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Mechanical Behaviour of a Stressed Structural Plate Element Under Odd Energy Functional.

Uzoukwu C. S., Ibearugbulem O. M., Ikpa P. N., Nwokorobia G.C., Iroegbu R. U, Ogbonna S. N., Efiok E. N., Emejulu D. A., Chike K. O., Anyanwu V. K.

Civil Engineering Department, Federal University of Technology Owerri, Imo State, Nigeria
Email: cskambassadors@gmail.com, Phone: +2348130131114

ABSTRACT

The structural rectangular Clamped-Simple-Clamped-Simple was used as case study in the research work. Unlike in the cases of Ritz and Garlerkin, 3rd order energy functional was adopted in the formulation of the critical Buckling load of the plate element under consideration. From the first principle, the shape functions were derived, the integral values of the differentiated shape functions of the various boundary cases were also derived. Further integration of these differential values gave rise to the, the stiffness coefficients of the various boundary conditions. Upon the addition of external load on the formulated strain energy, the Third Order Total Potential Energy Functional was generated. The integration of this last value with respect to the amplitude gave rise to the Governing equation. The final stage of the work is the minimization of the governing equation from where the critical buckling load equations were formulated. Adopting aspect ratios of different ranges, the non-dimensional buckling load parameters were obtained for the plate element under consideration.

Key words: Total Potential Energy Functional, Buckling Coefficient, Flexural Rigidity 3rd Order Functional.

Ew *Deflection in horizontal Direction*

Ns *Deflection in Vertical Direction*

Introduction

A CLAMPED SIMPLE CLAMPED SIMPLE isotropic thin plate may buckle in a various ways depending on its dimensions, the loading, and the method of support. The simplest form of buckling arises when compressive loads are applied to plates with simply supported opposite edges and the unloaded edges are free. The buckling analysis of a CLAMPED SIMPLE CLAMPED SIMPLE isotropic thin plate enables one to determine the critical buckling loads of the plate element and also to determine the corresponding buckling configuration of equilibrium.

The analysis is applicable to plate either CLAMPED SIMPLE CLAMPED SIMPLE isotropic plate element having either straight or curves boundaries, and also possesses three dimensions known as the primary, secondary and tertiary dimension. The CSCS plate posses tertiary dimension very small compared to other dimensions. They also display uniform material properties in all directions , making it direction independent plate. Stability analysis has been a subject of study in solid structural mechanics long before now. This is the study of plate element as it changes from stable condition to unstable state passing through neutral state. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time, other researchers have gotten solution using both the Second and Fourth the Order energy functional for Buckling of plate. None of the scholars have any work on buckling of plate using odd order energy functional. The resolution of the buckling tendency of CLAMPED SIMPLE CLAMPED SIMPLE isotropic plate using 3rd order energy functional is the gap the work tends to fill. The plates was arranged on simple support both on the east west direction while clamped on the north south direction

1.3.2 Analysis of the East-West Orientation of the Plate

The case of horizontal Direction (x-x axis)

Also introducing the boundary conditions on the horizontal component

On the left support, i = 0

When $Ew = 0$

$Ew_x = p_o + p_1 i + p_2 i^2 + p_3 i^3 + p_4 i^4$ 1

The first derivative

$Ew_x^1 = p_1 + 2p_2 i + 3p_3 i + 4p_4 i$ 2

similarly for the second derivative

$Ew_x^2 = 2p_2 + 6p_3 i + 12p_4 i^2$ 3

considering the boundary conditions,

when i = 0,

$Ew_x = 0 = p_o + 0 + 0 + 0 + 0$ 4

leaving $n_o = 0$

Also when $Ew_x^2 = 0$ 5

$Ew_x^2 = 0 = 2p_2 + 0 + 0 + 0$ 6

leaving $2p_2 = 0$ 7

$n_2 = 0$ 8

at the right support, i =1

$Ew_x = 0 = 0 + 0 + 0 + p_3 + p_4$ 9

$p_3 = - p_4$

$Ew_x^2 = 2p_2 + 6p_3 + 12p_4$ 10

$0 = 0 + 6p_3 + 12p_4$ 11

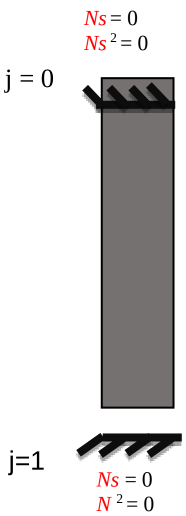
This implies that $6p_3 = -12p_4$

$p_3 = -2p_4$

Substituting back the derived values into the general equation for the vertical

Components gives $Ew_x = p_5(i - 2i^3 + i^4)$ 12

The case of North-South Arrangement (Y- Y axis)



