = 27N_{124} - 6.75(N_{112}+N_{122}+N_{114}+N_{144}+N_{224}+N_{244}) + 2.25(N_1+N_2+N_4)$ **(15)**

$\beta_{125} = 27N_{125} - 6.75(N_{112}+N_{122}+N_{115}+N_{155}+N_{225}+N_{255}) + 2.25(N_1+N_2+N_5)$ **(16)**

$\beta_{126} = 27N_{126} - 6.75(N_{112}+N_{122}+N_{116}+N_{166}+N_{226}+N_{266}) + 2.25(N_1+N_2+N_5)$ **(17)**

$\beta_{134} = 27N_{134} - 6.75(N_{113}+N_{133}+N_{114}+N_{144}+N_{334}+N_{344}) + 2.25(N_1+N_3+N_4)$ **(18)**

$\beta_{135} = 27N_{135} - 6.75(N_{113}+N_{133}+N_{115}+N_{155}+N_{335}+N_{355}) + 2.25(N_1+N_3+N_5)$ **(19)**

$\beta_{136} = 27N_{136} - 6.75(N_{113}+N_{133}+N_{116}+N_{166}+N_{336}+N_{366}) + 2.25(N_1+N_3+N_6)$ **(20)**

$\beta_{145} = 27N_{145} - 6.75(N_{114}+N_{144}+N_{115}+N_{155}+N_{445}+N_{455}) + 2.25(N_1+N_4+N_5)$ **(21)**

$\beta_{146} = 27N_{146} - 6.75(N_{114}+N_{144}+N_{116}+N_{166}+N_{446}+N_{466}) + 2.25(N_1+N_4+N_6)$ **(22)**

$\beta_{156} = 27N_{156} - 6.75(N_{115}+N_{155}+N_{116}+N_{166}+N_{556}+N_{566}) + 2.25(N_1+N_5+N_6)$ **(23)**

$\beta_{234} = 27N_{234} - 6.75(N_{223}+N_{233}+N_{224}+N_{244}+N_{334}+N_{344}) + 2.25(N_2+N_3+N_4)$ **(24)**

$\beta_{235} = 27N_{235} - 6.75(N_{223}+N_{233}+N_{225}+N_{255}+N_{335}+N_{355}) + 2.25(N_2+N_3+N_5)$ **(25)**

$\beta_{236} = 27N_{236} - 6.75(N_{223}+N_{233}+N_{226}+N_{266}+N_{336}+N_{366}) + 2.25(N_2+N_3+N_6)$ **(26)**

$\beta_{245} = 27N_{245} - 6.75(N_{224}+N_{244}+N_{225}+N_{255}+N_{445}+N_{455}) + 2.25(N_2+N_4+N_5)$ **(27)**

$\beta_{246} = 27N_{246} - 6.75(N_{224}+N_{244}+N_{226}+N_{266}+N_{446}+N_{466}) + 2.25(N_2+N_4+N_6)$ **(28)**

$\beta_{256} = 27N_{256} - 6.75(N_{225}+N_{255}+N_{226}+N_{266}+N_{556}+N_{566}) + 2.25(N_2+N_5+N_6)$ **(29)**

$\beta_{345} = 27N_{345} - 6.75(N_{334}+N_{344}+N_{335}+N_{355}+N_{445}+N_{455}) + 2.25(N_3+N_4+N_5)$ **(30)**

$\beta_{346} = 27N_{346} - 6.75(N_{334}+N_{344}+N_{336}+N_{366}+N_{446}+N_{466}) + 2.25(N_3+N_4+N_6)$ **(31)**

$\beta_{356} = 27N_{356} - 6.75(N_{335}+N_{355}+N_{336}+N_{366}+N_{556}+N_{566}) + 2.25(N_3+N_5+N_6)$ **(32)**

$\beta_{456} = 27N_{456} - 6.75(N_{445}+N_{455}+N_{446}+N_{466}+N_{556}+N_{566}) + 2.25(N_4+N_5+N_6)$ **(33)**

$\gamma_{12} = 2.25(3N_{112}+3N_{122}-N_1+N_2)$; $\gamma_{13} = 2.25(3N_{113}+3N_{133}-N_1+N_3)$; $\gamma_{14} = 2.25(3N_{114}+3N_{144}-N_1+N_4)$; **34(a-c)**

$\gamma_{15} = 2.25(3N_{115}+3N_{155}-N_1+N_5)$; $\gamma_{16} = 2.25(3N_{116}+3N_{166}-N_1+N_6)$; $\gamma_{23} = 2.25(3N_{223}+3N_{233}-N_2+N_3)$; **35(a-c)**

$\gamma_{24} = 2.25(3N_{224}+3N_{244}-N_2+N_4)$; $\gamma_{25} = 2.25(3N_{225}+3N_{255}-N_2+N_5)$; $\gamma_{26} = 2.25(3N_{226}+3N_{266}-N_2+N_6)$; **36(a-c)**

$\gamma_{34} = 2.25(3N_{334}+3N_{344}-N_3+N_4)$; $\gamma_{35} = 2.25(3N_{335}+3N_{355}-N_3+N_5)$; $\gamma_{36} = 2.25(3N_{336}+3N_{366}-N_3+N_6)$; **37(a-c)**

$\gamma_{45} = 2.25(3N_{445}+3N_{455}-N_4+N_5)$; $\gamma_{46} = 2.25(3N_{446}+3N_{466}-N_4+N_6)$; $\gamma_{56} = 2.25(3N_{556}+3N_{566}-N_4+N_6)$ **38(a-c)**

where N_i = Response Function for the Pure Component, i

2.2.5. HPNFRC SCHEFFE'S (6, 3) MIXTURE DESIGN MODEL

When we substitute Eqns. (8)-(38) into Eqn. (7), we obtain the mixture design model for HPNFRC based on Scheffe's (6,3) lattice.

