



## Permeability Analysis of Unconsolidated Porous Media by Darcy Flow Model

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### ABSTRACT

The study of porous media is important to many areas, from engineering to biomedical area. The properties of these porous media are of fundamental importance, which will provide data and information that will influence the development and viability of projects related to the study of porous media, such as the recovery of gas and oil, preparation of catalysts, migration of contaminants in the soil and other things more. The present study aimed to evaluate more specific ranges of permeability of unconsolidated porous media through the studies of flow versus pressure. Permeability measurements were then performed using the Darcy model in three types of unconsolidated materials: sand with particle size of 1 to 2 mm and greater than 2 mm and glass spheres with particle size of 3 mm. The experimental apparatus used for the study of Darcy's Law model was a closed-circuit permeameter. The use of this apparatus provided the simplest and most objective handling of permeability measurements through flow versus pressure studies. It was possible to obtain the permeability values in more specific study ranges according to the granulometry of the material that composes the porous medium beds.

Keywords: Permeability, porous media, Darcy model

### 1. Introduction

Porous media are present in several areas, from engineering to biomedical. According to Dullien (1979), porous structures have many applications ranging from hip prostheses, dental implants, intervertebral discs prostheses. Also, can be cited applications of porous media in different areas such as: preparation of catalysts, membranes and filters; migration of contaminants and fertilizers in soils etc.

According to Cabral and Da Silva (2018), the macroscopic properties of rocks, such as porosity and permeability, are important for the oil industry, as they influence the profitability of an oil deposit. "A reservoir, from an economic point of view, is considered to have good productivity if it has, in addition to a large amount of oil, optimal conditions for recovering the fluids, which are directly associated with porosity".

Flow in porous media is found on a large scale. According to Collins (1961), one of the essential properties of flow in porous media is permeability, which characterizes the ability of the fluid to flow through a given porous material. The water existing in the voids of the porous media can only flow if there are a network of connected pores which allows it to move freely through the soil or rock (Franciss, 1980).

The permeability is dependent on several soil properties, mainly density and macro and microporosity (Mesquita & Moraes, 2004). The granulometry and structure of the soil influence the pore space, its total porosity and distribution of pores, which affects the water movement to a greater or lesser extent. According to Neves (1987), the permeability coefficient can be determined in different ways, such as: empirical formulas; laboratory tests (using permeameter) and field tests.

Permeability and porosity are two typical attributes of sedimentary rocks, which is why sedimentary basins (porous layers or sheets of sand, sandstone or limestone) are the main exploratory regions for the search of oil and natural gas. Porous rocks, when they have high permeability, form the ideal scenario for the occurrence of economically viable oil and natural gas reservoirs (Meirelles, 2007). The improvement of studies, through improvements in the methods and processes used, helps to better understand the phenomena and their properties.

In 1856, a French hydraulic engineer, Henry Darcy, began the study of hydrogeology as a quantitative science with the publication of his work from experiments carried out in the city of Dijon, France. He obtained the Darcy's law through experimental models,

$$Q = \frac{kA\Delta P}{\mu L} \quad (1)$$

where Q [m<sup>3</sup>/s] is the flow rate, k [m<sup>2</sup>] is the permeability, ΔP [Pa] is the pressure gradient applied to the bed, μ [Pa.s] is the viscosity of the fluid, L [m] is the length of the bed and A [m<sup>2</sup>] is the total transversal area of the bed.

The flow process in porous media is of great interest to a wide variety of engineers and scientists. The one-dimensional law discovered empirically in 1856 by Darcy served as a starting point for numerous practical applications and as a constant challenge for theorists. While the original conditions studied by Darcy are found in several practical situations, their extensions to more general cases require a special theoretical analysis, as they are situations in which the experiments are difficult to carry out (Whitaker, 1986).

According to Bird et al., 2007 the simulation of flow phenomena in porous media becomes difficult due to the complex nature of the geometry and topology of the porous media and the heterogeneity in the chemical composition of their internal walls. An alternative path to follow is the study of flow in porous media at the macroscopic scale, that is, at the observation scale.

