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Heteroscedasticity Detection in the existence of Outliers in Discrete-Time Series

[1]A. Bishir, [2]M.U. Adehi, [3]Bilkisu Maijamaa and [4]Abubakar, Muhammad Auwal

[1]Department of Statistics, Abubakar Tafawa Balewa University, Bauchi, Nigeria.
[1]Department of Statistics, Nasarawa State University, Keffi, Nigeria.
[2,3,4]Department of Statistics, Nasarawa State University, Keffi, Nigeria.
Email: abashir@atbu.edu.edu.in
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ABSTRACT:

This study determined the impacts of outliers on the detection of heteroscedasticity in the daily closing share price returns series of Zenith Bank using correlogram, Ljung-Box test and Lagrange Multiplier test. The daily stock closing price of the bank were collected for the period 03/01/2006 to 31/12/2023, and comprises 4398 observations. About Seven hundred and ninety-eight (798) outliers were identified in the return series, and their effects were removed to achieve an outlier adjusted series for the bank under study. Meanwhile, heteroscedasticity was found to exist in the two (the outlier contaminated and the outlier-adjusted) series. However, the outcome of our findings revealed that outliers could hide significant heteroscedasticity in correlogram, amplifying the power of Ljung-Box test and Lagrange Multiplier test. The implication is that failure to account for outliers would result in compromised or spurious heteroscedasticity detection in discrete-time series. Thus, the strength of this study is in emphasizing the negative impact that outliers have on the detection of heteroscedasticity.

1. Introduction

Heteroscedasticity means changing variance. It is a phenomenon that occurs when the assumption of constant variance is violated. The existence of heteroscedasticity commonly called the ARCH effect is a very common occurrence in time series data especially financial time series data. The inability of linear stationary models to take shifting variance into account when applied to financial data (returns series) is a significant drawback. The variance-covariance matrix of linear models that ignore heteroscedasticity is no longer the typical one, even though the estimates of ARIMA parameters obtained using ordinary least squares are still consistent and asymptotically normally distributed. Because of this, the t-statistics are deemed incorrect and cannot be applied to assess the importance of the model's individual explanatory variables [1,2]. Also, Neglecting heteroscedasticity can lead to over-parameterization of an ARIMA model and inadequate statistical power, among other disadvantages. Furthermore, ignoring heteroscedasticity might result in erroneous nonlinearity in the conditional mean and make it challenging to calculate the forecast's confidence interval [3,4,1,2].

Nonetheless, the existence of outliers is a highly typical feature of time series data. When a homoscedastic model has outliers, the model becomes heteroscedastic, which distorts the diagnostic tools for heteroscedasticity and may make it difficult to identify. Similarly, [5] confirmed and upheld that outliers have an impact on GARCH model estimation and conditional heteroscedasticity identification. Furthermore, [6] makes clear that outliers significantly affect the estimators of the heteroscedastic models and the current heteroscedasticity tests. The effects of outliers on the heteroscedasticity diagnostic tools are clearly outlined in [7]. They demonstrated how outliers, particularly additive outliers, have a negative impact on the asymptotic size and power features of the Lagrange Multiplier (LM) test for ARCH/GARCH. Additionally, [8] discovered that outliers had an unanticipated effect on the t-statistics, associated p-values, and order of identification of the estimations of GARCH parameters. Thus, one may argue that anytime heteroscedasticity is represented, it is beneficial to account for the existence of outliers.

This study provides a unique starting point because earlier research in Nigeria did not account for outliers when modelling heteroscedasticity in stock returns. For instance, [9] used three different heteroscedastic process models GARCH(1,1), EGARCH(1, 1), and GJR-GARCH(1, 1) models to examine the time series behaviors of the daily stock returns of four companies listed on the Nigerian Stock Market from January 2, 2002 to December 31, 2006. United Bank for Africa, Unilever, Guinness, and Mobil were the four companies whose share prices were analyzed. The leverage effect, leptokurtosis, volatility clustering, and negative skewness that characterize the majority of economic financial time series are all present in the return series. The estimated findings showed that the GJR-GARCH (1, 1) provides a superior assessment for both in-sample and out-of-sample predictions and fits the data better.