2.2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE HPNFRC SCHEFFE'S (6,3) DESIGN LATTICE AT INITIAL EXPERIMENTAL TEST POINT[IETP] AND EXPERIMENTAL (CONTROL) TEST POINT[ECTP]

A. AT THE INITIAL EXPERIMENTAL TEST POINTS [IETP]

Based on Eqn. (1), the requirement of simplex lattice design criteria makes it impossible to use the conventional mix ratios such as 1:2:4 etc., at a given water/cement ratio for the actual mix ratio. Thus, there is need for the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix ratios are always chosen for the six vertices of Scheffe's (6,3) lattice as follows :

A_1 (0.67:1:1.7:2:0.5:0.5); A_2 (0.56:1:1.6:1.8:0.8:0.8); A_3 (0.5:1:1.2:1.7:1:1); A_4 (0.7:1:1:1.8:1.2:1.2);

A_5 (0.75:1:1.3:1.2:1.5:1.5), and A_6 (0.80:1:1.3:1.2:0.9:0.9) **(39)**

Which represent Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate, Polypropylene Fibre and Nylon Fibre respectively. For the pseudo mix ratio, the following corresponding mix ratios at the vertices for six component mixtures from Eqn. (3) are always chosen:

$A_1(1{:}0{:}0{:}0{:}0{:}0)$, $A_2(0{:}1{:}0{:}0{:}0{:}0)$, $A_3(0{:}0{:}1{:}0{:}0{:}0)$, $A_4(0{:}0{:}0{:}1{:}0{:}0)$, $A_5(0{:}0{:}0{:}0{:}1{:}0)$ and $A_6(0{:}0{:}0{:}0{:}0{:}1)$ **(40)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa, Eqns. (5) and (6) are used. Substituting the mix ratios from point A_1 in Eqn.(39) into Eqn. (5) yields:

$$\begin{pmatrix} 0.67 \\ 1 \\ 1.7 \\ 2 \\ 0.5 \\ 0.5 \end{pmatrix} = \begin{pmatrix} A_{111} & A_{112} & A_{113} & A_{114} & A_{115} & A_{116} \\ A_{221} & A_{222} & A_{223} & A_{224} & A_{225} & A_{226} \\ A_{331} & A_{332} & A_{333} & A_{334} & A_{335} & A_{336} \\ A_{441} & A_{442} & A_{443} & A_{444} & A_{445} & A_{446} \\ A_{551} & A_{552} & A_{553} & A_{554} & A_{555} & A_{556} \\ A_{661} & A_{662} & A_{663} & A_{664} & A_{665} & A_{666} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ \mathbf{0} \\ 0 \\ 0 \\ 0 \end{pmatrix} \qquad \textbf{(41)}$$

Transforming the R.H.S matrix and solving, we obtain: $A_{111}= 0.67$; $A_{221}= 1$; $A_{331}= 1.7$; $A_{441}= 2$; $A_{551}= 0.5$; $A_{661}= 0.5$. The same approach is used to obtain the remaining values as shown in Eqn. (42)

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 & 1.5 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 & 1.5 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} \qquad \textbf{(42)}$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(42) will yield the corresponding actual mix ratios. For instance, considering point A_{112} we have:

$$\begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \\ Z_6 \end{pmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 & 0.75 \\ 1.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 1.7 & 1.6 & 1.2 & 1.0 & 1.3 & 1.3 \\ 2.0 & 1.8 & 1.7 & 1.8 & 1.2 & 1.2 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 & 1.5 \\ 0.5 & 0.8 & 1.0 & 1.2 & 1.5 & 1.5 \end{pmatrix} \begin{pmatrix} 0.67 \\ 0.33 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0.63 \\ 1.0 \\ 1.67 \\ 1.90 \\ 1.60 \\ 1.60 \end{pmatrix} \qquad \textbf{(43)}$$

Solving, $Z_1 = 0.63$; $Z_2 = 1.00$; $Z_3 = 1.67$’ $Z_4 = 1.90$; $Z_5 = 1.60$. And for the remaining mid-point mix ratios, the same approach goes. Thus, fifty-six (56) experimental tests will be carried out in order to generate the polynomial coefficients. Table 1 depicts the corresponding mix ratios for the HPNFRC based on Scheffe’s (6,3) Lattice.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio For HPNFRC Based On Scheffe’s (6,3) Lattice For IETP

S/N	**IETP**	**PSEUDO COMPONENT**						**RESPONSE SYMBOL**	**ACTUAL COMPONENT**					
		X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**Z_6**
1.	**E_1**	1	0	0	0	0	0	N_1	0.67	1.00	1.70	2.0	0.5	0.5
2.	**E_2**	0	1	0	0	0	0	N_2	0.56	1.00	1.60	1.8	0.8	0.8
3.	**E_3**	0	0	1	0	0	0	N_3	0.50	1.00	1.20	1.7	1.0	1.0
4.	**E_4**	0	0	0	1	0	0	N_4	0.70	1.00	1.00	1.8	1.2	1.2
5.	**E_5**	0	0	0	0	1	0	N_5	0.75	1.00	1.30	1.2	1.5	1.5
6.	**E_6**	0	0	0	0	0	1	N_6	0.63	1.00	1.67	1.9	1.6	1.6
7.	**E_{112}**	0.67	033	0	0	0	0	N_{112}	0.63	1.00	1.67	1.9	1.6	1.6

