International Journal of Research Publication and Reviews, Vol 5, no 6, pp 2929-2940 June 2024

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Evaluation of Compressive Strength Property of Plastic Fibre Reinforced Concrete [PLFRC] Based on Scheffe's Model

K. C. Nwachukwu[1]*, O. Edike* [2] *, C.C. Mathew*[3] *B.O. Mama*[4] *and O. Oguaghamba* [5]

[1]Department Of Civil Engineering, Federal University Of Technology, Owerri, Imo State, Nigeria.
[2] Department Of Civil Engineering, Nigeria Maritime University, Okerenkoko, Delta State, Nigeria.
[3]Department Of Engineering Mechanics , Virginia Polytechnic Institute And State University, USA
[4,5]Department Of Civil Engineering, University of Nigeria, Nsukka, Enugu State, Nigeria.
E- Mail: knwachukwu@gmail.com, knwachukwu@futo.edu.ng .Whatsapp: +234-8036670993

ABSTRACT

When there is urgent need to reduce the overall construction cost [OCC] due to expensive nature of conventional steel (reinforcement), incorporation of Plastic Fibres [PLF] is one sure way of achieving this feat. PLF are capable of replacing welded mesh, light reinforcement bars and sometimes conventional reinforcement with good mechanical strength output such as reduced structural weight and improved compressive strength. This research work is therefore focused at using Scheffe's (5,2) Model to evaluate the optimized compressive strength of Plastic Fibre Reinforced Concrete [PLFRC]. While there are many types of PLF, the PLF of interest in this work is the Polyester Fibre [POF]. Using Scheffe's Simplex method, the compressive strength of PLFRC was evaluated for fifteen different mix ratios. Fifteen control experiments were also carried out and the compressive strength determined. Thereafter, the adequacy of the model was tested using Student's t-test and the test statistics found the model adequate. Maximum compressive strength for the Scheffe's (5,2) model was obtained as 29.35MPa. Since structural concrete elements are generally made with concrete having a compressive strength of 20 to 35 MPa according to the American Concrete Institute [ACI], this implies that optimized PLFRC based on Scheffe's Second model can produce the required compressive strength needed in light weight and some major construction projects such as Suspended floors and roof elements, Large scale industrial floors, Lightweight applications, Architecturally sensitive buildings , Construction of walkways, Pavement slabs, Bridges etc, at the best economic, aesthetic and safety advantages.

Keywords: PLFRC, POF , Compressive strength, Scheffe's (5,2) Optimization Model, Mixture Design

1 1.INTRODUCTION

In today's world, the management and disposal of the plastic wastes constitute the main environmental problem. Plastic has been used widely in packaging, automotive and industrial applications, medical delivery systems, artificial implants, other healthcare applications, land/soil conservation, water desalination, flood prevention, preservation and distribution of food, housing, communication materials, security systems, and other uses. Large applications of plastics in all part of daily activities increase the volume of plastic waste. Again, there has been also clear evidence that the use of plastics in various places as packing materials and the products such as bottles, polythene sheets, containers, packing strips etc., are increasing day by day. Consequently, this has resulted in production of plastic wastes from all sorts of livings from industrial manufacturers to domestic users. One way to circumvent this pollution crisis is to ensure that many products are produced from reusable waste plastics and through recycling. On the other side, the construction industry is facing problems due to insufficient and unavailability of construction materials, and sometimes, even when the materials are available, their costs are beyond the reach of common man. To remedy the situation, plastics waste are converted into plastic fibres to find use in the construction industry as either partial or total substitute for cement, aggregate and conventional reinforcement in concrete works. The work of Ishaya and others (2016) is a typical work that converted waste bottle to useful construction material. Thus, the plastic waste that could block drainage openings leading to devastating flooding can now be harnessed as construction materials. Plastic Fibres (PLF) are plastics that have been spun into fibres or filaments and used to make fabrics, string, ropes, and cables, even optical fibres. Some of the most recognizable plastic fibres are polyester, nylon, rayon, acrylic, and spandex, etc .Typical example of plastic fibre is shown in Figure 1.

Incorporation and utilization of the PLF shown in Figure 1 can be best carried out through optimization. Generally, an optimization problem is one requiring the determination of the optimal value of a given function, known as the objective function, subject to a set of stated constraints placed on the concerned variables. In every optimization problem there is always need for an objective function which might be to maximize profit or benefit, to minimize cost or to minimize the use of material resources. In the area of concrete production to be precise, optimization of the concrete mixture design is a process of search for a mixture for which the sum of the costs of the ingredients is lowest, yet satisfying the required performance of concrete, such as strength, workability and durability etc. The objective of mix design, according to Shacklock (1974), is to determine the most appropriate proportions in which to use the constituent materials to meet the needs of construction work By definition, concrete mix design according to Jackson and Dhir (1996) remains the procedure which, for any given set of condition, the proportions of the constituent materials are chosen so as to produce a concrete with all the required properties for the minimum cost. From the above definition, it can be envisaged that the cost of any concrete includes, in addition to that of the materials themselves, the cost of the mix design, of batching, mixing and placing the concrete and of the site supervision. Thus, the empirical mix design methods and procedures proposed by Hughes (1971), ACI- 211(1994) and DOE (1988) seems to be more complex and time consuming as they involve a lot of trial mixes and complex statistical calculations before the desired strength of the concrete can be reached. Therefore, optimization of the concrete mixture design proves to be the fastest method, best option, most convenient and the most efficient way of selecting concrete mix ratios /proportions for better efficiency and better performance of concrete when compared with usual empirical methods .Typical examples of well-known optimization model is Scheffe's Polynomial Model which can be in the form of Scheffe's Second Degree Model or Scheffe's Third Degree Model. Thus, in this present study, Scheffe's Second Degree Model for five components mixtures (namely Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Plastic Fibre will be in focus.

In the construction industry, no material is most widely used than concrete, except water. Due to its vast plethora of applications in construction compared with other materials, as well as its availability and global impact, concrete most especially, in the long run, is universally preferred. Concrete, according to Oyenuga (2008) is a composite inert material comprising of a binder course (cement), mineral filter or aggregates and water. Concrete, being a homogeneous mixture of cement, sand, gravel and water is very strong in carrying compressive forces and hence is gaining increasing importance as building materials throughout the world (Syal and Goel, 2007). Again, concrete, according to Neville (1990), plays an important part in all building structures owing to its several advantages that ranges from low built in fire resistance, high compressive strength to low maintenance. However, concrete, especially the plain type also has its own limitation. According to Shetty (2006), plain concrete possesses a very low tensile strength, limited ductility, and little resistance to cracking to mention but few. That is to say that unreinforced (plain) concrete is brittle in nature, and is characterized by low tensile strength but high compressive strength. As a result of this situation, stakeholders in the construction industries have been in continuous search for the improvement and upgrading of the concrete properties in critical areas. In line with this, attempts have been made in the past to improve the tensile properties of concrete members by way of using conventional reinforced steel bars. Although both these methods provide tensile strength to the concrete members, they however, do not increase the inherent tensile strength of concrete itself. Following further researches and recent developments in concrete technology, it has been established that the addition of fibres to concrete would act as crack arrester and would substantially improve its static as well as dynamic properties. This type of concrete is known as Fibre reinforced concrete (FRC). It is a composite material consisting of mixtures of cement, mortar or concrete and discontinuous, discrete, uniformly dispersed. Combining fibres with concrete can produce a range of materials which possess enhanced tensile strength, compressive strength, elasticity, toughness, and durability etc. This is accomplished by limiting or controlling the start, spread, or spread persistence of cracks. Plastic Fibre Reinforced Concrete (PLFRC) is concrete mixture where the conventionally steel reinforcement in concrete production is partially or wholly replaced with Plastic Fibre (PLF). The mechanical property of PLFRC of interest here is the compressive strength. Compressive strength of concrete is the strength of hardened concrete measured by the compression test. It is a measure of the concrete's ability to resist loads which tend to compress it. It is measured by crushing cylindrical concrete specimens in a universal testing machine (UTM). Further, the compressive strength of the concrete cube test further provides an idea about all the characteristics of concrete under investigation.