[10] used the daily closing prices of the Nigerian Stock Exchange (NSE) to study how volatility responded to both beneficial and bad news. by analyzing the NSE daily stock return series from January 2, 1996, to December 30, 2011, using the EGARCH (1, 1) and GJR-GARCH (1, 1) models. They

discovered substantial evidence for asymmetric impacts in NSE stock returns, but no evidence for a leverage effect. In particular, the EGARCH model estimates revealed a positive and large asymmetric volatility coefficient. Similarly, the GJR-GARCH results demonstrated a substantial and negative asymmetric volatility coefficient, corroborating the presence of positive asymmetric volatility. The study's overall findings supported the theory that good news will cause more volatility in the near future than bad news of the same shape in Nigeria.

[11] used data from January 4, 2005, to August 31, 2012, to study the modelling and forecasting of daily returns volatility of Nigerian bank stocks. The volatility pattern of the bank stocks was captured using two asymmetric models, EGARCH (1, 1) and TARCH (1, 1), and three symmetric models, ARCH (1), ARCH (2), and GARCH (1, 1). The study's conclusions showed that the return series had the ARCH effect and were stationary but not normally distributed. Furthermore, compared to symmetric heteroscedastic models, the results of the post-estimation evaluation showed that asymmetric conditional heteroscedastic models are better suited for modelling the daily returns volatility of Nigerian Banks stocks.

[12] examined how the ARMA and ARCH-type models may be combined to create an ARMA-ARCH model, which would fully represent the linear and non-linear aspects of financial data. First Bank of Nigeria plc's closing share prices every day from January 4, 2000, to December 31, 2013 were taken into account. Evidence from the study demonstrated that the ARCH (1) model was sufficient to describe the changing conditional variance in the returns, whereas the ARMA (2, 2) model was sufficient to model the linear dependency in the returns. Thus, the First Bank of Nigeria's returns series was fully modelled using the ARMA (2, 2)-ARCH (1) model.

[13] investigate on Modeling and Forecast of Ghana’s GDP Using ARIMA-GARCH Model and concluded that the ARIMA-GARCH model is effective in removing the error variance and improving forecasts. In terms of performance, shows that the combined model outperforms the classic ARIMA model and suggested that further research might investigate the efficiency and accuracy of the ARIMA-GARCH model.

Through an examination of the share price returns of Zenith bank plc received from the Nigerian Stock Exchange between January 4, 2006 and May 26, 2015, [14] identified and modelled the asymmetric GARCH effects in a discrete-time series. In order to determine the asymmetric GARCH effects, the study used the sign and size tests, which were represented by EGARCH and TGARCH, respectively, with regard to the normal distribution. The study's conclusions showed that the TGARCH (0, 1) and EGARCH (0, 1) models effectively reflected the asymmetric impact. However, they failed to consider the existence of anomalies.

This study specifically aims to determine how outliers affect the instruments (correlogram, Ljung-Box test, and Lagrange Multiplier test) used to detect heteroscedasticity. Furthermore, the remaining portion of this work is structured as follows: section 2 covers the methods to be investigated, section 3 analysis and discusses the results, and section 4 concludes with the overall results.

2. Materials and Methods

2.1 Return

The return series R_t can be obtained given that P_t is the price of a unit share at time, *t* and P_{t-1} is the share time at *t-1* as follows:

$$Rt = \nabla lnp_t = (1-B)p_t = lnp_t - lnp_{t-1} \tag{1}$$

The R_t in equation (1) is defined as a natural logarithmic transformed share price series, P_t, which is necessary to achieve stationarity, that is, both mean and variance of the series are expected to be stable [15]. The letter *B* is the backshift operator.

