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MHD MIXED CONVECTION FLOW OF A VISCOUS FLUID THROUGH POROUS MEDIUM WITH PRESSURE STRESS WORK

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ABSTRACT

Magnetohydrodynamic mixed convection flow of a viscous fluid through porous medium is studied taking into account the viscous dissipation and pressure stress work of the fluid. Stream functions and similarity variables were introduced to transform the partial differential equations representing the flow into ordinary differential equations. The governing time-dependent boundary layer equations are solved analytically using perturbation expansion series technique. The velocity profile, temperature profile and the rate of heat transfer are computed with detailed discussion presented for various values of Grashof number (Gr), Eckert number (Ec), Prandlth number (Pr) and pressure stress work parameter. The numerical results revealed that increase in suction decreases the fluid temperature and increase in both suction and magnetic parameter result to increase in fluid velocity.

Keywords: Magnetohydromagnetic flow, viscous fluid, mixed convection, pressure stress work

1INTRODUCTION :

Without doubt, a large portion of the earth is in fluid state. It is therefore, imperative for researchers to stive to understand the concept of fluid. Much attention has been paid to the study of mixed convective flow along a rigid plate due to its vast applications, some of which are the cooling of nuclear reactors during an unexpected failure, solar cells to mention a few. Researchers like Hamza *et.al* (2019) investigated MHD mixed convection of a viscous dissipating fluid flow within a vertical cylinder and observed that slight increase in mixed convection increase the fluid velocity. Again, Hamza *et al* (2023) studied mixed convection flow of viscous reactive fluids which involves thermal diffusion and radial magnetic field in a vertical porous annulus and recently Samaila *et al* (2023) numerically studied the impacts of exponentially growing/decaying pressure gradient on mixed convection flow of viscous reactive fluid in a vertical tube and noted that raising the values of Frank–Kamenetskii (λ), mixed convection (Gre) as well as exponential growing pressure term increases the velocity fluid. The analysis of the problem of the motion of dusty viscous fuid through a circular pipe with insulation wall under the influence of a transverse magnetic field has been presented by Sastry and Seetharamaswami (1982), Sastry and Bhadram (1978). Gupta and Gupta (1985) also investigated convective flow of dusty viscous fluids. Sato (1961) studied Hall effect on steady hydromagnetic flow between two parallel plates. Pande and Hatzikonstantinou (1984) have investigated the problem of unsteady hydromagnetic thermal boundary layer flow of a strongly ionized gas. Acroyed (1974) was the first who treated the problem of natural convection along a heated vertical plate taking into account both the viscous dissipation and the pressure stress work. Also, some researchers had applied perturbation method to the problem of natural convection over a vertical isothermal plate taking into account both the viscous dissipation and the pressure stress work in the energy equation using two non-dimensional numbers. Seddeek (2000), studied the effect of variable viscosity on hydromagnetic flow and heat transfer past a continuously moving porous boundary with radiation. Youn (2000) studied unsteady MHD convection flow of polar fluid past a vertical moving porous plate in a porous medium. In this research, we investigated MHD mixed convection flow of a viscous fluid through porous medium with pressure stress work

2 MATHEMATICAL FORMULATION

Consider laminar mixed convection along a vertical porous plate in a calm environment along a vertical porous plate placed in calm environment with u and v respectively representing the velocity component in the x and y directions with x vertically upwards and y perpendicular to x. For steady two-dimensional flow, the boundary equations including viscous dissipation and pressure work are:

Continuity Equation

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum Equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2}+\beta g(T-T_\infty)-\left(\sigma\frac{B_0^2}{\rho}+\frac{\upsilon}{\kappa}\right)u \qquad (2)$$

Energy Equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\beta T}{\rho c_p}u\frac{dP}{dx} \qquad (3)$$

P is the pressure. Recall that the fluid pressure consists of the hydrostatic and motion pressure