This recent work examines the application of Scheffe's Second Degree Polynomial Model in the optimization of the compressive strength of PLFRC. There are a lot of done researches related to plastic fibres and general optimization applications, but none has been able to address the subject matter in detail. For example, Zhang and others (2013) investigated the mechanical properties of plastic concrete containing bentonite. Sanjaykumar and Daule (2017) examined the use of plastic fibre in the concrete. Adda and Slimane (2019) research investigation focused on the study of concrete reinforced by plastic fibres based on local materials. Finally, Yin and others (2015) investigated the use and review of macro plastic fibres in concrete. Coming to the use of optimization application in concrete mixtures, recent works show that many researchers have used Scheffe's method to carry out one form of optimization work or the other. For example, Nwakonobi and Osadebe (2008) used Scheffe's model to optimize the mix proportion of Clay- Rice Husk Cement Mixture for Animal Building. Egamana and Sule (2017) carried out an optimization work on the compressive strength of periwinkle shell aggregate concrete. Ezeh and Ibearugbulem (2009) applied Scheffe's model to optimize the compressive cube strength of River Stone Aggregate Concrete. Scheffe's model was used by Ezeh and others (2010a) to optimize the compressive strength of cement- sawdust Ash Sandcrete Block. Again Ezeh and others (2010b) optimized the aggregate composition of laterite/ sand hollow block using Scheffe's simplex method. The work of Ibearugbulem (2006) and Okere (2006) were based on the use of Scheffe' model in the optimization of compressive strength of Perwinkle Shell- Granite Aggregate Concrete and optimization of the Modulus of Rupture of Concrete respectively. Obam (2009) developed a mathematical model for the optimization of strength of concrete using shear modulus of Rice Husk Ash as a case study. The work of Obam (2006) was based on four component mixtures, that is Scheffe's (4,2) and Scheffe's (4,3) where comparison was made between second degree model and third degree model. Nwachukwu and others (2017) developed and employed Scheffe's Second Degree Polynomial model to optimize the compressive strength of Glass Fibre Reinforced Concrete (GFRC). Also, Nwachukwu and others (2022a) developed and used Scheffe's Third Degree Polynomial model, Scheffe's (5,3) to optimize the compressive strength of GFRC where they compared the results with their previous work, Nwachukwu and others (2017). Nwachukwu and others (2022c) used Scheffe's (5,2) optimization model to optimize the compressive strength of Polypropylene Fibre Reinforced Concrete (PFRC). Again, Nwachukwu and others (2022d) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Nylon Fibre Reinforced Concrete (NFRC). Nwachukwu and others (2022b) applied Scheffe's (5,2) mathematical model to optimize the compressive strength of Steel Fibre Reinforced Concrete (SFRC). Furthermore, Nwachukwu and others (2022e) used Scheffe's Third Degree Regression model, Scheffe's (5,3) to optimize the compressive strength of PFRC. Nwachukwu and others (2022f) applied Modified Scheffe's Third Degree Polynomial model to optimize the compressive strength of NFRC. Again, Nwachukwu and others (2022g) applied Scheffe's Third Degree Model to optimize the compressive strength of SFRC. In what is termed as introduction of six component mixture and its Scheffe's formulation ,Nwachukwu and others (2022h) developed and use Scheffe's (6,2) Model to optimize the compressive strength of Hybrid- Polypropylene – Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2022 i) applied Scheffe's (6,2) model to optimize the Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2022j) applied Scheffe's (6,2) model in the Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete (HPNFRC) . Nwachukwu and others (2022k) applied the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of Mussel Shell Fibre Reinforced Concrete (MSFRC). Nwachukwu and others (2022 l) carried out an optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model. Nwachukwu and others (2023a) applied Scheffe's Third Degree Regression Model to optimize the compressive strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023b) applied Scheffe's (6,3) Model in the Optimization Of Compressive Strength of Concete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA). Nwachukwu and others (2023c) applied Scheffe's (6,2) model to optimize the Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023d) made use of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC). Nwachukwu and others (2023e) used optimization techniques in the Flexural Strength And Split Tensile Strength determination of Hybrid Polypropylene - Steel Fibre Reinforced Concrete (HPSFRC). Nwachukwu and others (2023f) applied Scheffe's Optimization model in the evaluation of Flexural Strength And Split Tensile Strength Of Plastic Fibre Reinforced Concrete (PLFRC). Nwachukwu and H.E. Opara (2023) in the their paper presented at the Conference Proceedings of the Nigeria Society of Engineers, demonstrated the use of Snail Shells Ash (SSA) in the partial replacement of cement using Scheffe's (5,2) optimization model. Nwachukwu and others (2024a) applied the use of Scheffe's (6,2) model to evaluate the optimum flexural and split tensile strengths of Periwinkle Shells Ash (PSA)- Mussel Shells Ash (MSA)- Cement Concrete (PMCC). Finally, Nwachukwu and others (2024b) applied the use of Scheffe's (6,2) model to evaluate the optimum compressive strength of Periwinkle Shells Ash (PSA)- Snail Shells Ash (SSA)- Cement Concrete (PSCC). Based on the works reviewed so far, it can be envisaged that no work has been done on the use of Scheffe's Second Degree Polynomial Model to optimize the compressive strength of PLFRC. Henceforth, the need for this present research work.

Fig. 1: Typical Example Of Plastic Fibre.

2 2. METHODOLOGY

2.1.1 2.1 MATERIALS FOR PLFRC MIXTURES

In this present work, the component materials under examination in line with Scheffe's (5, 2) model are Water/Cement ratio, Cement, Fine and Coarse Aggregates and PLF. The water is procured from potable water clean water source and was applied in accordance with ASTM C1602/C1602M-22 (2022). The cement is Dangote cement, a brand of Ordinary Portland Cement obtained from local distributors, which conforms to British Standard Institution BS 12 (1978). Fine aggregate of sizes that range from 0.05 - 4.5mm was purchased from the local river. Crushed granite as a coarse aggregate of 20mm size was purchased from a local stone market and was later downgraded to 4.75mm. Both fine and coarse aggregates were procured and prepared in accordance with ASTM C33/C33M-18 (2018). Plastic Fibres (PLF) are plastics that have been spun into fibres or filaments and used to make fabrics, string etc. For the purpose of this research, (PLF) as shown in Figure 1, with diameter: 2mm; and Length: 50mm are procured in the local market.

