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Formulation of Rayleigh- Ritz Based Peculiar Total Potential Energy Functional (TPEF) For Asymmetric Multi - Cell (ASM) Thin- Walled Box Column (TWBC) Cross- Section.

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ABSTRACT

It is a well-known phenomenon that the carrying capacity of Thin -Walled Columns (TWC) is often governed by instability. Thus, the determination of the critical buckling load is an important exercise in the analysis of instability in Thin-Walled Structures (TWS) in general and in TWC in particular. This research work is therefore aimed at evaluating and formulating the peculiar Total Potential Energy Functional (TPEF) for Asymmetric Multi - cell (ASM)- Thin -walled Box Column (TWBC) cross sections based on Rayleigh - Ritz Method (RRM). This is to pave way to the eventual stability analysis of ASM cross section. By using Thin-walled assumptions, the cross sectional properties of ASM were first evaluated and the cross sectional area, A^{ASM} and Moments of Inertia, I^{ASM} obtained. Then , based on the formulated governing RRM based TPEF for the TWBC by Nwachukwu and others (2017) as well as other related works done by Nwachukwu and others (2021 a & b), the peculiar TPEF for ASM- TWBC cross sections was now formulated for different boundary conditions. The formulated RRM- based Energy Functional Equations support the stability analysis of an ASM thin-walled box (closed) column cross-section. The derived expressions will now be used to formulate series of stability matrices in subsequent ASM- TWBC research works where the critical buckling load will be determined.

KEYWORD: *Bulkling/ Stability Analysis, ASM, TPEF, Thin -Walled Box Column (TWBC) or Thin-Walled Column (TWC), Rayleigh- Ritz Method (RRM),*

1. INTRODUCTION

In general, a thin-walled structure (TWS), according to Murray (1984), is one which is made from thin plates joined along their edges. The plate thickness for the TWS however is small compared to other cross sectional dimensions which are in turn often small compared with the overall length of the member or structure. According to Sudhir and others (2014), TWS have a high load-carrying capacity, despite their small thickness. Thin-walled columns (TWC) as well as other TWS are presumed to be very light compared with alternative structures and thus, they are used extensively in long-span bridges and other structures where weight and cost are prime considerations. They are also widely used in the construction industry because of their light weight and economy, particularly for long span floors in industrial and public buildings and for storage structures for liquids and bulk materials, such as tanks, hoppers, silos, and coal houses. TWS are especially suitable for use in constructing Aircrafts, exhibition pavilions, concert halls, and sports arenas because the variety of available shapes permits great architectural expression, the covering of wide areas, and flexibility in the choice of construction materials, including steel, aluminum, reinforced concrete, and laminated plastic.

As a result of numerous applications of TWC/TWS and the resulted instability due their high carrying capacity, the study of stability becomes necessary. According to Srinath (2009), stability represents a fundamental problem in solid mechanics, which must be mastered to ensure the safety of structures against collapse. The theory of stability is of crucial importance for TWS applications to structural engineering, aerospace engineering, nuclear engineering, offshore, and ocean engineering. According to Bazant (2000), the theory of stability also plays an important role in certain problems of space structures, geotechnical structures, geophysics and material science. The continued importance and vitality of research on structural stability problem is due to technical and economic developments that demand the use of ever stronger and ever higher structures in an increasingly wider range of applications. According to Mohri and others (2008), such an expansion of use is made possible by developments in manufacturing, fabrication technology, computer-aided- design, economic competition and construction efficiency. These developments continually do not only change the way in which traditional structures are designed and built, but they also make possible the economic use of materials in other areas of application, such as offshore structures, transportation vehicles, and outer- space structures.

ASM are common examples of TWC cross –sections .According to Simao and Simoes da silva (2004), the use of very slender thin-walled cross-sections members have become increasingly in demand due to their high stiffness/weight ratio, in recent years. For about a century many branches of the industry have sought stronger and at the same time lighter structural solutions which optimize the effectiveness and the cost of the structures (Andreassen, 2012). Such industries cut across civil, offshore, mechanical, naval, and aerospace industries.