8.	$\mathbf{E}_{122}$	0.67	0	0.33	0	0	0	N_{122}	0.61	1.00	1.54	1.9	0.6	0.6
9.	$\mathbf{E}_{113}$	0.67	0	0	0.33	0	0	N_{113}	0.56	1.00	1.37	1.8	0.8	0.8
10.	$\mathbf{E}_{133}$	0.67	0	0	0	0.33	0	N_{133}	0.68	1.00	1.47	1.9	0.7	0.7
11.	$\mathbf{E}_{114}$	0.67	0	0	0	0	0.33	N_{114}	0.69	1.00	1.23	1.8	0.9	0.9
12.	$\mathbf{E}_{144}$	0	0.67	0.33	0	0	0	N_{144}	0.70	1.00	1.57	1.7	0.8	0.8
13.	$\mathbf{E}_{115}$	0	0.67	0	0.33	0	0	N_{115}	0.72	1.00	1.43	1.4	1.1	1.1
14.	$\mathbf{E}_{155}$	0	0.67	0	0	0.33	0	N_{155}	0.55	1.00	1.40	1.7	0.8	0.8
15.	$\mathbf{E}_{116}$	0	0.67	0	0	0	0.33	N_{116}	0.52	1.00	1.20	1.7	0.9	0.9
16.	$\mathbf{E}_{166}$	0	0	0.67	0.33	0	0	N_{166}	0.61	1.00	1.67	1.8	0.9	0.9
17.	$\mathbf{E}_{223}$	0	0	0.67	0	0.33	0	N_{223}	0.66	1.00	1.73	1.8	1.0	1.0
18.	$\mathbf{E}_{233}$	0	0	0.67	0	0	0.33	N_{233}	0.63	1.00	1.50	1.6	0.7	0.7
19.	$\mathbf{E}_{224}$	0	0	0	0.67	0.33	0	N_{224}	0.69	1.00	1.40	1.4	0.6	0.6
20.	$\mathbf{E}_{244}$	0	0	0	0.67	0	0.33	N_{244}	0.57	1.00	1.13	1.7	1.0	1.0
21.	$\mathbf{E}_{225}$	0	0	0	0	0.67	0.33	N_{225}	0.64	1.00	1.07	1.7	1.1	1.1
22.	$\mathbf{E}_{255}$	0.50	0.50	0	0	0	0	N_{255}	0.68	1.00	1.70	2.0	0.5	0.5
23.	$\mathbf{E}_{226}$	0.50	0	0.50	0	0	0	N_{226}	0.58	1.00	1.60	1.8	0.8	0.8
24.	$\mathbf{E}_{266}$	0.50	0	0	0.50	0	0	N_{266}	0.52	1.00	1.20	1.7	1.0	1.0
25.	$\mathbf{E}_{334}$	0.50	0	0	0	0.50	0	N_{334}	0.76	1.00	1.00	1.8	1.2	1.2
26.	$\mathbf{E}_{344}$	0.50	0	0	0	0	0.50	N_{344}	0.77	1.00	1.30	1.2	1.5	1.5
27.	$\mathbf{E}_{335}$	0	0.50	0.50	0	0	0	N_{335}	0.67	1.00	1.67	1.9	1.6	1.6
28.	$\mathbf{E}_{355}$	0	0.50	0	0.50	0	0	N_{355}	0.69	1.00	1.67	1.9	1.6	1.6
29.	$\mathbf{E}_{336}$	0	0.50	0	0	0.50	0	N_{336}	0.69	1.00	1.54	1.9	0.6	0.6
30.	$\mathbf{E}_{366}$	0	0.50	0	0	0	0.50	N_{366}	0.58	1.00	1.37	1.8	0.8	0.8
31.	$\mathbf{E}_{445}$	0	0	0.50	0.50	0	0	N_{445}	0.69	1.00	1.47	1.9	0.7	0.7
32.	$\mathbf{E}_{455}$	0	0	0.50	0	0.50	0	N_{455}	0.69	1.00	1.23	1.8	0.9	0.9
33.	$\mathbf{E}_{446}$	0	0	0.50	0	0	0.50	N_{446}	0.74	1.00	1.57	1.7	0.8	0.8
34.	$\mathbf{E}_{466}$	0	0	0	0.50	0.50	0	N_{466}	0.74	1.00	1.43	1.4	1.1	1.1
35.	$\mathbf{E}_{556}$	0	0	0	0.50	0	0.50	N_{556}	0.57	1.00	1.40	1.7	0.8	0.8
36.	$\mathbf{E}_{566}$	0	0	0	0	0.50	0.50	N_{566}	0.56	1.00	1.20	1.7	0.9	0.9
37.	$\mathbf{E}_{123}$	0.80	0.20	0	0	0	0	N_{123}	0.64	1.00	1.67	1.8	0.9	0.9
38.	$\mathbf{E}_{124}$	0.80	0	0.20	0	0	0	N_{124}	0.66	1.00	1.73	1.8	1.0	1.0
39.	$\mathbf{E}_{125}$	0.80	0	0	0.20	0	0	N_{125}	0.66	1.00	1.50	1.6	0.7	0.7
40.	$\mathbf{E}_{126}$	0.80	0	0	0	0.20	0	N_{126}	0.71	1.00	1.40	1.4	0.6	0.6
41.	$\mathbf{E}_{134}$	0.80	0	0	0	0	0.20	N_{134}	0.59	1.00	1.13	1.7	1.0	1.0
42.	$\mathbf{E}_{135}$	0	0.80	0.20	0	0	0	N_{135}	0.66	1.00	1.07	1.7	1.1	1.1
43.	$\mathbf{E}_{136}$	0	0.80	0	0.20	0	0	N_{136}	0.65	1.00	1.70	2.0	0.5	0.5
44.	$\mathbf{E}_{145}$	0	0.80	0	0	0.20	0	N_{145}	0.54	1.00	1.60	1.8	0.8	0.8

45.	**E_{146}**	0	0.80	0	0	0	0.20	N_{146}	0.53	1.00	1.20	1.7	1.0	1.0
46.	**E_{156}**	0	0	0.80	0.20	0	0	N_{156}	0.71	1.00	1.00	1.8	1.2	1.2
47.	**E_{234}**	0	0	0.80	0	0.20	0	N_{234}	0.73	1.00	1.30	1.2	1.5	1.5
48.	**E_{235}**	0	0	0.80	0	0	0.20	N_{235}	0.62	1.00	1.67	1.9	1.6	1.6
49.	**E_{236}**	0	0	0	0.80	0.20	0	N_{236}	0.62	1.00	1.68	1.9	1.6	1.6
50.	**E_{245}**	0	0	0	0.80	0	0.20	N_{245}	0.63	1.00	1.54	1.9	0.6	0.6
51.	**E_{246}**	0	0	0	0	0.80	0.20	N_{246}	0.57	1.00	1.37	1.8	0.8	0.8
52.	**E_{256}**	0.60	0.40	0	0	0	0	N_{256}	0.69	1.00	1.47	1.9	0.7	0.7
53.	**E_{345}**	0.60	0	0.40	0	0	0	N_{345}	0.73	1.00	1.23	1.8	0.9	0.9
54.	**E_{346}**	0.60	0	0	0.40	0	0	N_{346}	0.73	1.00	1.57	1.7	0.8	0.8
55.	**E_{356}**	0.60	0	0	0	0.40	0	N_{356}	0.70	1.00	1.43	1.4	1.1	1.1
56.	**E_{456}**	0.60	0	0	0	0	0.40	N_{456}	0.53	1.00	1.40	1.7	0.8	0.8

B. AT THE HPNFRC EXPERIMENTAL (CONTROL) TEST POINTS [ECTP]

Fifty - six (56) different control mix ratios will be made available for prediction of compressive strength which according to Scheffe’s (1958), their summation should not be greater than one. The same approach for component transformation adopted for the initial experimental points are also adopted for the control points and the results are shown in Table 2.