2.1.2 2.2 FUNDAMENTALS OF PLFRC SCHEFFE'S OPTIMIZATION MODEL

According to Aggarwal (2002) perspective, a simplex lattice can be defined as a structural representation of lines joining the atoms of a mixture, and these atoms in turn are the constituent components of the mixture. For instance, when we consider the present mixture, PLFRC , the constituent elements are the water, cement, fine aggregate (as sand), coarse aggregate and plastic fibre (PLF). Consequently, a simplex of five-component mixture is a four-dimensional solid as illustrated by Nwachukwu and others (2017). Again, according to Obam (2009), mixture components are subject to the constraint that the sum of all the components must be equal to 1. That is:

$$X_1 + X_2 + X_3 + \dots + X_q = 1 \; ; \quad \Rightarrow \sum_{i=1}^{q} X_i = 1 \qquad \textbf{(1)}$$

where $X_i \geq 0$ and $i = 1, 2, 3\dots q$, and q = the number of mixtures

2.1.3 2.2.1. POSSIBLE DESIGN POINTS FOR PLFRC SCHEFFE'S (5,2) MIXTURE

The Scheffe's (q, m) such as Scheffe' s (5,2) simplex lattice design are characterized by the symmetric arrangements of points within the experimental region and a well-chosen polynomial equation to represent the response surface over the entire simplex region . The (q, m) simplex lattice design given by Scheffe, according to Nwakonobi and Osadebe (2008) contains ${}^{q+m-1}C_m$ points where each components proportion takes (m+1) equally spaced values $X_i = 0, \frac{1}{m}, \frac{2}{m}, \frac{3}{m}, \dots, 1; \quad i = 1, 2, \dots, q$ ranging between 0 and 1 and all possible mixture with these component proportions are used, and m is scheffe's polynomial degee, which in this present study is 2. For example a (3, 2) lattice consists of ${}^{3+2-1}C_2$ i.e. ${}^{4}C_2$ = 6 points. Each X_i can take m+1 = 3 possible values; that is $x = 0, \frac{1}{2}, 1$ with which the possible design points are: $(1,0,0), (0,1,0), (0,0,1), \left(\frac{1}{2}, \frac{1}{2}, 0\right), \left(0, \frac{1}{2}, \frac{1}{2}\right), \left(\frac{1}{2}, 0, \frac{1}{2}\right)$. To evaluate the number of coefficients/ or terms/ or design points required for a given lattice , the following general formula is applied:

$$k = \frac{(q+m-1)!}{(q-1)!\,.\,m!} \quad \text{Or} \quad {}^{q+m-1}C_m \qquad \textbf{2(a-b)}$$

Where k = number of coefficients/ terms / design points ,q = number of components/mixtures = 5 in this present study, m = number of degree of polynomial = 2 in this present work.Using either of Eqn. (2), $k_{(5,2)} = 15$

This implies that the possible design points for Scheffe's (5,2) lattice can be as follows:

A_1 (1,0,0,0,0,); A_2 (0,1,0,0,0,); A_3 (0,0,1,0,0,); A_4 (0,0,0,1,0,), A_5 (0,0,0,0,1,); A_{12} (0.5, 0.5, 0, 0, 0); A_{13} (0.5, 0, 0.5, 0, 0); A_{14} (0.5, 0, 0, 0.5, 0); A_{15} (0.5, 0, 0, 0, 0.5); A_{23} (0, 0.5, 0.5, 0,0); A_{24} (0, 0.5, 0, 0.5, 0); A_{25} (0, 0.5, 0, 0, 0.5); A_{34} (0, 0, 0.5, 0.5, 0); A_{35} (0, 0, 0.5, 0, 0.5) and A_{45} (0, 0, 0, 0.5, 0.5) **(3)**

According to Obam (2009), a Scheffe's polynomial function of degree, m in the q variable $X_1, X_2, X_3, X_4 \ldots X_q$ is given in form of: $Y = b_0 + \sum bi\, xi + \sum bij\, xj + \sum bi\, jxjxk + + \sum bi^{\,j}{}_2 + \ldots i_n xi_2 xi_n$ **(4)**

where ($1 \leq i \leq q$, $1 \leq i \leq j \leq k \leq q$, $1 \leq i_1 \leq i_2 \leq \ldots \leq i_n \leq q$ respectively) , b = constant coefficients and Y is the response (the response is a polynomial function of pseudo component of the mix) which represents the property under study, which ,in this case is the compressive strength. This research work is based on the (5, 2) simplex. The actual form of Eqn. (4) has already been developed by Nwachukwu and others (2017) and will be applied subsequently.

2.1.4 2.2.2. PSEUDO AND ACTUAL COMPONENTS IN PLFRC SCHEFFE'S THEORY

In every Scheffe's mixture design, the relationship between the pseudo components and the actual components is given as:

$$Z = A * X \qquad \textbf{(5)}$$

where Z is the actual component; X is the pseudo component and A is the coefficient of the relationship

Re-arranging Eqn. (5), we have :

$$X = A^{-1} * Z \qquad \textbf{(6)}$$

2.1.5 2.2.3. FORMULATION OF POLYNOMIAL EQUATION FOR PLFRC SCHEFFE'S (5, 2) LATTICE

The polynomial equation by Scheffe (1958), describing the response is given in Eqn.(4). But, for Scheffe's (5,2) simplex lattice, the polynomial equation for five component mixtures has been derived from Eqn.(4) by Nwachukwu and others (2017) . Eqn.(7) gives the simplified version :

$$Y = ß_1X_1 + ß_2X_2 + ß_3X_3 + ß_4X_4 + ß_5X_5 + ß_{12}X_1X_2 + ß_{13}X_1X_3 + ß_{14}X_1X_4 + ß_{15}X_1X_5 +$$

$$ß_{23}X_2X_3 + ß_{24}X_2X_4 + ß_{25}X_2X_5 + ß_{34}X_3X_4 + ß_{35}X_3X_5 + ß_{45}X_4X_5 \qquad \textbf{(7)}$$

2.1.6 2.2.4. COEFFICIENTS EVALUATION OF THE PLFRC SCHEFFE'S (5, 2) POLYNOMIAL

From the work of Nwachukwu and others (2022h), the simplified equations for the coefficients of the Scheffe's (5, 2) polynomial are expressed as follows. :

$$\beta_1 = Y_1;\ \beta_2 = Y_2;\ \beta_3 = Y_3;\ \beta_4 = Y_4;\ \beta_5 = Y_5;\ \beta_{12} = 4Y_{12} - 2Y_1 - 2Y_2;\ \beta_{13} = 4Y_{13} - 2Y_1 - 2Y_3; \qquad \textbf{8(a-g)}$$

$$\beta_{14} = 4Y_{14} - 2Y_1 - 2Y_4;\ \beta_{15} = 4Y_{15} - 2Y_1 - 2Y_5;\ \beta_{23} = 4Y_{23} - 2Y_2 - 2Y_3;\ \beta_{24} = 4Y_{24} - 2Y_2 - 2Y_4; \qquad \textbf{9(a-d)}$$

$$\beta_{25} = 4Y_{25} - 2Y_2 - 2Y_5;\ \beta_{34} = 4Y_{34} - 2Y_3 - 2Y_4;\ \beta_{35} = 4Y_{35} - 2Y_3 - 2Y_5;\ \beta_{45} = 4Y_{45} - 2Y_4 - 2Y_5 \qquad \textbf{10(a-d)}$$

Where Y_i = Response Function (or Compressive Strength) for the pure component, i

2.1.7 2.2.5. PLFRC SCHEFFE'S (5, 2) MIXTURE DESIGN MODEL

If we substitute Eqns. (8)-(10) into Eqn. (7), we obtain the mixture design model for the PLFRC mixture based on Scheffe's (5, 2) lattice.