2.2 Autoregressive Integrated Moving Average (ARIMA) Model

[16] consider the extension of ARMA model to deal with the homogenous non-stationary time series in which R_t, itself is non-stationary but its d^{th}difference is a stationary ARMA model. Denoting the d^{th} difference of R_t by

$$\varphi(B)\hat{R}_t = \varphi(B)\nabla^d \hat{R}_t = \theta(B)a_t, \tag{2}$$

where $\varphi(B)$ is a nonstationary autoregressive operator such that *d* of the roots of $\varphi(B) = 0$ are unity of the remainder lie outside the unit cycle. $\varphi(B)$ is a stationary autoregressive operator.

2.3 Tools for identification of heteroscedasticity

Correlogram: If at least one lag term in both ACF and PACF of squared residual series is found to be statistically significant, then the presence of ARCH effect is confirmed [14,17]

Ljung - Box Test is given as

$$Q(m) = T(T+2)\sum_{l=1}^{m} \frac{\hat{p}_l^2 a_t'^2}{T-t} \tag{3}$$

where *m* is the sample size, m is a properly chosen number of autocorrelations used in the test, $\hat{p}_l^2(\,a_t^2)$ is the lag-l ACF of a_t^2 [18]. If they consider linear model is adequate, $Q(m)$ is asymptotically a Chi-squared random variable with *m – p – q* degrees of freedom [19].

Lagrange Multiplier Test: Alternatively, the Lagrange Multiplier (LM) test of ARCH(q) can be used to assess the ARCH/GARCH effect (also known as heteroscedasticity in the transforming conditional variance) against the null hypothesis of no ARCH effects to the $\{a_t^2\}$ series.

. The LM test is carried out by computing $\chi^2 = TR^2$ in the regression of a_t^2 on a constant and q lagged values. T is the sample size and R^2 is the coefficient of determination. Under the null hypothesis of no ARCH effects, the statistic has a Chi-square distribution with q degrees of freedom. If the LM test statistic is larger than the critical value, then, there is evidence of the presence of ARCH effect [20].

Outliers in Time Series: An observation on a sample that deviates from the general pattern is called an outlier. A time series may typically contain k or more outliers of various kinds, and the general outlier model is as follows;

$$Y_t = \sum_{k=1}^{n} t_k V_k(B) I_t^{(T)} + X_t, \quad (4)$$

where $X_t = \frac{\theta(B)}{\varphi(B)} a_t$, $V_k(B) = 1$ for an AO, and $V_k(B) = \frac{\theta(B)}{\varphi(B)}$ for a IO at $t = T_k$, $V_k(B) = (1-B)^{-1}$ for a LS, $V_k(B) = (1-\delta B)^{-1}$ for a TC, and τ is the size of outlier. For more details on the types of outliers and estimation of the outlier effects see [21,22,16,19,23, 24, 25].

Additionally, the residual series, a_t, in financial time series is considered to be uncorrelated with its own past; as a result, additive, innovative, temporary change, and level shift outliers coincide, and in instances where the variance and mean equations evolve simultaneously, we obtain

$$R_t - \mu_t = \bar{a}_t + wI_t^{(T)} \quad (5)$$

$$\bar{a}_t = \sigma_t e_{t,} \quad (6)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 \bar{a}_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (7)$$

where $\bar{a}_t$ is the outliers contaminated residuals.

3 Results and Discussion

This study considers the daily closing share prices of Zenith bank in Nigeria and were obtained from the Nigerian Stock Exchange through the data range from January 3, 2006 to December 31, 2023 and comprises 4398 observations.

3.1 Time Series Plot Interpretation

Fig. 1 shows the bank's share price series. It is evident that there is no common mean around which the share prices of the banks move. As a result, this strongly suggests that there is a stochastic tendency in the share prices, indicating non-stationarity.