Table 2: Actual & Pseudo Mix Ratio Component Of HPNFRC Based On Scheffe ‘s (6,3) Lattice For ECTP

S/N	**ECTP**	**PSEUDO COMPONENT**						**RESPONSE SYMBOL**	**ACTUAL COMPONENT**					
		X_1	**X_2**	**X_3**	**X_4**	**X_5**	**X_6**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**	**Z_6**
1.	C_1	0.25	0.25	0.25	0.25	0	0	N_1	0.61	1	1.38	1.83	0.5	0.50
2.	C_2	0.25	0.25	0.25	0	0.25	0	N_2	0.62	1	1.45	1.68	0.8	0.8
3.	C_3	0.25	0.25	0	0.25	0.25	0	N_3	0.67	1	1.40	1.70	1	1
4.	C_4	0.25	0	0.25	0.25	0.25	0	N_4	0.66	1	1.30	1.68	1.2	1.2
5.	C_5	0	0.25	0.25	0.25	0.25	0	N_5	0.63	1	1.28	1.63	1.5	1.5
6.	C_6	0.20	0.20	0.20	0.20	0.20	0	N_6	0.64	1	1.36	1.70	0.65	0.65
7.	C_{112}	0.30	0.30	0.30	0.10	0	0	N_{112}	0.59	1	1.45	1.83	0.75	0.75
8.	C_{122}	0.30	0.30	0.30	0	0.10	0	N_{122}	0.59	1	1.48	1.77	0.85	0.85
9.	C_{113}	0.30	0.30	0	0.30	0.10	0	N_{113}	0.61	1	1.38	1.83	0.5	0.50
10.	C_{133}	0.30	0	0.30	0.30	0.10	0	N_{133}	0.62	1	1.45	1.68	0.8	0.8
11.	C_{114}	0	0.30	0.30	0.30	0.10	0	N_{114}	0.67	1	1.40	1.70	1	1
12.	C_{144}	0.10	0.30	0.30	0.30	0	0	N_{144}	0.66	1	1.30	1.68	1.2	1.2
13.	C_{115}	0.30	0.10	0.30	0.30	0	0	N_{115}	0.63	1	1.28	1.63	1.5	1.5
14.	C_{155}	0.30	0.10	0.30	0.30	0	0	N_{155}	0.64	1	1.36	1.70	0.65	0.65
15.	C_{116}	0.10	0.20	0.30	0.40	0	0	N_{116}	0.59	1	1.45	1.83	0.75	0.75
16.	C_{166}	0.30	0.20	0.10	0.40	0	0	N_{166}	0.59	1	1.48	1.77	0.85	0.85
17.	C_{223}	0.20	0.20	0.10	0.10	0.40	0	N_{223}	0.61	1	1.38	1.83	0.5	0.50

18.	C_{233}	0.30	0.10	0.30	0.20	0.10	0	N_{233}	0.62	1	1.45	1.68	0.8	0.8
19.	C_{224}	0.25	0.25	0.15	0.15	0.20	0	N_{224}	0.67	1	1.40	1.70	1	1
20.	C_{244}	0.30	0.30	0.20	0.10	0.10	0	N_{244}	0.66	1	1.30	1.68	1.2	1.2
21.	C_{225}	0.10	0.30	0.30	0.30	0	0	N_{225}	0.63	1	1.28	1.63	1.5	1.5
22.	C_{255}	0.25	0.25	0.25	0.25	0	0	N_{255}	0.65	1	1.38	1.83	0.5	0.50
23.	C_{226}	0.25	0.25	0.25	0	0.25	0	N_{226}	0.60	1	1.45	1.68	0.8	0.8
24.	C_{266}	0.25	0.25	0	0.25	0.25	0	N_{266}	0.69	1	1.40	1.70	1	1
25.	C_{334}	0.25	0	0.25	0.25	0.25	0	N_{334}	0.68	1	1.30	1.68	1.2	1.2
26.	C_{344}	0	0.25	0.25	0.25	0.25	0	N_{344}	0.65	1	1.28	1.63	1.5	1.5
27.	C_{335}	0.20	0.20	0.20	0.20	0.20	0	N_{335}	0.68	1	1.36	1.70	0.65	0.65
28.	C_{355}	0.30	0.30	0.30	0.10	0	0	N_{355}	0.65	1	1.45	1.83	0.75	0.75
29.	C_{336}	0.30	0.30	0.30	0	0.10	0	N_{336}	0.64	1	1.48	1.77	0.85	0.85
30.	C_{366}	0.30	0.30	0	0.30	0.10	0	N_{366}	0.68	1	1.38	1.83	0.5	0.50
31.	C_{445}	0.30	0	0.30	0.30	0.10	0	N_{445}	0.69	1	1.45	1.68	0.8	0.8
32.	C_{455}	0	0.30	0.30	0.30	0.10	0	N_{455}	0.68	1	1.40	1.70	1	1
33.	C_{446}	0.10	0.30	0.30	0.30	0	0	N_{446}	0.68	1	1.30	1.68	1.2	1.2
34.	C_{466}	0.30	0.10	0.30	0.30	0	0	N_{466}	0.63	1	1.28	1.63	1.5	1.5
35.	C_{556}	0.30	0.10	0.30	0.30	0	0	N_{556}	0.64	1	1.36	1.70	0.65	0.65
36.	C_{566}	0.10	0.20	0.30	0.40	0	0	N_{566}	0.59	1	1.45	1.83	0.75	0.75
37.	C_{123}	0.30	0.20	0.10	0.40	0	0	N_{123}	0.59	1	1.48	1.77	0.85	0.85
38.	C_{124}	0.20	0.20	0.10	0.10	0.40	0	N_{124}	0.64	1	1.38	1.83	0.5	0.50
39.	C_{125}	0.30	0.10	0.30	0.20	0.10	0	N_{125}	0.65	1	1.45	1.68	0.8	0.8
40.	C_{126}	0.25	0.25	0.15	0.15	0.20	0	N_{126}	0.69	1	1.40	1.70	1	1
41.	C_{134}	0.30	0.30	0.20	0.10	0.10	0	N_{134}	0.65	1	1.30	1.68	1.2	1.2
42.	C_{135}	0.10	0.30	0.30	0.30	0	0	N_{135}	0.66	1	1.28	1.63	1.5	1.5
43.	C_{136}	0.25	0.25	0.25	0.25	0	0	N_{136}	0.64	1	1.38	1.83	0.5	0.50
44.	C_{145}	0.25	0.25	0.25	0	0.25	0	N_{145}	0.66	1	1.45	1.68	0.8	0.8
45.	C_{146}	0.25	0.25	0	0.25	0.25	0	N_{146}	0.68	1	1.40	1.70	1	1
46.	C_{156}	0.25	0	0.25	0.25	0.25	0	N_{156}	0.69	1	1.30	1.68	1.2	1.2
47.	C_{234}	0	0.25	0.25	0.25	0.25	0	N_{234}	0.67	1	1.28	1.63	1.5	1.5
48.	C_{235}	0.20	0.20	0.20	0.20	0.20	0	N_{235}	0.68	1	1.36	1.70	0.65	0.65
49.	C_{236}	0.30	0.30	0.30	0.10	0	0	N_{236}	0.63	1	1.45	1.83	0.75	0.75
50.	C_{245}	0.30	0.30	0.30	0	0.10	0	N_{245}	0.72	1	1.48	1.77	0.85	0.85
51.	C_{246}	0.30	0.30	0	0.30	0.10	0	N_{246}	0.68	1	1.38	1.83	0.5	0.50
52.	C_{256}	0.30	0	0.30	0.30	0.10	0	N_{256}	0.60	1	1.45	1.68	0.8	0.8
53.	C_{345}	0	0.30	0.30	0.30	0.10	0	N_{345}	0.65	1	1.40	1.70	1	1
54.	C_{346}	0.10	0.30	0.30	0.30	0	0	N_{346}	0.65	1	1.30	1.68	1.2	1.2