2.1.8 2.2.6. ACTUAL AND PSEUDO MIX RATIOS FOR THE PLFRC SCHEFFE'S (5,2) DESIGN LATTICE AT INITIAL EXPERIMENTAL POINT[IETP] AND EXPERIMENTAL CONTROL TEST POINT[ECTP]

2.1.9 A. AT THE PLFRC INITIAL EXPERIMENTAL TEST POINTS [IETP]

The requirement of simplex lattice design from Eqn.(1) makes it impossible to use the conventional mix ratios such as 1:2:4, etc., at a given water/cement ratio for the actual mix ratio. This necessitates the transformation of the actual components proportions to meet the above criterion. Based on experience and previous knowledge from literature, the following arbitrary prescribed mix proportions were chosen for the five points/vertices.

A_1 (0.67:1: 1.7: 2.0:0.5); A_2 (0.56:1:1.6:1.8:0.8); A_3 (0.5:1:1.2:1.7:1.0); A_4 (0.7:1:1:1.8:1.2) and A_5 (0.75:1:1.3:1.2:1.5), **(11)**

which represent water/cement ratio, cement, fine aggregate, coarse aggregate and plastic fibre.

For the pseudo mix ratio, we have the following corresponding mix ratios at the vertixes:

A_1(1:0:0:0:0), A_2(0:1:0:0: 0), A_3(0:0:1:0:0), A_4(0:0:0:1:0), and A_5(0:0:0:0:1) **(12)**

For the transformation of the actual component, Z to pseudo component, X, and vice versa , Eqns.(5)and (6) are used..

If we substitute the mix ratios from point A_1 into Eqn. (5), we obtain:

$$\begin{Bmatrix} 0.67 \\ 1 \\ 1.7 \\ 2.0 \\ 0.5 \end{Bmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} \\ A_{21} & A_{22} & A_{23} & A_{24} & A_{25} \\ A_{31} & A_{32} & A_{33} & A_{34} & A_{35} \\ A_{41} & A_{42} & A_{43} & A_{44} & A_{45} \\ A_{51} & A_{52} & A_{53} & A_{54} & A_{55} \end{pmatrix} \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \textbf{(13)}$$

Solving, $Z_1 = 0.67$, $Z_2 = 1$, $Z_3 = 1.70$, $Z_4 = 2.0$, $Z_5 = 0.5$. The same approach is used in obtaining the remaining values at the five vertices as shown in Eqn. (14).

$$\begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \\ Z_5 \end{Bmatrix} = \begin{pmatrix} 0.67 & 0.56 & 0.5 & 0.7 & 0.75 \\ 1 & 1 & 1 & 1 & 1 \\ 1.7 & 1.6 & 1.2 & 1 & 1.3 \\ 2 & 1.8 & 1.7 & 1.8 & 1.2 \\ 0.5 & 0.8 & 1 & 1.2 & 1.5 \end{pmatrix} \begin{Bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{Bmatrix} \qquad \textbf{(14)}$$

Considering mix ratios at the mid points from Eqn.(3) and substituting these pseudo mix ratios in turn into Eqn.(14) yields the corresponding actual mix ratios as follows:

At point A_{12} we have: A_{12} (0.5, 0.5, 0, 0, 0); Then substituting into Eqn.(14), we have:

$Z_1 = 0.62$, $Z_2 = 1$, $Z_3 = 1.65$, $Z_4 = 1.9$, $Z_5 = 0.65$.

The same approach goes for the remaining mid-point mix ratios. Hence, in order to generate the fifteen coefficients, fifteen (15) experimental tests was carried out and the corresponding mix ratios are as displayed in Table 1.

Table 1: Pseudo (X) and Actual (Z) Mix Ratio For PLFRC Based On Scheffe's (5,2) Lattice For IETP

S/N	**IETP**	**PSEUDO COMPONENT**					**RESPONSE**	**ACTUAL COMPONENT**				
		X_1	**X_2**	**X_3**	**X_4**	**X_5**	**SYMBOL**	**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**
1	E_I	1	0	0	0	0	Y_1	0.67	1	1.70	2.00	0.50
2	E_2	0	1	0	0	0	Y_2	0.56	1	1.60	1.80	0.80
3	E_3	0	0	1	0	0	Y_3	0.50	1	1.20	1.70	1.00
4	E_4	0	0	0	1	0	Y_4	0.70	1	1.00	1.80	1.20
5	E_5	0	0	0	0	1	Y_5	0.75	1	1.30	1.20	1.50

6	E_{12}	0.50	0.50	0	0	0	Y_{12}	0.62	1	1.65	1.90	0.65
7	E_{13}	0.50	0	0.50	0	0	Y_{13}	0.59	1	1.45	1.85	0.75
8	E_{14}	0.50	0	0	0.50	0	Y_{14}	0.69	1	1.35	1.90	0.85
9	E_{15}	0.50	0	0	0	0.50	Y_{15}	0.71	1	1.50	1.60	1.00
10	E_{23}	0	0.50	0.50	0	0	Y_{23}	0.53	1	1.40	1.75	0.90
11	E_{24}	0	0.50	0	0.50	0	Y_{24}	0.63	1	1.30	1.80	1.00
12	E_{25}	0	0.50	0	0	0.50	Y_{25}	0.66	1	1.45	1.50	1.15
13	E_{34}	0	0	0.50	0.50	0	Y_{34}	0.60	1	1.10	1.75	1.10
14	E_{35}	0	0	0.50	0	0.50	Y_{35}	0.63	1	1.25	1.45	1.25
15	E_{45}	0	0	0	0.50	0.50	Y_{45}	0.73	1	1.15	1.50	1.50

B. AT THE PLFRC EXPERIMENTAL (CONTROL) TEST POINTS [ECTP]

Here, fifteen (15) different control mix ratios were predicted and listed in Table 2, which according to Scheffe's (1958), their summation should not be greater than one. The same approach for component transformation adopted for the IETP are also adopted for the ECTP and the results are shown in Table 2.