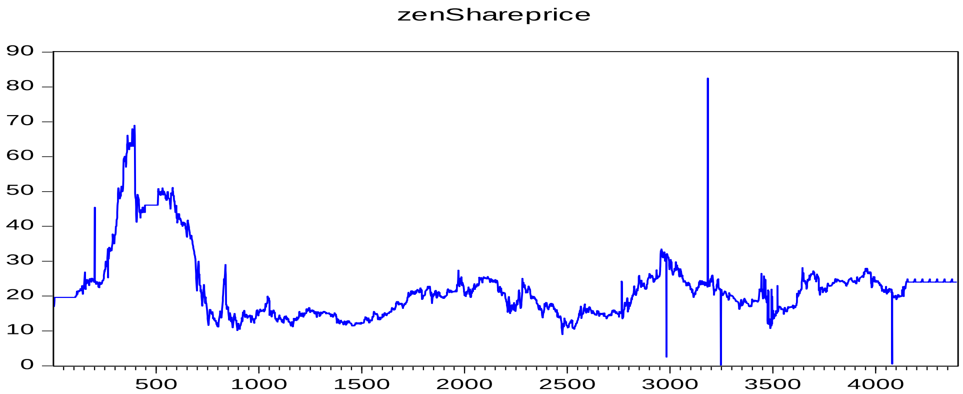

Figure 1: Share Price Series of Zenith Bank

Given the non-stationary nature of the share price series, a stationary (returns) series can be obtained by taking the first difference of the share price series' natural logarithm. The variance is stabilized through the use of the log transformation. It is suggested by Fig. 2 that volatility clustering is clearly visible in the various series and that the returns series appear to be stationary.

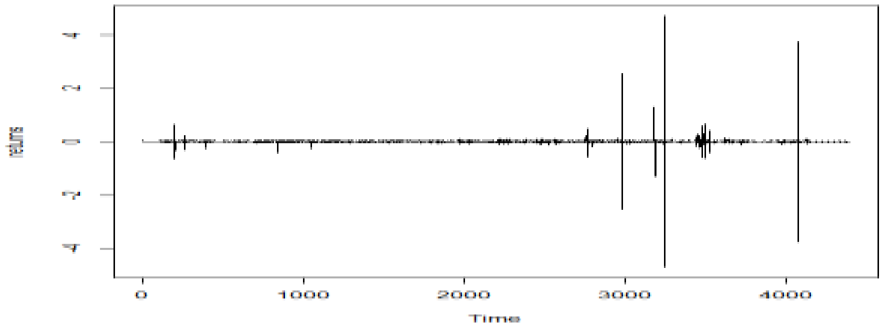

Figure 2: Return Series of Zenith Bank

From Figs. 3 and 4, both ACF and PACF indicate that mixed model could be entertained. The following models, ARIMA (1, 1, 1), ARIMA (1, 1, 2), ARIMA (2, 1, 1) and ARIMA (2, 1, 2) are entertained tentatively.