55.	C_{356}	0.30	0.10	0.30	0.30	0	0	N_{356}	0.67	1	1.28	1.63	1.5	1.5
56.	C_{456}	0.30	0.10	0.30	0.30	0	0	N_{456}	0.63	1	1.36	1.70	0.65	0.65

2.2.7. MEASUREMENT OF QUANTITIES OF HPNFRC MATERIALS IN THE LABORATORY

The actual component as transformed from Table (1) and (2) can now be used to measure out the quantities of Water/Cement Ratio(Z_1),Cement(Z_2),Fine Aggregate(Z_3),Coarse Aggregate(Z_4),Polypropylene Fibre (Z_5) and Nylon Fibre (Z_6) respectively in their respective ratios for the concrete cube strength test using a weighing balance of 50kg capacity at the laboratory.

Mathematically, from the works of Nwachukwu and others (2024a), Measured Quantity, M^Q of HPNFRC Mixture is given by Eqn.(44)

$$M^Q = \frac{X}{T} * Y \qquad \textbf{(44)}$$

Where, X = Individual mix ratio at each test point = 1.7 for Z_3 at E_1 in Table 1, for example.

T = Sum of mix ratios at each test point = 6.37 at E_1 in Table 1, for example

And Y = Average weight of Concrete cube/beam/cylinder

Samples of measured quantities for compressive strength test can be seen from the works of Nwachukwu and others (2024 b).

2.3. METHOD

2.3.1. HPNFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR COMPRESSIVE STRENGHT TEST

The specimens for the compressive strength are usually concrete cubes. The cubes were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 112 mix ratios were to be used to produce 224 prototype concrete cubes. Fifty- six (56) out of the 112 mix ratios were used as control mix ratios to produce 112 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (38). Curing commenced 24hours after moulding. The specimens were removed from the moulds and were placed in clean water for curing. After 28days of curing the specimens were taken out of the curing tank for the determination of the compressive strength

2.3.2. HPNFRC COMPRESSIVE STRENGTH TEST PROCEDURE/CALCULATION

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline .Two samples were crushed for each mix ratio and in each case, the compressive strength was then calculated using Eqn.(44)

$$\text{Compressive Strength}[\textbf{MPa}] = \frac{\text{Average failure Load,P (N)}}{\text{Cross- sectional Area, A (mm}^2\text{)}} \qquad \textbf{(44)}$$

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3. RESULTS AND DISCUSSION

3.1. HPNFRC RESPONSES FOR THE INITIAL EXPERIMENTAL TESTS POINTS [IETP] AND EXPERIMENTAL (CONTROL) TEST POINTS [ECTP].

The results of the compressive strength ($R_{esponse,} N_i$) of HPNFRC based on a 28-days strength is presented in Table 3. The initial experimental test responses are calculated from Eqn..(44)

Table 3: 28th Day Compressive Strength (Responses) Test Results for HPNFRC Based on Scheffe's (6, 3) Model for the IETP and ECTP.

S/N	**POINTS**		**EXPERIMENTAL NUMBER**		**RESPONSES [MPa]**		**RESPONSE SYMBOL**	**AVERAGE RESPONSE [MPa]**	
	IETP	**ECTP**	**IETP**	**ECTP**	**AT IETP**	**AT ECTP**		**AT IETP**	**AT ECTP**

1.	**E_1**	C_1	HPNFRC/ E_I A	HPNFRC/ C_I A	**64.77**	**60.66**	N_1	**62.00**	**60.85**
			HPNFRC/ E_I B	HPNFRC/ C_I B	**65.23**	**61.03**			
2.	**E_2**	C_2	HPNFRC/ E_2 A	HPNFRC/ C_2 A	**63.44**	**61.11**	N_2	**63.34**	**61.07**
			HPNFRC/ E_2 B	HPNFRC/ C_2 B	**63.23**	**61.08**			
3.	**E_3**	C_3	HPNFRC/ E_3 A	HPNFRC/ C_3 A	**58.98**	**56.88**	N_3	**58.77**	**56.82**
			HPNFRC/ E_3 B	HPNFRC/ C_3 B	**58.56**	**56.76**			
4.	**E_4**	C_4	HPNFRC/ E_4 A	HPNFRC/ C_4 A	**56.45**	**54.44**	N_4	**56.79**	**54.42**
			HPNFRC/ E_4 B	HPNFRC/ C_4 B	**57.12**	**54.39**			
5.	**E_5**	C_5	HPNFRC/ E_5 A	HPNFRC/ C_5 A	**51.12**	**50.54**	N_5	**50.89**	**50.65**
			HPNFRC/ E_5 B	HPNFRC/ C_5 B	**50.65**	**50.75**			
6.	**E_6**	C_6	HPNFRC/ E_6 A	HPNFRC/ C_6 A	**71.38**	**69.49**	N_6	**71.36**	**69.49**
			HPNFRC/ E_6 B	HPNFRC/ C_6 B	**71.34**	**69.48**			
7.	**E_{112}**	C_{112}	HPNFRC/ E_7 A	HPNFRC/ C_7 A	**42.56**	**45.56**	N_{112}	**42.84**	**46.00**
			HPNFRC/ E_7 B	HPNFRC/ C_7 B	**43.11**	**46.43**			
8.	**E_{122}**	C_{122}	HPNFRC/ E_8 A	HPNFRC/ C_8 A	**56.43**	**57.43**	N_{122}	**56.77**	**57.50**
			HPNFRC/ E_8 B	HPNFRC/ C_8 B	**57.11**	**57.56**			
9.	**E_{113}**	C_{113}	HPNFRC/ E_9 A	HPNFRC/ C_9 A	**47.23**	**49.67**	N_{113}	**47.18**	**49.78**
			HPNFRC/ E_9 B	HPNFRC/ C_9 B	**47.12**	**49.88**			
10.	**E_{133}**	C_{133}	HPNFRC/ E_{10} A	HPNFRC/ C_{10} A	**53.33**	**52.23**	N_{133}	**53.16**	**52.00**