This present study is an attempt to evaluate and formulate the TPEF for ASM cross-sections. It is the follow up of the works by Nwachukwu and others (2017) and Nwachukwu and others (2021a) where the governing equation for the TPEF for a TWBC applicable to RRM and peculiar TPEF for DSS cross – section were derived respectively . Of recent, many researchers have carried out one form of analysis or the other on thin- walled box columns and related topics. For instant, Krolak and others (2009) presented a theoretical, numerical and experimental analysis of the stability and ultimate load of multi-cell thin-walled columns of rectangular and square cross-sections subjected to axial compression. Shanmugam and others (1989) presented a numerical method to investigate the ultimate strength behavior of thin-walled steel box columns subjected to axial loads and biaxial end moments. The work of Ezeh (2009) involved a theoretical formulation based on Vlasov's theory as modified by Varbanov, in analyzing flexural, flexural-torsional, and flexural-torsional-distortional buckling modes of thin-walled closed columns. Chidolue and Osadebe (2012), also used Vlasov's theory to carryout Torsional- Distortional analysis of thin- walled box girder bridges. Chidolue and Aginam (2012) investigated the effects of shape factor on the Flexural-Torsional-Distortional behavior of thin- walled box girder structures using Vlasov's Theory. Ezeh (2010) also investigated the buckling behavior of axially compressed multi- cell doubly symmetric thin- walled column using Vlasov's theory. The works of Osadebe and Chidolue (2012a), Osadebe and Chidolue (2012b), Osadebe and Ezeh (2009a), Osadebe and Ezeh (2009b) were also based on Vlasov's method. Nwachukwu and others (2017) and Nwachukwu and others (2021a) derived the RRM based governing TPEF equation for the TWBC applicable to RRM and evaluated and formulated the peculiar TPEF for DSS cross – section respectively . Nwachukwu and others (2021b) evaluated and formulated the TPEF for DSM and MSM cross section. Again, Nwachukwu and others (2022a) have evaluated and formulated the peculiar TPEF for MSS TWC cross section. Finally, Nwachukwu and others (2022b) have also evaluated and formulated the peculiar TPEF for ASS TWC cross section. Thus in the area of stability analysis of thin-walled box (closed) columns, little or no effort has been done to use RRM to evaluate and formulate the peculiar TPEF for ASM cross section. Henceforth, the need for this recent research work. The derived energy functional will now be used to analyze an ASM thin- walled box (closed) columns of different boundary conditions in subsequent works.

2. BACKGROUND INFORMATION ON ASM- RRM BASED TWBC STABILTY ANALYSIS

The major objective of this work is to formulate the individual TPEF for ASM TWBC cross section. Already, following the work of Nwachukwu and others (2017), the general TPEF based on RRM has been formulated as shown in Eqn.(1).

$$\pi = k_1 \int_L v^2\, x^2 (2L - x)^2 dx + \mathrm{k}_2 \int_L (v^I)^2\, dx + k_3 \int_L (v^{II})^2\, \mathrm{dx} - k_4 \int_L (v^I)^2\, \mathrm{dx}. \qquad \textbf{(1)}$$

In Eqn.(1), $k_1 = \frac{Ap^2}{8EI^2}$; $k_2 = \frac{\mathrm{AG}}{2}$; $k_3 = \frac{EI}{2}$; $and\ k_4 = \frac{P}{2}$ **2(a-d)**

Where P is critical buckling load, A is Cross sectional area, E is young modulus of elasticity, G is shear modulus, I is moment of inertia, and L is length of the column.

Also, in Eqn.(1), v = the displacement function, which is a function of polynomial shape function, ϕ

Now, according to Rayleigh- Ritz Theory: v = $\sum_i^n c_i\, \phi_i = c_1\phi_1 + c_2\phi_2 + c_3\phi_3 + \ldots\ldots + c_n\phi_n$ **(3)** From Eqn.(3), c = undetermined coefficient / unknown constant and ϕ = Polynomial shape function. Fortunately, the Polynomial Shape Function, ϕ has been generated by Nwachukwu and others (2021a) for the S-S, C-C and C-S boundary conditions. Again Nwachukwu and others (2021a) has used Eqn. (1) in combination with the generated polynomial shape function to formulate the peculiar/individual TPEF for Doubly Symmetric Single (DSS) cell TWBC for different boundary conditions. The same goes for Nwachukwu and others (2021b) and Nwachukwu and others (2022 a & b) for the formulation of peculiar TPEF for DSM/MSM and MSS and ASS cross- section respectively. For instance, considering the DSS cross-section, the RRM- based peculiar TPEF for Fixed-Fixed or Clamped-Clamped (C-C) boundary condition is as stated in Eqns.(4 and 5).