Table 2:Actual & Pseudo Component Of PLFRC Based On Scheffe 's (5,2) Lattice For ECTP

S/N	**ECTP**	**PSEUDO COMPONENT**					**RESPONSE SYMBOL**	**ACTUAL COMPONENT**				
		X_1	**X_2**	**X_3**	**X_4**	**X_5**		**Z_1**	**Z_2**	**Z_3**	**Z_4**	**Z_5**
1	C_1	0.25	0.25	0.25	0.25	0.00	Y_1	0.61	1	1.38	1.83	0.50
2	C_2	0.25	0.25	0.25	0.00	0.25	Y_2	0.62	1	1.45	1.68	0.80
3	C_3	0.25	0.25	0.00	0.25	0.25	Y_3	0.67	1	1.40	1.70	1.00
4	C_4	0.25	0.00	0.25	0.25	0.25	Y_4	0.66	1	1.30	1.68	1.20
5	C_5	0.00	0.25	0.25	0.25	0.25	Y_5	0.63	1	1.28	1.63	1.50
6	C_{12}	0.20	0.20	0.20	0.20	0.20	Y_{12}	0.64	1	1.36	1.70	0.65
7	C_{13}	0.30	0.30	0.30	0.10	0.00	Y_{13}	0.59	1	1.45	1.83	0.75
8	C_{14}	0.30	0.30	0.30	0.00	0.10	Y_{14}	0.59	1	1.48	1.77	0.85
9	C_{15}	0.30	0.30	0.00	0.30	0.10	Y_{15}	0.65	1	1.42	1.80	1.00
10	C_{23}	0.30	0.00	0.30	0.30	0.10	Y_{23}	0.64	1	1.30	1.77	0.90
11	C_{24}	0.00	0.30	0.30	0.30	0.10	Y_{24}	0.60	1	1.27	1.71	1.00
12	C_{25}	0.10	0.30	0.30	0.30	0.00	Y_{25}	0.60	1	1.31	1.79	1.15
13	C_{34}	0.30	0.10	0.30	0.30	0.00	Y_{34}	0.62	1	1.33	1.83	1.10

14	C_{35}	0.30	0.30	0.10	0.30	0.00	Y_{35}	0.63	1	1.41	1.85	1.25
15	C_{45}	0.10	0.20	0.30	0.40	0.00	Y_{45}	0.61	1	1.25	1.79	0.50

2.1.10 2.2.7. MEASUREMENT OF QUANTITIES OF PLFRC MATERIALS IN THE LABORATORY

The actual components as obtained from Tables 1 and 2 were used to measure out the quantities of Water/Cement Ratio (Z_1), Cement (Z_2), Fine Aggregate (Z_3), Coarse Aggregate (Z_4), and Plastic Fibre (Z_5) in their respective ratios for the concrete cube strength test using a weighing balance of 50kg capacity in their respective ratios for the Concrete Cube Strengths test at the laboratory.

Mathematically, from the works of Nwachukwu and others (2024a), Measured Quantity, M^Q of PLFRC Mixture is given by Eqn.(15)

$$M^Q = \frac{X}{T} * Y \qquad \textbf{(15)}$$

Where, X = Individual mix ratio at each test point = 0.67 for Z_1 at E_1 in Table 1, for example.

T = Sum of mix ratios at each test point = 5.87 at E_1 in Table 1, for example

And Y = Average weight of Concrete cube/beam/cylinder

For the Compressive Strength concrete cube mould of 15cm*15cm*15cm, Average Y from experience = 8kg

Samples of measured quantities for compressive strength test can be seen from the works of Nwachukwu and others (2024 b).

2.1.11 2.3. METHOD

2.1.12 2.3.1. PLFRC SPECIMEN PREPARATION / BATCHING/ CURING FOR COMPRESSIVE STRENGHT TEST

The specimen for the compressive strength is concrete cubes. They were cast in steel mould measuring 15cm*15cm*15cm. The mould and its base were damped together during concrete casting to prevent leakage of mortar. Thin engine oil was applied to the inner surface of the moulds to make for easy removal of the cubes. Batching of all the constituent material was done by weight using a weighing balance of 50kg capacity based on the adapted mix ratios and water cement ratios. A total number of 30 mix ratios were to be used to produce 60 prototype concrete cubes. Fifteen (15) out of the 30 mix ratios were as control mix ratios to produce 30 cubes for the conformation of the adequacy of the mixture design given by Eqn. (7), whose coefficients are given in Eqns. (8) – (10). Twenty-four (24) hours after moulding, curing commenced. Thereafter, specimens were removed from the moulds and were placed in clean water for curing. After 28 days of curing the specimens were taken out of the curing tank for compressive strength determination.

2.1.13 2.3.2. PLFRC COMPRESSIVE STRENGTH TEST PROCEDURE/CALCULATION

Compressive strength testing was done in accordance with BS 1881 – part 116 (1983) - Method of determination of compressive strength of concrete cube and ACI (1989) guideline. In this present study, two samples were crushed for each mix ratio. In each case, the compressive strength was then calculated using Eqn.(16)

$$\text{Compressive Strength } \textbf{(MPa)} = \frac{\text{Average failure Load (N)}}{\text{Cross- sectional Area (mm}^2\text{)}} \qquad \frac{P}{A} \qquad \textbf{(16)}$$

---

3 3. RESULTS PRESENTATION AND DISCUSSION

3.1.1 3.1. PLFRC RESPONSES FOR THE INITIAL EXPERIMENTAL TESTS POINTS [IETP] AND EXPERIMENTAL (CONTROL) TEST *POINTS [ECTP].*

The results of the compressive strength ($R_{esponse,}$ Y_i) of PLFRC based on a 28-days strength is presented in Table 3. The initial experimental test responses are calculated from Eqn..(16)

Table 3: 28th Day Compressive Strength (Responses) Test Results for PLFRC Based on Scheffe's (5, 2) Model for the IETP and ECTP.

S/N	**POINTS**		**EXPT. NO.**		**RESPONSES [MPa]**		**RESPONSE SYMBOL**	**AVERAGE RESPONSE [MPa]**	
	IETP	**ECTP**	**IETP**	**ECTP**	**AT IETP**	**AT ECTP**		**AT IETP**	**AT ECTP**