$Ns_y = q_o + q_1 i + q_2 i^2 + q_3 i^3 + q_4 i^4$ 13

The first derivative on Y axis gives

$Ns_y^1 = q_1 + 2\, q_2\, I + 3q_3 I^2 + 4q_4 I^3$ 14

Considering the boundary conditions on the clamped ends gives

At I = 0,

$Ns_y = 0 = q_o + 0 + 0 + 0 + 0$ 15

Leaving $q_o = 0$ 16

Also

$Ns_y^1 = q_1 + 0 + 0 + 0 + 0$ 17

$q_1 = 0$ 18

At I = 1,

$Ns_y = 0 = 0 + 0 + q_2 + q_3 + q_4$ 19

$m_2 + m_3 = - m_4$ 20

$Ns_y^1 = 0 = 0 + 2q_2 + 3q_3 + 4q_4$ 21

$q_2 = - q_3 - q_4$ 22

Putting Equation 40 into the first derivatives gives

$Ns_y^1 = 0 = 0 + 2(- q_3 - q_4) + 3q_3 + 4q_4$ 23

Opening the bracket gives

$q_3 + 2q_4 = 0$ 24

That means $q_3 = -2q_4$ 25

Putting it back into Equation 40 gives

$q_2 = - (-2q_4) - q_4$ 26

$q_2 = +q_4$ 27

Substituting the derived values into Equation 30 gives

$Ns_y = q_4 j^2 - 2q_4 j^3 + q_4 j^4$ 28

$Ns_y = m_4(I^2 - 2i^3 + j^4)$ 29

That means

$dfl = Ew_x * Ns_y = m_5\, (i - 2i^3 + i^4) * n_5 (j^2 - 2j^3 + j^4)$ 30

Factorizing further gives the Amplitude and the shape function

$dfl = m_5 n_5\; m_5\, (i - 2i^3 + i^4) * n_5 (j^2 - 2j^3 + j^4)$ 31

Strain Energy Equation.

The summation of all the products of the stress and strain along the plate continuum gave the strain energy equation. But the Overall potential energy, O_p is gotten when External Work is added to the Strain energy, Є . Mathematically expressed as E_w

given as: $O_p = Є + E_w$ 32

To derive the strain energy,Є the product of normal stress and normal strain in

x direction is considered as $\S_x ð_x = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + \mu\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2\right)$ 33

while their product in y direction is considered as

$\S_y ð_y = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial y^2}\right]^2 + \mu\left[\frac{\partial^2 fk}{\partial x \partial y}\right]^2\right)$ 34

And finally the product of the in-plane shear stress and in-plane shear strain is given as: $\tau_{xy}\gamma_{xy} = 2\,\frac{Ez^2(1-\mu)}{(1-\mu^2)}\left[\frac{\partial^2 fk}{\partial x\partial y}\right]^2$ 1iv

adding all together gives

$\S_x ð_x + \S_y ð_y + \tau_{xy}\gamma_{xy} \quad = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fk}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 fk}{\partial y^2}\right]^2\right)$ 1v

But $\epsilon = \frac{1}{2}\iint_{xy}\overline{\epsilon}\,\mathrm{dxdy}$ where $\overline{\epsilon} = \frac{\mathrm{Ez}^2}{1-\mu^2}\int\left(\left[\frac{\partial^2 fk}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fk}{\partial x\partial y}\right]^2 + \left[\frac{\partial^2 fk}{\partial y^2}\right]^2\right)$ 1vi

Upon minimisation of the expressions above, the third order strain energy equation is given as

$\epsilon = \frac{G}{2}\int_0^n\int_0^m\left(\frac{\partial^3 fk}{\partial x^3}.\frac{\partial fk}{\partial x} + 2\,\frac{\partial^3 fk}{\partial x\,\partial y^2}.\frac{\partial \mathrm{fk}}{\partial x} + \frac{\partial^3 fk}{\partial y^3}.\frac{\partial \mathrm{fk}}{\partial y}\right)\mathrm{dxdy}$ 2i

with the external load as $v = -\frac{Bx}{2}\int_0^n\int_0^m\left(\frac{\partial \mathrm{fk}}{\partial x}\right)^2\mathrm{dxdy}$ 2ii

The third order total potential energy functional is expressed mathematically as

$O_p = \frac{G}{2}\int\int\left(\frac{\partial^3 fk}{\partial x^3}.\frac{\partial \mathrm{fk}}{\partial x} + 2\,\frac{\partial^3 fk}{\partial x^2\,\partial y}.\frac{\partial \mathrm{fk}}{\partial y} + \frac{\partial^3 fk}{\partial y^3}.\frac{\partial \mathrm{fk}}{\partial y}\right)\mathrm{dxdy} - \frac{Bx}{2}\int\int\frac{\partial^2 fk}{\partial x^2}\mathrm{dxdy}$ 2ii

Rearranging the total potential energy equation in terms of non dimensional parameters, the buckling load equation is gotten as

$$B_x = \frac{\frac{G}{a^2}\int_0^1\int_0^1.\left(\left[\frac{\partial^3 \mathrm{fk}}{\partial J^3}\right].\frac{\partial \mathrm{fk}}{\partial J} + 2\frac{1}{p^2}\left[\frac{\partial^3 \mathrm{fk}}{\partial J\partial I^2}\right].\frac{\partial \mathrm{fk}}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 \mathrm{fk}}{\partial I^3}\right].\frac{\partial \mathrm{fk}}{\partial I}\right)\mathrm{dJdI}}{\int_0^1\int_0^1\left(\frac{\partial \mathrm{fk}}{\partial J}\right)^2\mathrm{dJdI}}$$