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A class of new solutions in integers to Ternary Quadratic Diophantine Equation.

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ABSTRACT

This paper aims at presenting many non-zero distinct solutions in integers to ternary quadratic Diophantine equation $12(x^2+y^2)-23xy+2x+2y+4=56z^2$ .The substitution strategy is employed to obtain the solution patterns.

Keywords : Ternary quadratic equation ,Non-homogeneous quadratic equation, Integer solutions

Introduction

It is well-known that the subject of diophantine equations occupies a pivotal role in the Number theory. There is a vast general theory for quadratic equations in many variables and it is a topic for research even today. While collecting problems on quadratic diophantine equations having three variables ,the paper [1] was noticed and the authors have presented only four sets of solutions belonging to a single pattern. It is worth to mention that the equation presented in [1] has many more fascinating patterns of solutions in integers.

In this paper ,the process of obtaining some more choices of solutions in integers is illustrated.

Technical procedure

The non-homogeneous ternary quadratic Diophantine equation to be solved is

$$12(x^2+y^2)-23xy+2x+2y+4=56z^2 \quad (1)$$

To start with , it is observed by scrutiny that (1) is satisfied by

$$x=\pm 4k-2\ ,y=\pm 2k-2\ ,z=k$$

To obtain the other choices of patterns of integer solutions to (1) ,the substitution of the linear transformations given below

$$\begin{aligned} x &= 6a+56b+2c-2\ , \\ y &= 6a-56b-2c-2\ , \\ z &= 2c+47b \end{aligned} \quad (2)$$

in (1) leads to the homogeneous ternary quadratic equation

$$c^2=a^2+658\,b^2 \quad (3)$$

Note that (3) is of the form $z^2=y^2+D\,x^2$,where $D>0$ &square-free.

Employing the most cited solutions to the above equation ,it is seen that (3) is satisfied by

$$b=2pq\ ,a=658p^2-q^2\ ,c=658p^2+q^2 \quad (4)$$

In view of (2) , the corresponding integer solutions to (1) are given by

$$x = 6\,(658\,p^2 - q^2) + 112\,p\,q + 2(658\,p^2 + q^2) - 2\,,$$
$$y = 6\,(658\,p^2 - q^2) - 112\,p\,q - 2(658\,p^2 + q^2) - 2\,,$$
$$z = 94\,p\,q + 2(658\,p^2 + q^2).$$

Note 1

The integer solutions to (3) may also be taken as

$$b = 2\,p\,q\,, a = p^2 - 658\,q^2\,, c = p^2 + 658\,q^2$$

For this choice , the corresponding integer solutions to (1) are given by

$$x = 6\,(p^2 - 658\,q^2) + 112\,p\,q + 2(p^2 + 658\,q^2) - 2\,,$$
$$y = 6\,(p^2 - 658\,q^2) - 112\,p\,q - 2(p^2 + 658\,q^2) - 2\,,$$
$$z = 94\,p\,q + 2(p^2 + 658\,q^2).$$

Note 2

Apart from the linear transformations given by (2) ,one may also consider

$$x = 6a - 56b + 2c - 2\,,$$
$$y = 6a + 56b - 2c - 2\,,$$
$$z = 2c - 47b$$

which represent a different set of integer solutions to (1).

Further , one may represent (3) as the system of double equations as shown in Table 1 below :

Table 1- System of double equations

System	I	II	III	IV	V	VI	VII	VIII
$c + a$	b^2	$7b^2$	$47b^2$	$329b^2$	$658\,b$	$329\,b$	$94\,b$	$47\,b$
$c - a$	658	94	14	2	b	$2\,b$	$7\,b$	$14\,b$

Solving each of the above system of double equations ,the values of a, b, c are obtained. From (2) ,the respective integer solutions to (1) are found. For brevity, we present below the integer solutions to (1) :

Solutions from System I :

$$x = 6(2k^2 - 329) + 112k + 2(2k^2 + 329) - 2\,,$$
$$y = 6(2k^2 - 329) - 112k - 2(2k^2 + 329) - 2\,,$$
$$z = 2(2k^2 + 329) + 94k$$

Solutions from System II :

$$x = 6(14k^2 - 47) + 112k + 2(14k^2 + 47) - 2\,,$$
$$y = 6(14k^2 - 47) - 112k - 2(14k^2 + 47) - 2\,,$$
$$z = 2(14k^2 + 47) + 94k$$

Solutions from System III :

$$x = 6(94k^2 - 7) + 112k + 2(94k^2 + 7) - 2\,,$$
$$y = 6(94k^2 - 7) - 112k - 2(94k^2 + 7) - 2\,,$$
$$z = 2(94k^2 + 7) + 94k$$

Solutions from System IV :

$$x = 6(658k^2 - 1) + 112k + 2(658k^2 + 1) - 2\,,$$
$$y = 6(658k^2 - 1) - 112k - 2(658k^2 + 1) - 2\,,$$
$$z = 2(658k^2 + 1) + 94k$$

Solutions from System V :

$$x = 6(657k) + 112k + 2(659k) - 2,$$
$$y = 6(657k) - 112k - 2(659k) - 2,$$
$$z = 2(659k) + 94k$$

Solutions from System VI :

$$x = 6(327k) + 112k + 2(331k) - 2,$$
$$y = 6(327k) - 112k - 2(331k) - 2,$$
$$z = 2(331k) + 94k$$

Solutions from System VII :

$$x = 6(87k) + 112k + 2(101k) - 2,$$
$$y = 6(87k) - 112k - 2(101k) - 2,$$
$$z = 2(101k) + 94k$$

Solutions from System VIII :

$$x = 6(33k) + 112k + 2(61k) - 2,$$
$$y = 6(33k) - 112k - 2(61k) - 2,$$
$$z = 2(61k) + 94k$$

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