1.	E_I	C_1	PLFRC/ E_I A	PLFRC / C_I A	27.83	25.22	**Y_1**	27.42	24.78
	E_I	C_1	PLFRC/ E_I B	PLFRC / C_I B	26.98	24.33			
2.	E_2	C_2	PLFRC/ E_2A	PLFRC / C_2 A	24.48	22.10	**Y_2**	24.86	22.22
	E_2	C_2	PLFRC/ E_2 B	PLFRC / C_2 B	25.23	22.34			
3.	E_3	C_3	PLFRC/ E_3A	PLFRC / C_3 A	25.24	25.10	**Y_3**	25.69	24.65
	E_3	C_3	PLFRC/ E_3 B	PLFRC /C_3 B	26.14	24.20			
4.	E_4	C_4	PLFRC/ E_4A	PLFRC / C_4 A	20.43	19.30	**Y_4**	20.83	19.75
	E_4	C_4	PLFRC/ E_4 B	PLFRC / C_4 B	21.23	20.10			
5.	E_5	C_5	PLFRC/ E_5A	PLFRC / C_5 A	25.14	26.20	**Y_5**	25.61	26.25
	E_5	C_5	PLFRC/ E_5 B	PLFRC / C_5 B	26.08	26.30			
6.	E_{12}	C_{12}	PLFRC/ E_{12} A	PLFRC / C_{12}A	27.23	27.30	**Y_{12}**	26.83	27.07
	E_{12}	C_{12}	PLFRC/ E_{12} B	PSCC/ C_{12} B	26.43	26.71			
7.	E_{13}	C_{13}	PLFRC/ E_{I3}A	PLFRC / C_{I3} A	29.33	28.30	**Y_{13}**	29.35	28.36
	E_{I3}	C_{13}	PLFRC/ E_{I3}B	PLFRC / C_{I3}B	29.37	28.41			
8.	E_{I4}	C_{14}	PLFRC/ E_{I4}A	PLFRC / C_{I4} A	19.47	20.24	**Y_{14}**	19.85	20.04
	E_{I4}	C_{14}	PLFRC/ E_{I4}B	PLFRC / C_{I4} B	20.23	19.84			
9.	E_{I5}	C_{15}	PLFRC/ E_{I5}A	PLFRC / C_{I5} A	18.55	18.75	**Y_{15}**	18.60	19.00
	E_{I5}	C_{15}	PLFRC/ E_{I5}B	PLFRC / C_{I5} B	18.65	19.25			
10.	E_{23}	C_{23}	PLFRC/ E_{23}A	PLFRC / C_{23}A	20.00	21.22	**Y_{23}**	20.05	21.03
	E_{23}	C_{23}	PLFRC/ E_{23}B	PLFRC / C_{23} B	20.10	20.84			
11.	E_{24}	C_{24}	PLFRC/ E_{24}A	PLFRC /C_{24} A	21.27	22.08	**Y_{24}**	21.18	22.09
	E_{24}	C_{24}	PLFRC/ E_{24}B	PLFRC / C_{24} B	21.09	22.10			
12.	E_{25}	C_{25}	PLFRC/E_{25}A	PLFRC / C_{25}A	18.46	19.46	**Y_{25}**	18.44	19.15
	E_{25}	C_{25}	PLFRC/ E_{25}B	PLFRC /C_{25}B	18.42	18.84			
13.	E_{34}	C_{34}	PLFRC/ E_{34} A	PLFRC /C_{34} A	22.43	22.24	**Y_{34}**	22.14	22.34
	E_{34}	C_{34}	PLFRC/ E_{34}B	PLFRC /C_{34} B	21.85	22.44			

14.	E_{35}	C_{35}	PLFRC/ E_{35} A	PLFRC /C_{35} A	28.12	27.25	**Y_{35}**	28.15	27.30
	E_{35}	C_{35}	PLFRC/ E_{25} B	PLFRC /C_{35} B	28.18	27.35			
15.	E_{45}	C_{45}	PLFRC/ E_{45} A	PLFRC /C_{45} A	21.43	22.23	**Y_{45}**	21.79	22.66
	E_{45}	C_{45}	PSCC/ E_{45}B	PLFRC /C_{45} B	22.14	23.09			

3.1.2 3.2. SCHEFFE’ S (5, 2) POLYNOMIAL MODEL FOR THE PLFRC RESPONSES (COMPRESSIVE STRENGHT).

By substituting the values of the responses (compressive strengths) from Table 3 into Eqns.(8) through (12), we obtain the coefficients ($\beta_1, \beta_2 \ldots$ …. B_{45}) of the Scheffe’s Second degree polynomial for PLFRC as follows

$\beta_1 = 27.42$; $\beta_2 = 24.86$; $\beta_3 = 25.69$; $\beta_4 = 20.83$; $\beta_5 = 25.61$; $\beta_{12} = 14.04$; $\beta_{13} = 14.58$; $\beta_{14} = -8.80$;

$\beta_{15}= -26.06$; $\beta_{23}=-9.62$; $\beta_{24}= 4.52.$; $\beta_{25}=-20.34$; $\beta_{34}= 0.66$; $\beta_{35}= 7.40$; $\beta_{45}=-1.26$ **(17)**

Now, substituting the values of these obtained coefficients into Eqn. (7) yields the mixture design model for the optimization of the Compressive Strength, Y, of PLFRC (at the 28th day) based on Scheffe’s (5,2) lattice as stated under:

$$Y = 27.42X_1 + 24.86X_2 + 25.69X_3 + 20.83X_4 + 25.61X_5 + 14.04\, X_1X_2 + 14.04X_1X_3 -8.80X_1X_4 -26.06X_1X_5$$

$$-9.62X_2X_3 + 4.52X_2X_4 - 20.34\, X_2X_5 + 0.66X_3X_4 + 7.40X_3X_5 -1.26X_4X_5 \quad \textbf{(18)}$$

3.1.3 3.3. SCHEFFE’S (5, 2) MODEL RESPONSES (COMPRESSIVE STRENGHT) FOR PLFRC AT ECTP.

By substituting the pseudo mix ratio of points C_1, C_2, C_3, C_4, C_5, … C_{45} of Table 2 into Eqn.(18), we obtain the Scheffe’s Second degree model responses (compressive strength) for the ECTP of PLFRC.

3.1.4 3.4. VALIDATION OF PLFRC SCHEFFE’S (5, 2) MODEL RESULTS (FOR COMPRESSIVE STRENGHT) USING STUDENT’S – T -TEST

In order to determine the degree of closeness between the PLFRC compressive strengths results (lab responses at IETP) given in Tables 3 and model responses from the control points based on Session 3.3, the test of adequacy is performed using the Student’s – T – test. The result shows that there are no significant differences between the experimental results and model responses. Therefore, the model results are validated. The procedures involved in using the Student’s – T - test have been described by Nwachukwu and others (2022 c). Thus, the models are adequate for determining the compressive strengths of PLFRC based on Scheffe’s (5,2) simplex lattice.

3.1.5 3.5. RESULTS DISCUSSION

The maximum compressive strength of PLFRC based on Scheffe’s (5,2) lattice is **29.35MPa** . This corresponds to mix ratio of **0.59:1.00:1.45:1.85:0.75** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Plastic Fibre respectively. Similarly, the minimum compressive strength is **18.44MPa** which also correspond to the mix ratio of **0.66:1.00:1.45:1.50: 1.15** for Water/Cement Ratio, Cement, Fine Aggregate, Coarse Aggregate and Plastic Fibre respectively. The maximum value from the Scheffe’s model is greater than the minimum value specified by the American Concrete Institute [ACI] for the compressive strength of good concrete. Thus, the Scheffe’s model can be used to determine the PLFRC compressive strength of all points (1 - 45) in the simplex based on Scheffe’s Second Degree Model for six component mixtures.

4 4. CONCLUSION

So far in this recent work, Scheffe’s Second Degree Polynomial (5,2) was used to formulate a model for predicting the compressive strength of PLFRC cubes. It all started by using the Scheffe’s model to predict the mix ratio for evaluating the compressive strength of PLFRC. By using Scheffe’s (5,2) simplex model, the values of the compressive strength were determined at all 15 points (1- 45). The results of the student’s t-test validated the strengths predicted by the models and the corresponding experimentally observed results. The optimum attainable compressive strength predicted by the model based on Scheffe’s (5,2) model is as stated in the result discussion session. It is confirmed that the maximum value meets the minimum standard requirement (of 20 MPa) stipulated by American Concrete Institute [ACI], for the compressive strength of good concrete. Furthermore, with the Scheffe’s (5,2) model, any desired strength, given any mix proportions can be easily predicted and evaluated and vice versa. Therefore, the utilization of this Scheffe’s optimization model has solved the problem of having to go through vigorous, time-consuming and laborious empirical mixture design procedures in other to obtain the desired strength. Again, the use of Scheffe’s optimization techniques has not only helped us to find alternative replacement for conventional expensive steel reinforcement, but also has helped us to reduce pollution in the environment by allowing provision to the incorporation of waste plastic that could have block smooth running of drainage structures as either partial or total replacement for reinforcement in the Reinforced Concrete Production [RCP].