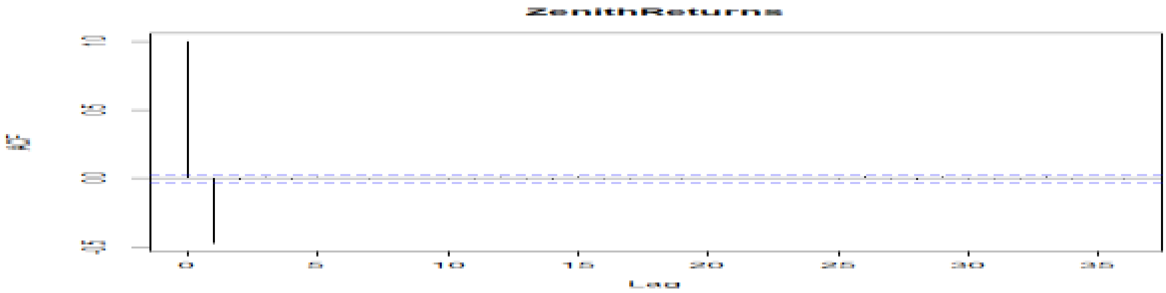

Figure 3: ACF of Return Series of Zenith Bank

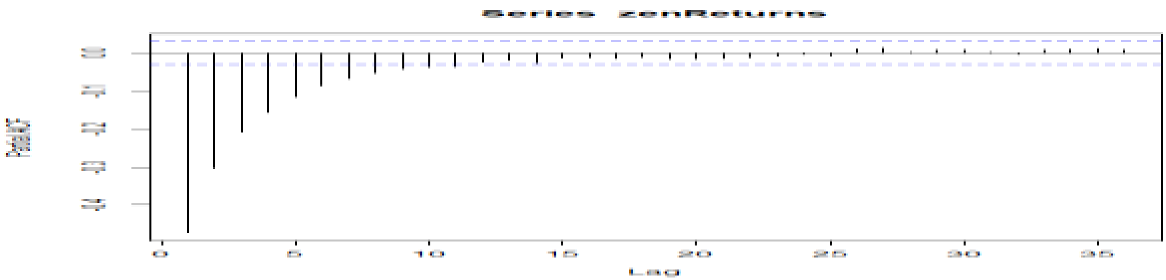

Figure 4:P ACF of Return Series of Zenith Bank

From Table 1, ARIMA (2, 1, 1) model is selected based on the ground of significance of the parameters and minimum AIC.

Table 1. ARIMA Models for Return Series of Zenith Bank

Model	Parameter	Estimate	s.e	z-ratio	P-value	AIC
ARIMA (1,1,1)	ϕ	-4.7294e-01	1.3290e-02	-35.5858	<2e-16	-5481.05
	θ	-1.0000e+00	8.4071e-04	-1189.4688	<2e-16	
ARIMA (1,1,2)	ϕ	3.3509e-02	1.9751e-02	1.6966	0.08978	-6356.63
	θ_1	-1.7905e+00	1.2860e-02	-139.2313	<2e-16	
	θ_2	7.9046e-01	1.2789e-02	61.8079	<2e-16	
ARIMA (2,1,1)	ϕ_1	-6.1607e-01	1.4379e-02	-42.8443	<2e-16	**-5900.62**
	ϕ_2	-3.0231e-01	1.4375e-02	-21.0305	<2e-16	
	θ	-1.0000e+00	8.5800e-04	-1165.5002	<2e-16	
ARIMA (2,1,2)	ϕ_1	4.1686e-02	2.1661e-02	1.9244	0.0543	

	ϕ_2	1.5191e-02	1.9341e-02	0.7855	0.4322	-6354.74
	θ_1	-1.7988e+00	1.5840e-02	-113.5580	< 2e-16	
	θ_2	7.9876e-01	1.5789e-02	50.5891	< 2e-16	

Furthermore, given the Q-statistic at Lags 1, 4, 8, and 16, namely, Q(1) = 0.00036, Q(4) = 0.65325, Q(8) = 0.90444, and Q(16) = 1.27890 with corresponding *(P =.9849), (P =.9570), (P =.9988),* and *(P = 1.0000)*, respectively, evidence from Ljung - Box Q-statistics demonstrates that the ARIMA (2, 1, 1) model is adequate at the 5% level of significance.

3.3 Identification heteroscedasticity in the return series of Zenith bank

The residual series of the ARIMA (2, 1, 1) model exhibits heteroscedasticity, as seen in Figs. 5 and 6, where the lags 1, 2, 3, 15, 16, 20 of the ACF and the lags 1, 2, 3 and 15 of the PACF are not within the significance boundaries.

Additionally, the Portmanteau-Q statistics Q(4) = 199.16, Q(8) = 199.18, Q(12) = 199.20, Q(16) = 199.22, Q(20) = 199.25, and Q(24) = 199.27, whose corresponding *(P =.0000), (P =.0000), (P =.0000)* and (P =.0000) are all less than 5% level of significance, are said to indicate the presence of heteroscedasticity in the residual series at lags 4, 8, 12, 16, 20, and 24. The Lagrange-Multiplier (LM) test statistics provide additional proof that the residual series of ARIMA (2,1,1) exhibit heteroscedasticity.