This work aimed to evaluate experimentally the permeability of unconsolidated porous media according to the grain size of the bed, utilizing three different types of unconsolidated beds: sand with grain sizes between 1 to 2 mm, sand with grain sizes larger than 2 mm and glass spheres with particle sizes of 3 mm. To obtain the experimental values, a mobile bench permeameter, in a closed circuit, was used, where measurements were handled through the flow rates and the pressure drop of the water in the column of the bed.

## 2. Materials and Methodology

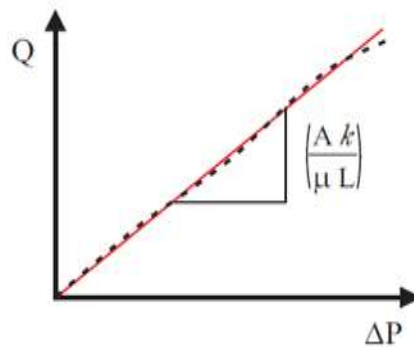


Figure 1: Determination of permeability by angular coefficient

The empirical Darcy model is the statistical average of the classical hydrodynamic equation over time through detailed variation in individual pores, resulting in a simplified macroscopic representation. It is basically a straight line in which the permeability is one of the terms that make up its angular coefficient as seen in Figure 1 below.

Thus, the angular coefficient can be expressed as follows:

$$\alpha = \frac{AK}{\mu L} \quad (2)$$

Substituting Eq. 2 into Eq. 1, the following equation was obtained:

$$Q = \alpha \Delta P \quad (3)$$

Where:  $Q$  [ $\text{m}^3/\text{s}$ ] is the flow,  $\alpha$  [ $\text{m}^3/\text{kg}$ ] is the angular coefficient and  $\Delta P$  [Pa] is the pressure gradient applied to the bed.

To standardize the results, it was also calculated the permeability coefficient for the temperature of 20 °C, as following:

$$k_{20} = k \frac{u}{u_{20}} \quad (4)$$

Where:  $u$  is the viscosity of the fluid at the test temperature and  $u_{20}$  is the viscosity of the fluid at 20 °C.

The basic unit used to measure permeability is the *darcy* which have the following unit:

$$1 \text{ darcy} = 1 \times 10^{-12} \text{ m}^2$$

The porous medium can be described as homogeneous and heterogeneous, and is classified as homogeneous when the permeability does not vary throughout the studied material and heterogeneous when there is a variation in permeability throughout the studied bed. Many porous media have structural qualities that depend incisively on the orientation chosen along the material and when faced with this type of material, the media is classified as anisotropic. Thus, in anisotropic media, permeability varies with the direction adopted along the material. Otherwise, the medium is said to be isotropic and has permeability independent of the direction considered (Brettas, 2013).

The development of the Darcy flow model is characterized by the following features (Amao, 2007): The Reynolds number will indicate whether the regime will be laminar or turbulent, and to employ Darcy's Law as a methodology in evaluating permeability, first it is necessary to ensure that the flow is behaving in the laminar regime field, thus being able to say that the flow is Darcian. In porous media, however, there is no clear limit or number that defines this transition. The nonlinearity of non-Darcian flow is not a result of turbulence, but of inertial effects, as mentioned previously, so it is known that non-Darcian flow occurs in porous media at a lower Reynolds number, and this phenomenon does not is initiated by the change in flow regime.

Ergun (1952) proposed a modification to Chilton Colburn's equation by including porosity ( $\phi$ ) and using the intrinsic velocity ( $v_s$ ) — flow velocity in the pores — which is obtained by the flow rate and the allowable flow area, in the calculation of the Reynolds number for porous media, which can be written as:

$$N_{RD} = \frac{\rho v_s d_m}{\mu} \frac{1}{(1-\phi)}, \quad (5)$$

Where:  $N_{RD}$  = Reynolds number for porous media;  $\phi$  = porosity;  $\rho$  [ $\text{kg}/\text{m}^3$ ] = fluid density;  $v_s$  [ $\text{m}/\text{s}$ ] = average flow velocity in the pores;

$d_m$  [ $\text{m}$ ] = average diameter of the particles of the porous medium through which the fluid flows;  $\mu$  [ $\text{Pa}\cdot\text{s}$ ] = viscosity of the fluid.