REFERENCES

[1] ACI Committee 211. (1994): "Standard Practice for Selecting Proportions for Normal, Heavyweight, and Mass Concrete (ACI 211.1R-91)"; American Concrete Institute, ACI Manual of Concrete Practice, Detroit, Michigan, USA.

[2] ACI Committee 544. (1982): "State-of-the-Report on Fibre Reinforced Concrete, (ACI 544.1R-82)";*Concrete International: Design and Construction*. Vol. 4, No. 5: Pp. 9-30, American Concrete Institute, Detroit, Michigan, USA.

[3] ACI Committee 544. (1989): "Measurement of Properties of Fibre Reinforced Concrete, (ACI 544.2R-89)";American Concrete Institute, Detroit, Michigan, USA.

[4] Adda ,H.M. and Slimane,M. (2019):"Study of Concrete reinforced by plastic fibres based on local materials; International Journal of Engineering Research in Africa , Vol.42, Pp. 100- 108.

[5] Aggarwal, M.L. (2002): "Mixture Experiments: Design Workshop Lecture Notes", University of Delhi, India.

[6] ASTM C33/C33M-18 (2018); Standard Specification for Concrete Aggregates. ASTM International: West Conshohocken, PA, USA.

[7] ASTM C1602/C1602M-22 (2022); Standard Specification for Mixing Water Used in the Production of Hydraulic Cement Concrete. ASTM International: West Conshohocken, PA, USA.

[8] British Standards Institution, BS 12 (1978): Ordinary and Rapid – Hardening Portland cement; London.

[9]British Standards Institution, BS 1881-Part 116 (1983). Methods of Determination of Compressive Strength of Concrete Cube, London.,

[10] Department of Environment, DOE(1988): Design of Normal Concrete Mixes , HMSO, London.

[11] Egamana, S and Sule, S.(2017): Optimization of compressive strength of periwinkle shell aggregate concrete, Nigerian Journal of Technology (NIJOTECH), Vol. 36, No. 1, Pp.32- 38.

[12] Ezeh, J.C.& Ibearugbulam, O.M. (2009):"Application of Scheffe's Model in Optimization of Compressive Cube Strength of River Stone Aggregate Concrete";*International Journal of Natural andAppllied Sciences;*Vol. 5, No. 4,Pp 303 – 308 .

[13] Ezeh, J.C., Ibearugbulam, O.M.& Anyaogu, L. (2010a):"Optimization of Compressive Strength ofCement- Sawdust Ash Sandcrete Block using Scheffe's Mathematical Model"; *Journal of Applied Engineering Research.* Vol.4, No.4, Pp 487–494.

[14] Ezeh, J.C., Ibearugbulam, O.M.&Anya, U. C (2010b):"Optimization of aggregate composition of laterite/sand hollow Block Using Scheffe's Simplex Method";*International Journal of Engineering.* Vol.4, No.4, Pp 471 – 478.

[15] Hughes, B.P. (197I): The Economic Utilization of Concrete Materials, Proceedings of the Symposium on Advances in Concrete , Concrete Society, London

[16] Ibearugbulam, O.M. (2006):"Mathematical Model for Optimization of Compressive Strength of Periwinkle

Shell-Granite Aggregate Concrete";*M.Eng.. Thesis,* Federal University of Technology, Owerri, Nigeria.

[17]. Ishaya, A., Oyemogun, I.M., Arinze, A.and Abah, J.C. (2016): Properties of Concrete Produced With Waste Bottle Caps As Partial Replacement Of Coarse Aggregate and Orange Leaves Powder As Plasticizer, Journal of Civil Engineering and Environmental Research ,Vol.8 , No.7, Pp. 22 – 44.

[18] Jackson,N. and Dhir, R.K. (1996): Civil Engineering Materials, 5th Edition, Palgrave, New York.

[19] Neville, A.M. (1990): Properties of Concrete; 3rd edition, Pitman, England.

[20] Nwachukwu,, K. C., Njoku, ,K.O., Okorie ,P. O, .Akosubo,, I.S. , Uzoukwu , C. S.,Ihemegbulem, E.O.and .Igbojiaku , A.U (2022a):" Comparison Between Scheffe's Second Degree (5,2) And Third Degree (5,3) Polynomial Models In The Optimization Of Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC)"; American Journal of Computing and Engineering (AJCE), Vol. 5, No. 1, Pp 1– 23.

[21] Nwachukwu, K.C., Okafor, M. ,Thomas, B., Oputa, A.A., Okodugha,D.A. and Osaigbovo, M.E. (2017): "An Improved Model For The Optimization Of The Compressive Strength Of Glass Fibre Reinforced Concrete (GFRC) Using Scheffe's Second Degree Polynomials"; Researchjournal's Journal of Civil Engineering, Vol. 3, No. 2

[22] Nwachukwu, K.C.,Okodugha, D.A., Uzoukwu , C. S.,Igbojiaku, A.U. and Okafor, M. (2022b): "Application Of Scheffe's Second Degree Mathematical Model For The Optimization Of Compressive Strength Of Steel Fibre Reinforced Concrete (SFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 3.

[23] Nwachukwu, K.C., Okorie, P.O.. Ihemegbulem , E.O., Kemebiye Dimaro , Uzoukwu , C. S., and Maduagwu, J.C.. (2022c): "The Use Of Scheffe's Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC)"; International Journal of Innovative Science and Research Technology (IJISRT), Vol. 7, No. 1

[24] Nwachukwu, K.C.,Ozioko, H.O., Okorie, P.O, and Uzoukwu , C. S. (2022d): "Application Of Scheffe's Mathematical Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC)"; International Journal of Advances in Engineering and Management (IJAEM) , Vol. 4, No. 2, Pp 850-862

[25] Nwachukwu, K. C., Okodugha, D.A., Igbojiaku , A.U. , Attah, U.G. And Ihemegbulem, E.O. (2022e): Application Of Scheffe's Third Degree Regression Model For The Optimization Of Compressive Strength Of Polypropylene Fibre Reinforced Concrete (PFRC); International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. 8, No. 2.

[26] Nwachukwu, K. C., Okodugha,, D.A., Edike, O., Okafor, M.. And Attah, U.G. (2022f): Application Of Modified Scheffe's Third Degree Polynomial Model For The Optimization Of Compressive Strength Of Nylon Fibre Reinforced Concrete (NFRC); International Journal of Trend In Scientific Research And Development (IJTSRD) , Vol. 6, No. 3.

[27] Nwachukwu, K.C., Okodugha, D.A, Akosubo, I.S. and Atulomah, F.K (2022g): Optimization of Compressive Strength of Steel Fibre Reinforced Concrete (SFRC) Using Scheffe's Third-Degree Regression Model; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 11.