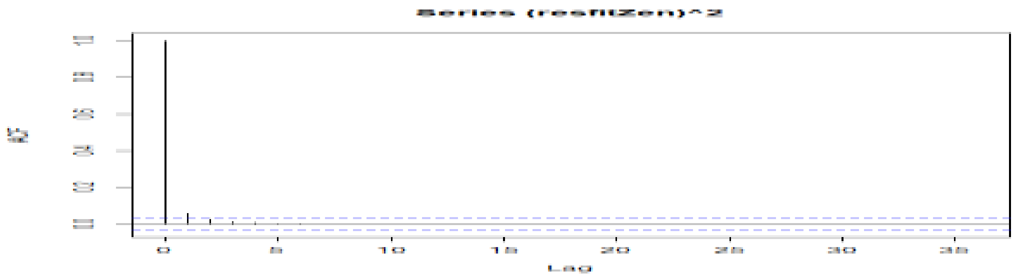

Figure 5: ACF of the Square Residuals of ARIMA (2,1,1) Model fitted to Return Series of Zenith Bank

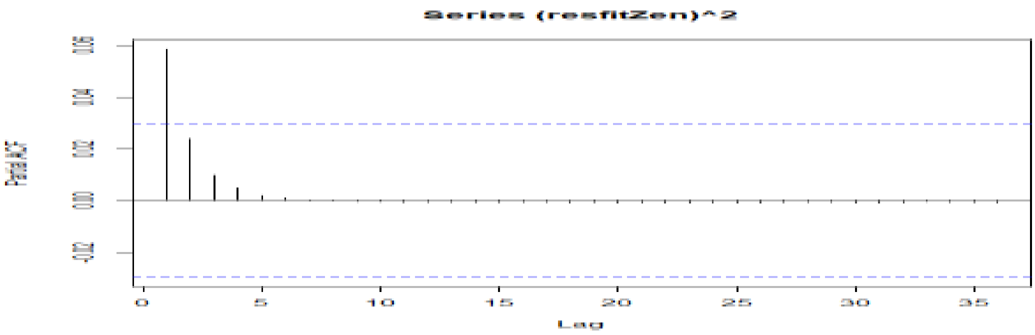

Figure 6: ACF of the Square Residuals of ARIMA (2,1,1) Model fitted to Return Series of Zenith Bank

3.4 Identification of outliers in the residual series of ARIMA (2, 1, 1) model fitted to the return series of Zenith bank

Eighty-five (85) are innovation outliers (IO), two hundred and sixty-nine (269) are additive outliers, and four hundred and forty-four (444) are temporary change outliers. A total of about (798) distinct outliers were found to have contaminated the residuals series of the ARIMA (2,1,1) model.

3.5 Building ARIMA (2, 1, 1) model for outlier adjusted return series of Zenith bank

Since it has been determined that the Zenith Bank return series is tainted by outliers, the outlier effects are eliminated from the series to create a new series that is devoid of outliers; we call these series "outlier adjusted series." Additionally, the ARIMA (2, 1, 1) model is fitted to the outlier adjusted series with all parameters significant at the 5% level [Table: 2]. Given the Q-statistics at Lags 1, 4, 8, and 16, namely Q(1) = 0.0144, Q(4) = 0.0525, Q(8) = 0.5225, and Q(16) = 1.2261 with corresponding *(P =.9144), (P =.6243), (P =.1982)* and *(P =.0842)*, it is determined to be sufficient at the 5% level of significance.

Table 2.: ARIMA (2,1,1) Model for Outliers Adjusted of Return Series of Zenith Bank

Model	Parameter	Estimate	Std. Error	z-value	P-value	AIC
	ϕ_1	-6.2603e-01	1.7163e-02	-36.4762	<2e-16	-3094.15

ARIMA (2,1,1)	ϕ_2	-3.0907e-01	1.7157e-02	-18.0149	<2e-16	
	θ	-1.0000e+00	1.0643e-03	-939.5724	<2e-16	