Following the analysis of Pantokratoras (2003), the fluid motion pressure is small compared to hydrostatic pressure and hence can be ignored. Therefore, the hydrostatic pressure becomes

$$\frac{dP_h}{dx}=-\rho g \qquad (5)$$

Substituting equation (5) in equation (3), the energy equation becomes:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\upsilon}{c_p}\left(\frac{\partial u}{\partial y}\right)^2+\frac{\beta T}{c_p}ug \qquad (6)$$

∂y ∂x

Using equation (7) in equations (1), (2) and (6) result to

$$f''' + ff'' - (f')^2 + Nf + Gr\theta = 0 \qquad (8)$$

$$\theta'' + \Pr f\theta' - \Pr f'\theta + \Pr Ec(f'')^2 - \Pr Ps f' = 0 \qquad (9)$$

subject to

$$f(0)=0,\ f'(0)=1 \text{ and } \theta(0)=0,\ \theta(\infty)=0 \qquad (10)$$

where

$$N = M^2+\chi^2$$

$$M^2=\frac{\sigma B_0^2}{a\rho}$$

$$\chi^2=\frac{\upsilon}{aR}$$

$$Gr = g\beta\frac{(T_w-T_\infty)}{a^2}$$

$$E_c=\frac{xa^2}{c_p(T_w-T_\infty)}$$

$$Ps=\frac{g\beta T}{c_p(T_w-T_\infty)}$$

Results and Discussion

The governing boundary layer equations (8) and (9) subject to the boundary conditions (10) are solved analytically using perturbation series expansion technique and numerically solved with MATHEMATICA for both the velocity and temperature profiles. The numerically computed results and graphs for the temperature profile, velocity profile and the magnetic field are presented below:

Table 3.1: Temperature profile for $\mathbf{Gr = 1.0}$; $\mathbf{Pr = 0.71}$; $\mathbf{Ec = 0.10}$; $\mathbf{Ps = 0.30}$ and

$\chi = 0.20$ with different values of Magnetic field parameter

n	M=1.0	M=1.2	M=1.4
0.0	0.986126	0.994732	0.997424
0.2	0.640568	0.561773	0.489217
0.4	0.413953	0.316193	0.239278
0.6	0.265946	0.177034	0.116397
0.8	0.169281	0.098179	0.055983
1.0	0.106147	0.053494	0.026281
1.2	0.064913	0.028174	0.011678
1.4	0.037982	0.013826	0.004498
1.6	0.020394	0.005695	0.000969
1.8	0.008906	0.001088	-0.00077
2.0	0.001403	-0.00152	-0.00162

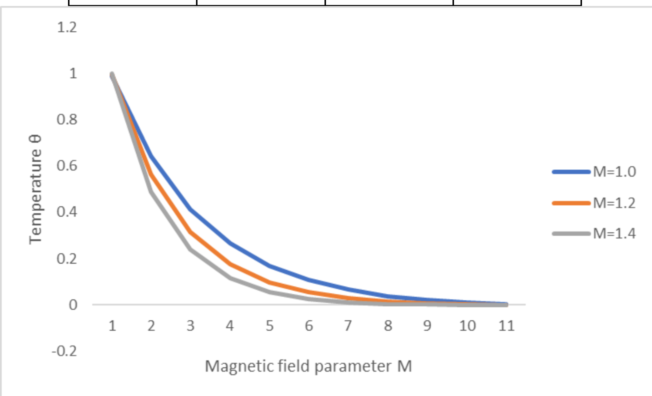

Figure 3.1: Temperature profile for $Gr = 1.0;\ Pr = 0.71;\ Ec = 0.10;\ Ps = 0.30$ and $\chi = 0.20$ with different values of Magnetic field parameter