[28] Nwachukwu, K.C.**,**Onwuegbuchulem,U.C., Atulomah· ,F.K. and Ihemegbulem, E.O.(2022h): Optimization of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC) ; Iconic Research And Engineering Journals (IRE), Vol. 5, No. 12

[29] Nwachukwu, K.C.**,**Thomas , B., Oguaghamba, O. ,Onwuegbuchulem, U.C., Igbojiaku, A.U., Atulomah, F.K and Okafor. M..(2022i): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's Second Degree Model; international Journal of Engineering Inventions (IJEI), Vol.11 , No.8,Pp.40-50.

[30] Nwachukwu, K.C , Oguaghamba, O, Akosubo, I.S., Atulomah, F.K. and Igbojiaku, A.U. (2022j): Optimization of Compressive Strength of Hybrid Polypropylene – Nylon Fibre Reinforced Concrete(HPNFRC); International Journal Of Advanced Research And Innovative Ideas In Education (IJARIIE), Vol. 8, No. 5. Pp. 376 -392.

[31] Nwachukwu, K.C.,.Edike, O. Oguaghamba, O., Onwuegbuchulem, U.C and Egbulonu, B.A. (2022k): The Application Of Scheffe's Model For The Optimization Of Mussel Shells Fibre Reinforced Concrete(MSFRC); International Journal of Engineering Inventions (IJEI), Vol. 11, No. 9. Pp. 47-57

[32]. Nwachukwu,K.C., Ozioko,H.O., Akosubo, I.S.,Oguaghamba, O.,Okafor, M.,Onyechere, I.C. and Ikpa, P.N (2022l): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Periwinkle Shells Ash (PSA) Using Scheffe's Second Degree Model; Paper presented at the Proceedings of the 1ST International Conference on Advances in cement and concrete research(ICACCR-2022) Organized by Ladoke Akintola University of Technology (LAUTECH), Nigeria. Theme: Cement and Concrete Innovative Materials For Sustainable Development.

[33] Nwachukwu, K.C., Ozioko, H. O., Mama, B.O., Oguaghamba, O., and Atulomah, F.K. (2023a): The Application of Scheffe's Third Degree Regression Model In The Optimization Of Compressive Strength of Hybrid- Polypropylene- Steel Fibre Reinforced Concrete (HPSFRC); International Journal of Research and Analytical Reviews (IJRAR), Vol. , No.. Pp.

[34]. Nwachukwu, K.C., Oguaghamba, O., Ozioko, H. O. and Mama, B.O. (2023b): Optimization Of Compressive Strength of Concrete Made With Partial Replacement Of Cement With Cassava Peel Ash (CPA) and Rice Husk Ash (RHA) Using Scheffe's (6,3) Model; International Journal of Trend In Scientific Research and Development (IJTSRD), Vol.7 , No.2, Pp. 737-754

[35] Nwachukwu, K. C., Mathew, C.C., Mama , B.O. , Oguaghamba, O. And Uzoukwu, C.S. (2023c): Optimization Of Flexural Strength And Split Tensile Strength Of Hybrid Polypropylene Steel Fibre Reinforced Concrete (HPSFRC) : International Journal Of Advance Research And Innovative Ideas In Education (IJARIIE) , Vol. , No.

[36] Nwachukwu, K. C., Oguaghamba, O, Akosubo, I.S., Egbulonu, B.A., Okafor, M. and Mathew, C.C.(2023d): The Use Of Scheffe's Second Degree Model In The Optimization Of Compressive Strength Of Asbestos Fibre Reinforced Concrete (AFRC); Research Inventy: International Journal of Engineering and Science (IJES) , Vol. , No. .

[37]. Nwachukwu, K.C., Nnegha, A.R., Senator, W.U., Okere, C.P. , Nmecha, T.M., Obi, C.G., Etukudoh, E.E. and Ezeike, U.C.(2024a): Determination Of Optimized Flexural And Split Tensile Strengths Of PSA-MSA- Cement Concrete (PMCC) Using Scheffe's (6, 2) Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 5, Pp.

[38]. Nwachukwu, K. C., Anyachi, C.M., Njoku, I.C., Olinya, I.A., Nkama, J., Egbara, T.E., Anike, V.A., Nwabueze, C.S and Ejiofor, H.C.(2024b): Investigation Of Compressive Strength Property Of PSA - SSA-Cement Concrete (PSCC) Using Scheffe's (6, 2) Model; International Journal of Research Publication and Reviews [IJRPR], Vol. 5, No. 6, Pp.

[39] Nwakonobi, T.U and Osadebe, N.N (2008):"Optimization Model for Mix Proportioning of Clay- Ricehusk- Cement Mixture for Animal Buildings"; *Agricultural Engineering International:the CIGR Ejournal,Manuscript BC 08 007*, Vol x

[40] Obam, S.O. (2006). The Accuracy of Scheffe's Third Degree over Second Degree Optimization Regression Polynomials, Nigerian Journal of Technology, Vol. 2, No.25, Pp 1 – 10.

[41] Obam, S.O.(2009):"A Mathematical model for Optimization of Strength of concrete : A case study for shear modulus of Rice Husk Ash Concrete." *Journal of Industrial Engineering International l;* Vol.5, No.9,Pp 76 – 84.

[42] Okere, C.E., (2006):"Mathematical Models for Optimization of the Modulus of rupture of concrete"; M.Eng. Thesis, Civil Engineering Department, Federal University of Technology, Owerri.

[43] Onwuka,D.O, Okere, C.E.,Arimanwa, J.I. and Onwuka, S.U.(2011):"Prediction of Concrete Mix ratios using Modified Regression Theory, *Computer Methods in Civil Engineering*, Vol. 2. No.1 Pp.95- 107.

[44] Oyenuga, V.O. (2008): Simplified Reinforced Concrete Design, 2nd Edition, Asros Ltd, Lagos, Nigeria.

[45] Sanjaykumar, B. and Daule, S.N. (2017):"Use of plastic fibre in the concrete; SSRG International Journal of Civil Engineering,Vol.4, No. 11. Pp. 4- 7

[46] Scheffe, H. (1958):Experiment with Mixtures"; *International Journal of Royal Statistics Society*, Series B, Vol.20, Pp. 344-360.

[47] Shacklock, B.W. (1974): Concrete Constituents And Mix Proportions, The Cement and Concrete Association, London.

[48] Shetty, M.S. (2006): Concrete Technology (Theory and Practice), S. Chand and Company Ltd, India.

[49] Syal, I.C. and Goel, A.K. (2007): Reinforced Concrete Structures, S.Chand , India.

[50] Yin, S, Tuladhar,R, Shi, F, Combe, M, Collister, T. and Sivakugan, N. (2015):" Use of macro plastic fibres in concrete: a review; Journal of Construction and Building Materials Applied, Vol.93, No. 15. Pp. 180- 188.

[51] Zhang, P, Guan, Q and Li, Q. (2013):"Mechanical Properties of Plastic Concrete Containing Bentonite; Journal of Applied Science, Engineering and Technology, Vol.5, No. 4. Pp. 1317- 1322