Aiming to study the water flow in an unconsolidated porous medium, under Darcian flow conditions, an experimental apparatus called closed-circuit permeameter was developed. Figure 2 shows the experimental apparatus.

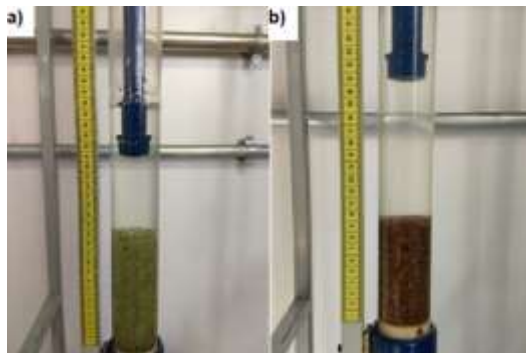


**Figure 2: Closed-circuit permeameter, used in the experiments.**

The experimental apparatus, Figure 2, has some points to be highlighted, which are: the water reservoir; the peripheral pump located on the bench that maintains a hydraulic load on the porous bed; the valves, which allow you to manually control the water flow to the acrylic tube; acrylic tube where all observation takes place, as the bed is located there and where the water column is formed; the rotameter, which is used to collect flows; the measuring tape where the height of the water column above the bed is collected.

In order to simulate porous media, unconsolidated beds of different bed column heights were used. For this purpose, beds composed of 3 mm glass spheres, sand with particle sizes between 1 and 2 mm and sand with particle sizes greater than 2 mm were used in the study.

After calculating the porosity of each bed, the material to form the respective bed was inserted into the permeameter. Figure 3 shows one of the beds generated for the experiments.



**Figure 3: Some bed utilized in the experiments – a) glass spheres, b) sand**

### 3. Results and Discussion

Table 1 shows the porosities of the beds used in the work. The table is organized into Bed porous media, Bed Column Height and Bed Porosity.

**Table 1 – Column height and porosity of the beds studied**

Bed	Column height (cm)	Porosity
Sand 1 (1mm-2mm)	5	43,9%
Sand 1 (1mm-2mm)	7	45,4%

Sand 1 (1mm-2mm)	9	45,6%
Sand 2 (>2mm)	7	43,6%
Sand 2 (>2mm)	9	43,4%
Glass spheres	all	33,0%

**Experiments with glass spheres**

To understand the permeability of the bed composed of glass spheres with a grain size of 3 mm, three column heights were used, L = 13.0 cm, L = 15.0 cm and L = 17.7 cm. In Fig. 4, for the bed composed of glass spheres, is plotted the data of flow versus pressure drop, collected in the experiments with bed columns of L = 13.0 cm, L = 15.0 cm and L = 17.7 cm. As it can be seen from Fig. 4, there is a proportional relation between flow and pressure drop which is an indication of Darcy flow regimen. For all the cases presented in Fig. 4, the determination coefficient was higher than 99.4% which shows the great adjust between the experiments and the model. The angular coefficient of the linear equations (Fig. 4), obtained from the correlation between the linear model and the experimental data, is proportional to the permeability as observed in Eq. 2.