↓**Table 3.2**: Temperature profile for $Gr = 1.0;\ Pr = 0.71;\ Ec = 0.10;\ Ps = 0.30$ and $\chi = 0.20$ with different values of suction parameter

n	**fw=1.3**	**fw=1.4**	**fw=1.5**
0.0	0.929765	0.986126	0.99561
0.2	0.68944	0.640568	0.562654
0.4	0.503325	0.413953	0.317058
0.6	0.363154	0.265946	0.17789
0.8	0.257637	0.169281	0.099029
1.0	0.178207	0.106147	0.054342
1.2	0.118415	0.064913	0.02902
1.4	0.073406	0.037982	0.014671
1.6	0.039524	0.020394	0.006539
1.8	0.014019	0.008906	0.001932
2.0	-0.00518	0.001403	-0.00068

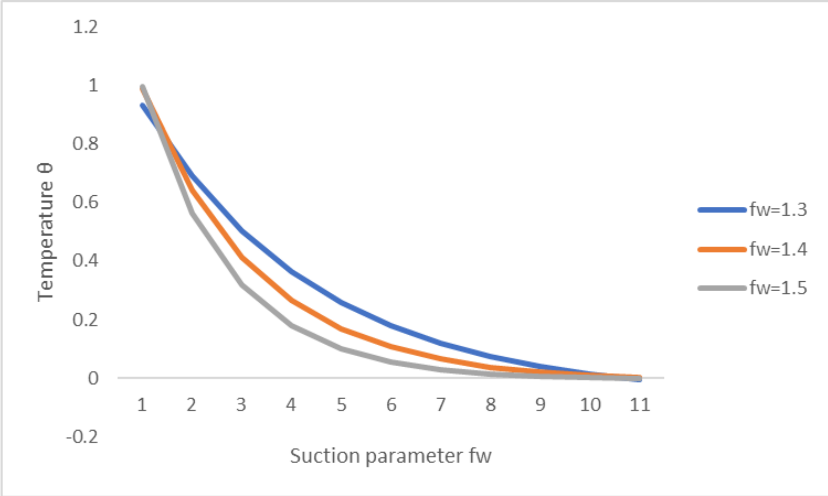

Figure 3.2: Temperature profile for Gr = 1.0; Pr = 0.71; Ec = 0.10; Ps = 0.30 and χ = 0.20 with different values of suction parameter

Figure 3.2 reveals that increase in suction parameter decreases the temperature of the fluid

Table 3.3: Temperature profile for Gr = 1.0; Pr = 0.71; Ec = 0.10; Ps = 0.30 and χ = 0.20 with different values of Magnetic field parameter

n	M=5.0	M=5.2	M=5.4
0.0	-0.01414	-0.05239	-0.03427
0.2	0.036519	0.031923	0.034613
0.4	0.041996	0.046826	0.045004
0.6	0.038013	0.044707	0.0419
0.8	0.03266	0.039073	0.036323
1.0	0.027684	0.033312	0.030879
1.2	0.023379	0.028188	0.026104
1.4	0.019724	0.023797	0.02203
1.6	0.016636	0.020075	0.018582
1.8	0.01403	0.016932	0.015672
2.0	0.011832	0.014279	0.013217