			HPNFRC/ E_{10} B	HPNFRC/ C_{10} B	**52.98**	**51.76**			
11.	**E_{114}**	C_{114}	HPNFRC/ E_{11}A	HPNFRC/ C_{11}A	**64.34**	**61.22**	N_{114}	**64.33**	**61.28**
			HPNFRC/ E_{11} B	HPNFRC/ C_{11} B	**64.32**	**61.34**			
12.	**E_{144}**	C_{144}	HPNFRC/ E_{12}A	HPNFRC/ C_{12}A	**58.34**	**57.43**	N_{144}	**58.39**	**57.44**
			HPNFRC/ E_{12}B	HPNFRC/ C_{12}B	**58.43**	**57.45**			
13.	**E_{115}**	C_{115}	HPNFRC/ E_{13} A	HPNFRC/ C_{13} A	**54.32**	**55.43**	N_{115}	**54.35**	**55.54**
			HPNFRC/ E_{13}B	HPNFRC/ C_{13}B	**54.38**	**55.65**			
14.	**E_{155}**	C_{155}	HPNFRC/ E_{14} A	HPNFRC/ C_{14} A	**59.32**	**60.43**	N_{155}	**59.35**	**60.44**
			HPNFRC/ E_{14} B	HPNFRC/ C_{14} B	**59.37**	**60.45**			
15.	**E_{116}**	C_{116}	HPNFRC/ E_{15} A	HPNFRC/ C_{15} A	**38.44**	**41.23**	N_{116}	**38.46**	**40.79**
			HPNFRC/ E_{15} B	HPNFRC/ C_{15} B	**38.47**	**40.34**			
16.	**E_{166}**	C_{166}	HPNFRC/ E_{16}A	HPNFRC/ C_{16}A	**57.43**	**59.32**	N_{166}	**57.82**	**59.34**
			HPNFRC/ E_{16} B	HPNFRC/ C_{16} B	**58.21**	**59.36**			
17.	**E_{223}**	C_{223}	HPNFRC/ E_{17} A	HPNFRC/ C_{17} A	**43.32**	**45.45**	N_{223}	**43.77**	**45.79**
			HPNFRC/ E_{17} B	HPNFRC/ C_{17} B	**44.21**	**46.12**			
18.	**E_{233}**	C_{233}	HPNFRC/ E_{18} A	HPNFRC/ C_{18} A	**37.67**	**37.34**	N_{233}	**37.84**	**37.36**
			HPNFRC/ E_{18} B	HPNFRC/ C_{18} B	**38.00**	**37.38**			
19.	**E_{224}**	C_{224}	HPNFRC/ E_{19} A	HPNFRC/ C_{19} A	**37.23**	**39.34**	N_{224}	**37.25**	**39.40**
			HPNFRC/ E_{19} B	HPNFRC/ C_{19} B	**37.27**	**39.45**			
20.	**E_{244}**	C_{244}	HPNFRC/ E_{20} A	HPNFRC/ C_{20} A	**48.32**	**50.32**	N_{244}	**48.72**	**50.28**

			HPNFRC/ E_{20} B	HPNFRC/ C_{20} B	**49.11**	**50.23**			
21.	**E_{225}**	C_{225}	HPNFRC/ E_{21} A	HPNFRC/ C_{21} A	**54.91**	**55.43**	N_{225}	**54.92**	**55.20**
			HPNFRC/ E_{21} B	HPNFRC/ C_{21} B	**54.92**	**54.97**			
22.	**E_{255}**	C_{255}	HPNFRC/ E_{22}A	HPNFRC/ C_{22}A	**50.33**	**51.33**	N_{255}	**50.33**	**51.34**
			HPNFRC/ E_{22} B	HPNFRC/ C_{22} B	**50.32**	**51.34**			
23.	**E_{226}**	C_{226}	HPNFRC/ E_{23} A	HPNFRC/ C_{23} A	**52.34**	**53.67**	N_{226}	**52.32**	**53.77**
			HPNFRC/ E_{23} B	HPNFRC/ C_{23} B	**52.32**	**53.87**			
24.	**E_{266}**	C_{266}	HPNFRC/ E_{24} A	HPNFRC/ C_{24} A	**64.31**	**68.00**	N_{266}	**64.71**	**67.69**
			HPNFRC/ E_{24} B	HPNFRC/ C_{24} B	**65.11**	**67.38**			
25.	**E_{334}**	C_{334}	HPNFRC/ E_{25} A	HPNFRC/ C_{25} A	**52.23**	**49.65**	N_{334}	**52.62**	**49.94**
			HPNFRC/ E_{25} B	HPNFRC/ C_{25} B	**53.00**	**50.23**			
26.	**E_{344}**	C_{344}	HPNFRC/ E_{26} A	HPNFRC/ C_{26} A	**49.65**	**50.34**	N_{344}	**49.88**	**50.44**
			HPNFRC/ E_{26} B	HPNFRC/ C_{26} B	**50.11**	**50.54**			
27.	**E_{335}**	C_{335}	HPNFRC/ E_{27} A	HPNFRC/ C_{27} A	**52.33**	**54.32**	N_{335}	**52.39**	**54.46**
			HPNFRC/ E_{27} B	HPNFRC/ C_{27} B	**52.45**	**54.38**			
28.	**E_{355}**	C_{355}	HPNFRC/ E_{28} A	HPNFRC/ C_{28} A	**49.22**	**49.23**	N_{355}	**49.67**	**49.13**
			HPNFRC/ E_{28} B	HPNFRC/ C_{28} B	**50.11**	**50.11**			
29.	**E_{336}**	C_{336}	HPNFRC/ E_{29} A	HPNFRC/ C_{29} A	**40.23**	**43.34**	N_{336}	**40.24**	**49.67**
			HPNFRC/ E_{29} B	HPNFRC/ C_{29} B	**40.24**	**44.12**			
30.	**E_{366}**	C_{366}	HPNFRC/ E_{30} A	HPNFRC/ C_{30} A	**53.32**	**54.22**	N_{366}	**53.33**	**54.67**