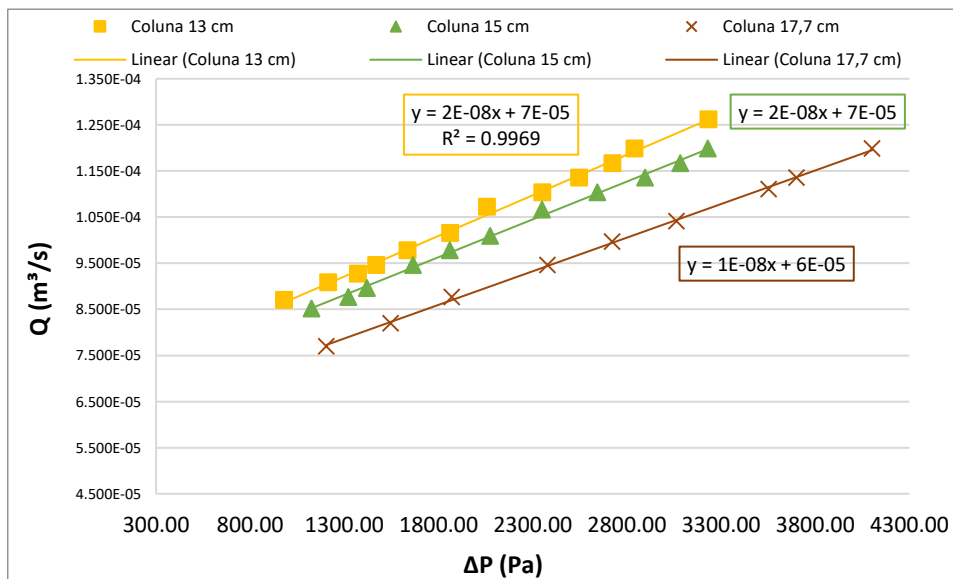
Table 2 presents the results of the experiments for beds of glass spheres. The table columns are organized as follows: experiment number; height of the bed;  $\alpha$  (the angular coefficient of the linear model); k (the permeability of the bed obtained through Equation 8);  $\bar{k}$  (the average permeability;  $\bar{k}_{20}$  (the average permeability at a temperature of 20 °C ) and  $\sigma$ , absolute error.

**Table 2: Permeabilities obtained in experiments with glass spheres.**

Nº	Bed heights [cm]	$\alpha$ [ $m^3/kg$ ]	$k$ [ $m^2$ ]	$\bar{k}$ [ $m^2$ ]	$\bar{k}_{20}$ [ $m^2$ ]	$\sigma$
1	13,0	1,647E-08	4,827E-12			4,136E-13
2	15,0	1,646E-08	5,316E-12	5,241E-12	5,047E-12	7,550E-14
3	17,7	1,464E-08	5,578E-12			3,381E-13

Permeability depends on the complexity of porous morphology and size distribution, porosity and pore connectivity. Usually, k and  $k_{20}$  are referred to as permeability constants, since they are supposedly independent of the permanent fluid or flow conditions, even though they may vary with temperature (Innocentini et al., 2009).

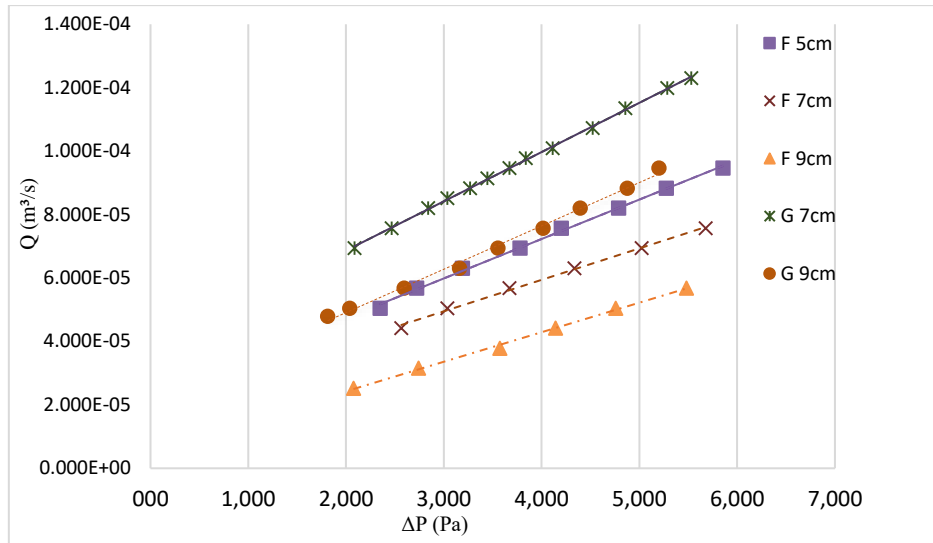
The permeability values ( $\bar{k}_{20}$ ) were used for comparison with the results from the experiments of Fetter, 1994. The permeability of the bed of glass spheres with particle size of 3 mm was  $\bar{k}_{20} = 5.047 E^{-12} m^2$ , which can then be classified as sand, according to Fetter classification, as the permeability result are within the range of  $10^{-14}$  to  $10^{-10} m^2$ .