3.6 Identification of heteroscedasticity in the outlier adjusted return series of diamond bank

From Figs. 7 and 8, it can be observed that there is heteroscedasticity in the residual series of the ARIMA (2, 1, 1) model considering some lags of the ACF and PACF of the squared residual series of the model fitted to the outlier adjusted return series of Zenith bank are outside the significance bounds. Furthermore, since the Portmanteau-Q statistics, Q(4) = 132.19, Q(8) = 132.23, Q(12) = 132.27, Q(16) = 132.29, Q(20) = 132.32, and Q(24) = 132.35, whose corresponding *(P =.0000), (P =.0000), (P =.0000),* and *(P =.0000)* are all less than 5% level of significance, indicate the presence of heteroscedasticity in the residual series at lags 4, 8, 12, 16, 20, and 24.

The Lagrange Multiplier test statistics, which are LM(4) = 2023.62, LM(8) = 9780.14, LM(12) = 6501.94, LM(16) = 4863.08, LM(20) = 3879.77, and LM(24) = 3224.20, with corresponding *(P =.0000), (P =.0000), (P =.0000),* and *(P =.0000)*, all show additional evidence of heteroscedasticity at lags 4, 8, 12, 16, 20, and 24.

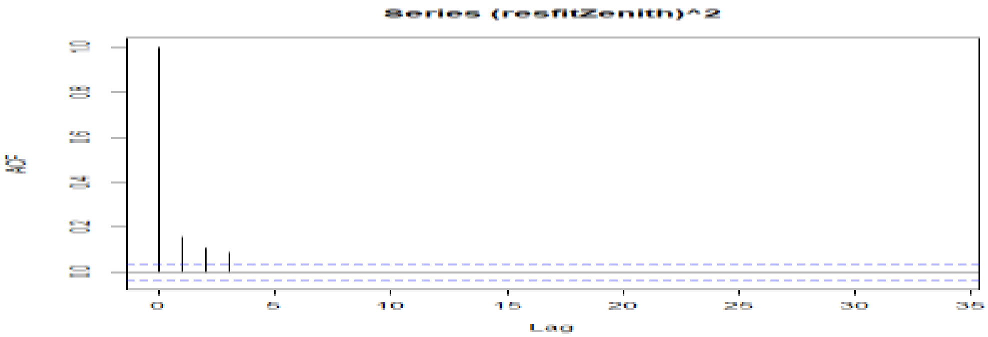

Figure 4.39: ACF of the Square Residuals of ARIMA (2,1,1) Model fitted to Outliers Adjusted Return Series of Zenith Bank

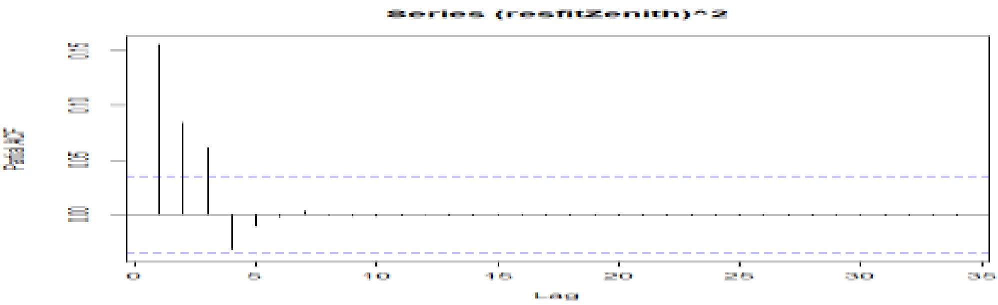

Figure 4.40: ACF of the Square Residuals of ARIMA (2,1,1) Model fitted to Outliers Adjusted Return Series of Zenith Bank

3.7 Effects of outliers on heteroscedasticity identification tools in the return series of Zenith bank

Correlogram: The ACF and PACF of the squared residuals of the ARIMA (2, 1, 1) model fitted the outlier adjusted return series of Zenith bank [Figs. 7 and 8] and the squared residuals of the ARIMA (2, 1, 1) model fitted the outlier contaminated return series of Zenith bank [Figs. 5 and 6] are clearly showing more and more significant lags in both ACF and PACF of the squared residuals of the ARIMA (2, 1, 1) model fitted the outlier adjusted return series. Therefore, it can be concluded that heteroscedasticity detection in the ACF and PACF of the return series of Zenith bank is hidden by the existence of outliers.