			HPNFRC/ E_{30} B	HPNFRC/ C_{30} B	**53.34**	**55.11**			
31.	**E_{445}**	C_{445}	HPNFRC/ E_{31} A	HPNFRC/ C_{31} A	**47.54**	**45.32**	N_{445}	**47.83**	**45.27**
			HPNFRC/ E_{31} B	HPNFRC/ C_{31} B	**48.12**	**45.22**			
32.	**E_{455}**	C_{455}	HPNFRC/ E_{32} A	HPNFRC/ C_{32} A	**52.21**	**55.23**	N_{455}	**52.67**	**55.29**
			HPNFRC/ E_{32} B	HPNFRC/ C_{32} B	**53.12**	**55.34**			
33.	**E_{446}**	C_{446}	HPNFRC/ E_{33} A	HPNFRC/ C_{33} A	**51.23**	**52.45**	N_{446}	**51.68**	**52.47**
			HPNFRC/ E_{33} B	HPNFRC/ C_{33} B	**52.12**	**52.48**			
34.	**E_{466}**	C_{466}	HPNFRC/ E_{34} A	HPNFRC/ C_{34} A	**54.34**	**50.67**	N_{466}	**53.79**	**50.62**
			HPNFRC/ E_{34}B	HPNFRC/ C_{34}B	**55.13**	**50.56**			
35.	**E_{556}**	C_{556}	HPNFRC/ E_{35}A	HPNFRC/ C_{35}A	**57.23**	**58.34**	N_{556}	**57.68**	**58.28**
			HPNFRC/ E_{35}B	HPNFRC/ C_{35}B	**58.12**	**58.21**			
36.	**E_{566}**	C_{566}	HPNFRC/ E_{36}A	HPNFRC/ C_{36}A	**50.43**	**48.34**	N_{566}	**50.72**	**48.33**
			HPNFRC/ E_{36} B	HPNFRC/ C_{36} B	**51.00**	**48.32**			
37.	**E_{123}**	C_{123}	HPNFRC/ E_{37}A	HPNFRC/ C_{37}A	**46.32**	**46.45**	N_{123}	**46.43**	**46.51**
			HPNFRC/ E_{37}B	HPNFRC/ C_{37}B	**46.54**	**46.56**			
38.	**E_{124}**	C_{124}	HPNFRC/ E_{38} A	HPNFRC/ C_{38} A	**50.32**	**54.33**	N_{124}	**50.77**	**54.82**
			HPNFRC/ E_{38} B	HPNFRC/ C_{38} B	**51.21**	**55.31**			
39.	**E_{125}**	C_{125}	HPNFRC/ E_{39} A	HPNFRC/ C_{39} A	**46.34**	**43.34**	N_{125}	**46.37**	**43.73**
			HPNFRC/ E_{39} B	HPNFRC/ C_{39} B	**46.39**	**44.12**			
40.	**E_{126}**	C_{126}	HPNFRC/ E_{40} A	HPNFRC/ C_{40} A	**40.32**	**41.23**	N_{126}	**40.43**	**41.67**

			HPNFRC/ E_{40} B	HPNFRC/ C_{40} B	**40.54**	**42.10**			
41.	**E_{134}**	C_{134}	HPNFRC/ E_{41} A	HPNFRC/ C_{41} A	**46.32**	**47.35**	N_{134}	**46.73**	**47.40**
			HPNFRC/ E_{41} B	HPNFRC/ C_{41} B	**47.13**	**47.45**			
42.	**E_{135}**	C_{135}	HPNFRC/ E_{42} A	HPNFRC/ C_{42} A	**52.23**	**54.32**	N_{135}	**52.68**	**54.72**
			HPNFRC/ E_{42} B	HPNFRC/ C_{42} B	**53.12**	**55.12**			
43.	**E_{136}**	C_{136}	HPNFRC/ E_{43} A	HPNFRC/ C_{43} A	**50.68**	**50.21**	N_{136}	**50.95**	**54.72**
			HPNFRC/ E_{43} B	HPNFRC/ C_{43} B	**51.21**	**50.23**			
44.	**E_{145}**	C_{145}	HPNFRC/ E_{44} A	HPNFRC/ C_{44} A	**51.12**	**49.32**	N_{145}	**51.39**	**49.39**
			HPNFRC/ E_{44}B	HPNFRC/ C_{44}B	**51.65**	**49.43**			
45.	**E_{146}**	C_{146}	HPNFRC/ E_{45} A	HPNFRC/ C_{45} A	**57.23**	**56.39**	N_{146}	**57.28**	**56.39**
			HPNFRC/ E_{45} B	HPNFRC/ C_{45} B	**57.33**	**56.38**			
46.	**E_{156}**	C_{156}	HPNFRC/ E_{46} A	HPNFRC/ C_{46} A	**70.33**	**65.54**	N_{156}	**70.68**	**65.95**
			HPNFRC/ E_{46} B	HPNFRC/ C_{46} B	**71.02**	**66.35**			
47.	**E_{234}**	C_{234}	HPNFRC/ E_{47} A	HPNFRC/ C_{47} A	**68.23**	**67.43**	N_{234}	**68.68**	**67.46**
			HPNFRC/ E_{47} B	HPNFRC/ C_{47} B	**69.12**	**67.48**			
48.	**E_{235}**	C_{235}	HPNFRC/ E_{48} A	HPNFRC/ C_{48} A	**43.23**	**43.56**	N_{235}	**43.40**	**43.61**
			HPNFRC/ E_{48} B	HPNFRC/ C_{48} B	**43.56**	**43.65**			
49.	**E_{236}**	C_{236}	HPNFRC/ E_{49} A	HPNFRC/ C_{49} A	**45.44**	**46.34**	N_{236}	**45.21**	**46.39**
			HPNFRC/ E_{49} B	HPNFRC/ C_{49} B	**44.98**	**46.43**			
50.	**E_{245}**	C_{245}	HPNFRC/ E_{50} A	HPNFRC/ C_{50} A	**67.39**	**67.78**	N_{245}	**67.76**	**67.51**