**Figure 4: Graph of flow versus pressure for the bed composed of glass spheres of 3 mm.**

**Experiments with sands**

In Figure 5, the experiments with sands are presented for two grain size ranges; the first one with grain sizes of 1 to 2 mm, and the second one with grain size greater than 2 mm. For the first range, particle size from 1 to 2 mm, it is presented in the graph index accompanied by the letter “F” followed by the height of the bed column. The second range, particle size greater than 2 mm, is accompanied by the letter “G ” followed by the height of the bed column.



**Figure 5: Graph of flow rate versus pressure for sand bed; Sizes of 1 to 2 mm (F) and greater than 2 mm (G)**

From Fig. 5 it can be seen the great adjust between the linear model and experimental data, around 99,4 and 99,9%. As it can be seen from the linear model, there is an increase in the linear coefficient with the increase in the size of the bed both for the sizes of 1 to 2mm (F) and greater than 2mm. The increase in the size of the bed increases the total resistance of the media reducing the flow. The angular coefficient is slightly higher for the high sand size (greater than 2mm, G) which is associated to a higher permeability.

Table 3 presents the results of the experiments for beds of sand. The table columns are organized as follows: experiment number; height of the bed;  $\alpha$  (the angular coefficient of the linear model);  $k$  (the permeability of the bed obtained through Equation 8);  $\bar{k}$  (the average permeability);  $\bar{k}_{20}$  (the average permeability at a temperature of 20 °C ) and  $\sigma$ , absolute error.

**Table 3- Permeabilities obtained in experiments with sand.**

Sand size	Bed size [cm]	$\alpha$ [m³/kg]	$k$ [m²]	$\bar{k}$ [m²]	$\bar{k}_{20}$ [m²]	$\sigma$
Sand 1<x<2 mm	5,0	1,280E-08	1,409E-12	1,579E-12	1,534E-12	1,696E-13
	7,0	9,522E-09	1,468E-12			1,112E-13
	9,0	9,383E-09	1,860E-12			2,808E-13
Sand x>2 mm	7,0	1,571E-08	2,423E-12	2,516E-12	2,444E-12	9,325E-14
	9,0	1,247E-08	2,609E-12			9,325E-14

From Table 3 it is clear the increase in the permeability for the sand with size higher than 2mm. Based on the results obtained in the present work, the height of the bed directly influences both the linear coefficient (flow rate at zero column height of water) as well as the angular coefficient (permeability). Other factors are important such as packing that is not the same for each experiment. An example is the comparison of the bed with 7 cm with sand size of 1 to 2 mm (F7 cm) with the bed with 7 cm with sand size larger than 2 mm (G7 cm). In this case there is a 65 % increase in the angular coefficient and approximately 66% increase in the linear coefficient for G7 bed compared to F7 bed.

**4. Conclusions**

From the experiments there was a need to obtain an average permeability due to the difference in permeability when changing the bed height, which is mainly related to the packing. This difference was more significant in the bed composed of sand grains, as the morphology is more complex and there is a greater randomness of grain sizes when compared to that of glass spheres. The structural complexity of the porous medium provided a greater difference in their permeabilities.

The experimental results for the bed composed of glass spheres with a grain size of 3 mm had an average permeability of  $5.047 \times 10^{-12} \text{ m}^2$  and porosity of 33%. The bed composed of sand with a grain size between 1 and 2 mm and greater than 2 mm resulted in an average permeability of  $1.534 \times 10^{-12} \text{ m}^2$  and  $2.444 \times 10^{-12} \text{ m}^2$ , respectively, and their porosities in the range of 43% to 45%.

Comparing the obtained permeabilities of glass spheres and sand, with Fetter classification, can be concluded that both materials can be classified as "sand soil". This shows the reliability of the experimental methodology applied. The permeability values found highlighted the scope and importance of Darcy's studies for the flow of fluids in non-consolidated porous media, since she was able to present consistent values for permeability. The mobile bench permeameter presented in this work showed great potential in measuring the permeability of unconsolidated porous media applying a water column height up to 70 cm.

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