Ljung-Box (Portmanteau) Q test: In order to determine the impact of outliers on the Ljung-Box (Portmanteau) Q-test, we conduct a comparison between the Q-statistic values obtained from the residuals of the ARIMA (2, 1, 1) model fitted to outlier-contaminated return series and the Q-statistic values obtained from the residuals of the model fitted to the outlier adjusted return series. Using Table 3's outlier-contaminated series as a guide, we were able to figure out that, for lags of 4, 8, 12, 16, 20, and 24, respectively, the presence of outliers reduces the power of the **Ljung-Box test:** by 33.62%, 33.61%, 33.61%, 33.60%, 33.59%, and 33.58%. It may be inferred that when there are outliers, the Ljung-Box test exhibits distortion and its power increases. Consequently, it becomes more difficult to identify actual heteroscedasticity.

Table 3: Effects of Outliers on Ljung-Box (Portmanteau) Q-test for Return Series of Zenith Bank

Lag	Value of Q-statistic on Residual Series of ARIMA (2,1,1) Model Fitted to Return Series of Zenith Bank	Value of Q-statistic on Residual Series of ARIMA (2,1,1) Model Fitted to Outlier Adjusted Return Series of Zenith Bank	Average Effect of Outlier Identified (%)
4	199.1500	132.1874	33.62
8	199.1765	132.2248	33.61
12	199.2013	132.2565	33.61
16	199.2236	132.2877	33.60
20	199.2455	132.3210	33.59
24	199.2729	132.3531	33.58

Lagrange Multiplier Test: In order to examine how outliers affect the Lagrange Multiplier (LM) test, we compare the values of the LM test statistic on the residuals of the ARIMA (2, 1, 1) model fitted to the outlier adjusted return series to the values of the LM test statistic on the residuals of the model fitted to the return series of contaminated with outliers. The existence of outliers raises the power of the Lagrange Multiplier test by 27.55%, 27.76%, 27.82%, 27.88%, 27.94%, and 28.01% at lags 4, 8, 12, 16, 20, and 24, respectively, according to Table 4, which we used as a reference point for the outlier contaminated series. It follows that when there are outliers, the Lagrange Multiplier test is distorted, and its power increases.

Table 4: Effect of Outliers on Lagrange Multiplier LM-test for Return Series of Zenith Bank

Lag	Value of LM on Residual Series of ARIMA (2,1,1) Model Fitted to Return Series of First Bank	Value of LM on Residual Series of ARIMA (2,1,1) Model Fitted to Outlier Adjusted Return Series of First Bank	Average Effect of Outlier Identified (%)
4	27930.456	20234.616	27.55
8	13537.883	9780.138	27.76
12	9007.992	6501.942	27.82
16	6743.222	4863.079	27.88
20	5384.346	3879.765	27.94
24	4478.372	3224.196	28.01

4 Conclusion

Thus far, the ARIMA (2, 1, 1) model has been successfully found and fitted to the Zenith Bank share price returns series. It was discovered that there were many outliers in the bank's series. The ARIMA (2, 1, 1) model was fitted to the outlier-adjusted series of the corresponding bank after the effects of outliers were eliminated from the series for the purpose of argument. Specifically, employing the correlogram, Ljung-Box test, and Lagrange multiplier, heteroscedasticity was found in both the outlier-contaminated and outlier-adjusted series of the corresponding bank. Our research showed that in the return series of the bank under investigation, outliers obscure, impede, and distort the detection of actual heteroscedasticity. Therefore, it can be concluded that the existence of outliers must be taken into account in order to properly detect and identify heteroscedasticity in discrete-time series. Additionally, the effects of outliers on parameter estimation in heteroscedastic models could be included by expanding the scope of this study.

Competing Interests

Authors have declared that no competing interests exist.

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