			HPNFRC/ E_{50} B	HPNFRC/ C_{50} B	**68.12**	**67.23**			
51.	**E_{246}**	C_{246}	HPNFRC/ E_{51} A	HPNFRC/ C_{51} A	**52.21**	**51.56**	N_{246}	**52.15**	**51.61**
			HPNFRC/ E_{51} B	HPNFRC/ C_{51} B	**52.08**	**51.65**			
52.	**E_{256}**	C_{256}	HPNFRC/ E_{52} A	HPNFRC/ C_{52} A	**49.34**	**56.34**	N_{256}	**49.79**	**56.73**
			HPNFRC/ E_{52} B	HPNFRC/ C_{52} B	**50.23**	**57.12**			
53.	**E_{345}**	C_{345}	HPNFRC/ E_{53} A	HPNFRC/ C_{53} A	**56.34**	**54.54**	N_{345}	**56.39**	**54.49**
			HPNFRC/ E_{53} B	HPNFRC/ C_{53} B	**56.43**	**54.43**			
54.	**E_{346}**	C_{346}	HPNFRC/ E_{54} A	HPNFRC/ C_{54} A	**37.34**	**38.22**	N_{346}	**37.79**	**38.26**
			HPNFRC/ E_{54} B	HPNFRC/ C_{54} B	**38.23**	**38.29**			
55.	**E_{356}**	C_{356}	HPNFRC/ E_{55} A	HPNFRC/ C_{55} A	**58.21**	**57.34**	N_{356}	**58.23**	**57.36**
			HPNFRC/ E_{55} B	HPNFRC/ C_{55} B	**58.24**	**57.38**			
56.	**E_{456}**	C_{456}	HPNFRC/ E_{56} A	HPNFRC/ C_{56} A	**63.54**	**62.28**	N_{456}	**63.21**	**62.53**
			HPNFRC/ E_{56} B	HPNFRC/ C_{56} B	**62.87**	**62.78**			

3.2. SCHEFFE'S (6,3) MATHEMATICAL MODEL FOR THE HPNFRC RESPONSES

Substituting the values of the compressive strengths (responses) from Table 3 into Eqns.(8) through (38), we obtain the coefficients ($\beta_1, \beta_2, \beta_3$ ....., β_{12}, β_{13}……..β_{123}, β_{124}, …..β_{456} …., γ_{12}, γ_{13}, γ_{56}), in MPa of the Scheffe's third degree model. Substituting the values of these coefficients into Eqn. (7), we obtain the mathematical model for the optimization of the compressive strength of HPNFRC based on Scheffe's (6,3) lattice.

3.3. SCHEFFE'S (6,3) MODEL RESPONSES FOR HPNFRC AT ECTP.

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4,… C_{112}, … C_{456} of Table 2 into revised Eqn.(7) , we obtain the third degree model responses for the control points of HPNFRC.

3.4 VALIDATION OF THE SCHEFFE'S (6,3) MODEL FOR HPNFRC

In order to check if there is any significant difference between the compressive strength results (lab responses) given in Table 3 and model responses from the control points determined through session 3.3, the Student's – T - test was adopted. The procedures for using the Student's – T - test have been explained by Nwachukwu and others (2022 c). The result of the test shows that there is no significant difference between the experimental results and model responses. Therefore, the Scheffe's model is validated and is very adequate for predicting the compressive strength of HPNFRC based on Scheffe's (6,3) lattice.

3.5. DISCUSSION OF RESULTS

The highest compressive strength of HPNFRC based on Scheffe's (6, 3) lattice is 71.36MPa .This value is higher than the maximum value obtained as 60.05MPa by Nwachukwu and others (2022j) based on Scheffe's second degree model. The maximum value also corresponds to mix ratio of 0.63:1.00:1.67:1.90:1.60:1.60 for water/cement ratio, cement, fine aggregate, coarse aggregate, polypropylene fibre and nylon fibre respectively. Similarly, the lowest compressive strength was found to be 37.25 MPa which corresponds to mix ratio of 0.69:1.00:1.40:1.40:0.60:0.60. The optimum values from both models are found to be greater than the minimum value specified by the American Concrete Institute for the compressive strength of good concrete and also minimum standard (of 4500psi or 30.75MPa) specified by the American Society of Testing and Machine, ASTM C 469 and ASTM C 39. Thus, using the model compressive strength of HPNFRC of all points (1 - 56) in the simplex can be evaluated based on Scheffe's third degree model**.**

4. CONCLUSION

So far, Scheffe's Third Degree Optimization/Regression Model, for six component mixtures, Scheffe's (6,3) has been presented . It was used to predict the mix proportions as well as a model for predicting the compressive strength of HPNFRC cubes. By using Scheffe's (6,3) simplex model, the values of the compressive strength were obtained for HPNFRC at all 56 points. The result of the student's t-test confirmed that there is a good correlation between the strengths predicted by the models and the corresponding experimentally observed results. The optimum (maximum) compressive strength of HPNFRC predicted by the Scheffe's (6,3) model is 71.36 MPa while the minimum value is 37.25 MPa. However, both values meet the minimum standard requirement (of 20 MPa and 30.75MPa) stipulated by American Concrete Institute (ACI) and American Society of Testing and Machine, ASTM C 469 or ASTM C 39 respectively, for the compressive strength of good concrete. Thus, with the Scheffe's (6,3) model, any desired strength of HPNFRC given any mix proportions can be easily predicted and evaluated and vice versa. By the utilization of this Scheffe's mathematical model, the problem of having to go through vigorous, time-consuming and laborious mixture design procedures to obtain a desiring strength of HPNFRC has been drastically reduced. Finally, using the Scheffe's optimization techniques has not only helped us to find alternative replacement for conventional expensive steel reinforcement. It has also enabled us to reduce pollution in the environment by allowing provision for the incorporation of waste polypropylene and nylon that could have posed as danger to the smooth running of drainage openings as replacement for conventional reinforcement in the Reinforced Concrete Production [RCP].

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