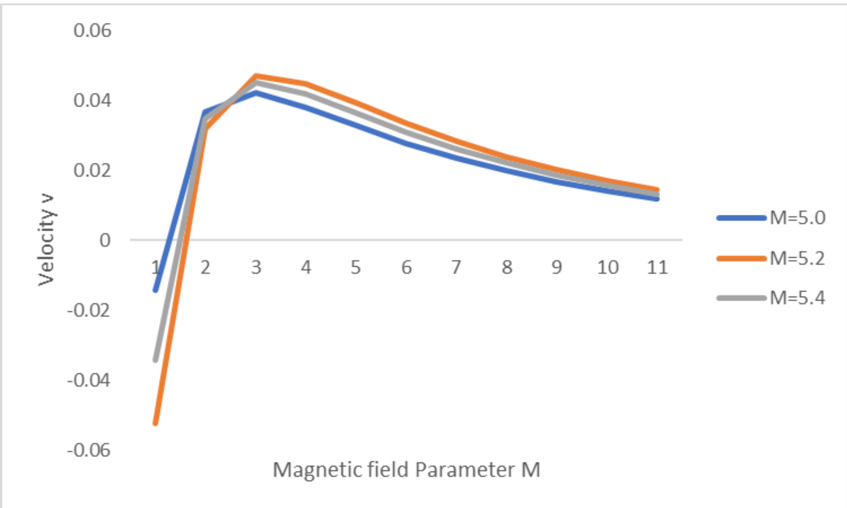

Figure 3.3: Velocity profile for Gr = 1.0; Pr = 0.71; Ec = 0.10; Ps = 0.30 and χ = 0.20 with different values of Magnetic field parameter

Figure 3.3 shows that increase in the magnetic field parameter decreases the velocity of the fluid

Table 3.4: Velocity profile for $\mathrm{Gr} = 1.0$; $\mathrm{Pr} = 0.71$; $\mathrm{Ec} = 0.10$; $\mathrm{Ps} = 0.30$ and $\chi = 0.20$ with different values of suction parameter

n	**fw=1.2**	**fw=1.4**	**fw=1.6**
0.0	-0.05239	0.003548	0.039851
0.2	0.031923	0.048835	0.056376
0.4	0.046826	0.04959	0.048965
0.6	0.044707	0.042647	0.039682
0.8	0.039073	0.035377	0.031729
1.0	0.033312	0.029087	0.025299
1.2	0.028188	0.023862	0.02016
1.4	0.023797	0.019564	0.016064
1.6	0.020075	0.016037	0.012799
1.8	0.016932	0.013146	0.010198
2.0	0.014279	0.010776	0.008125

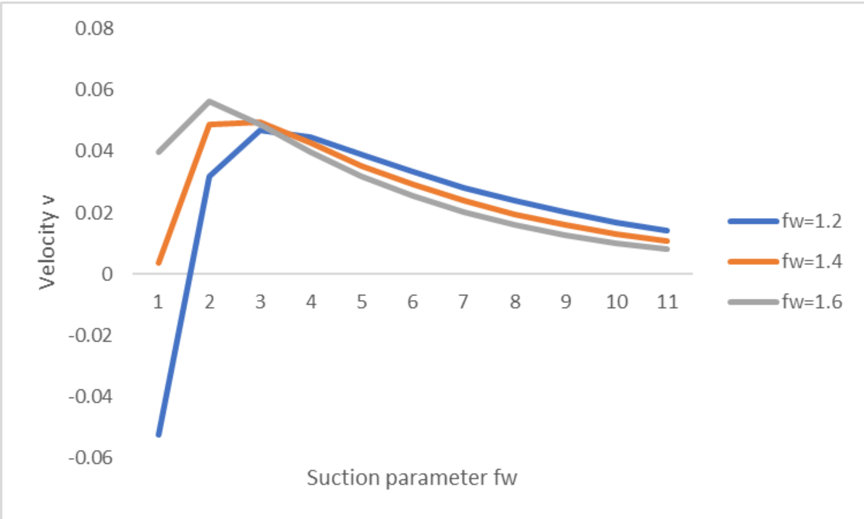

Figure 3.4: Velocity profile for $\mathrm{Gr} = 1.0$; $\mathrm{Pr} = 0.71$; $\mathrm{Ec} = 0.10$; $\mathrm{Ps} = 0.30$ and $\chi = 0.20$ with different values of suction parameter

Figure 3.4 shows that increase in the suction parameter increases the velocity of the fluid

Conclusion

MHD mixed convection flow of a viscous fluid through a porous medium with pressure stress work was considered. The continuity equation, momentum equation and energy equation were used to describe the fluid flow. Stream functions and similarity variables were introduced in order to transform the nonlinear partial differential equations into ordinary differential equations from which perturbation expansion series solution was applied to solve the problem.

The numerical result revealed that increase in magnetic parameter increases temperature. Also, increase in suction is inversely proportional to temperature. It also shows that the velocity fluctuates with increasing magnetic field which is similar to that of increase in suction

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