$$\pi_{DSS}^{C-C} = k_1{}^{DSS} \varphi_1{}^{C-C} + k_2{}^{DSS} \varphi_2{}^{C-C} + k_3{}^{DSS} \varphi_3{}^{C-C} - k_4{}^{DSS} \varphi_4{}^{C-C}. \qquad \textbf{(4)}$$

$$= k_1{}^{DSS} [360c_1{}^2 L^{12} - 1575c_1{}^2 L^{13} + 2870c_1{}^2 L^{14} - 2772c_1{}^2 L^{15} + \frac{16380c_1{}^2 L^{16}}{11}$$

$$- 420c_1{}^2 L^{17} + \frac{630c_1{}^2 L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\, L^{12}}{7} + 63c_1c_2\sqrt{53900}\, L^{13} - 162c_1c_2\sqrt{53900}\, L^{14} + \frac{2028c_1c_2\sqrt{53900}\, L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\, L^{16}}{11} + 87c_1c_2\sqrt{53900}\, L^{17} - 24c_1c_2\sqrt{53900}\, L^{18} + \frac{36c_1c_2\sqrt{53900}\, L^{19}}{14} + 3960c_2{}^2 L^{12} - 31185c_2{}^2 L^{13} + 105490c_2{}^2 L^{14} - 1995840c_2{}^2 L^{15} + 230580c_2{}^2 L^{16} - 166320c_2{}^2 L^{17} +$$

$$\frac{949410c_1{}^2 L^{18}}{13} - 17820c_2{}^2 L^{19} + 1848c_2{}^2 L^{20}]$$

$$+ k_2{}^{DSS} [840c_1{}^2 L^2 - 3780c_1{}^2 L^3 + 6552c_1{}^2 L^4 - 5040c_1{}^2 L^5 + 1440c_1{}^2 L^6$$

$$- 24c_1c_2\sqrt{53900} L^2 + 171c_1c_2\sqrt{53900} L^3 - 432c_1c_2\sqrt{53900} L^4 + 564c_1c_2\sqrt{53900} L^5 - 360c_1c_2\sqrt{53900} L^6 + 90c_1c_2\sqrt{53900} L^7 + 9240c_2{}^2 L^2 - 83160c_2{}^2 L^3 + 310464c_2{}^2 L^4 -$$

$$600600c_2{}^2 L^5 + 633600c_2{}^2 L^6 - 346500c_2{}^2 L^7 + 77000c_2{}^2 L^8]$$

$$+ k_3^{DSS} [\frac{2520c_1^2}{L^2} - \frac{8820c_1^2}{L} + 40320c_1^2 - 45360c_1^2L + 18144c_1^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L}$$

$$- 2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}L - 4320c_1c_2\sqrt{53900}L^2 +$$

$$1440c_1c_2\sqrt{53900}L^3 + \frac{27720c_2^2}{L^2} - \frac{332640c_2^2}{L} + 1884960c_2^2 - 5266800c_2^2L +$$

$$7650720c_2^2L^2 - 5544000c_2^2L^3 + 1584000c_2^2L^4]$$

$$-k_4^{DSS} [840c_1^2L^2 - 3780c_1^2L^3 + 6552c_1^2L^4 - 5040c_1^2L^5 + 1440c_1^2L^6$$

$$- 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240c_2^2L^2 - 83160c_2^2L^3 + 310464c_2^2L^4 -$$

$$600600c_2^2L^5 + 633600c_2^2L^6 - 346500c_2^2L^7 + 77000c_2^2L^8] \qquad \textbf{(5)}$$

Where k_1, k_2, k_3 and k_4 are all defined in Eqns.2 (a-d) respectively, but in terms of DSS cross-section. Thus, in the next section we will now examine the evaluation and derivation/ formulation of peculiar TPEF for ASM TWBC cross-section.

3. EVALUATION AND FORMULATION OF TPEF FOR ASM TWBC CROSS-SECTION

3.1. EVALUATION OF ASM CROSS- SECTIONAL PROPERTIES

Evaluation of the cross sectional properties here implies determining the Cross- Sectional Area for ASM, and its Moment of Inertia, I^{ASM}. An ASM thin walled box column cross section is shown in Fig.1.

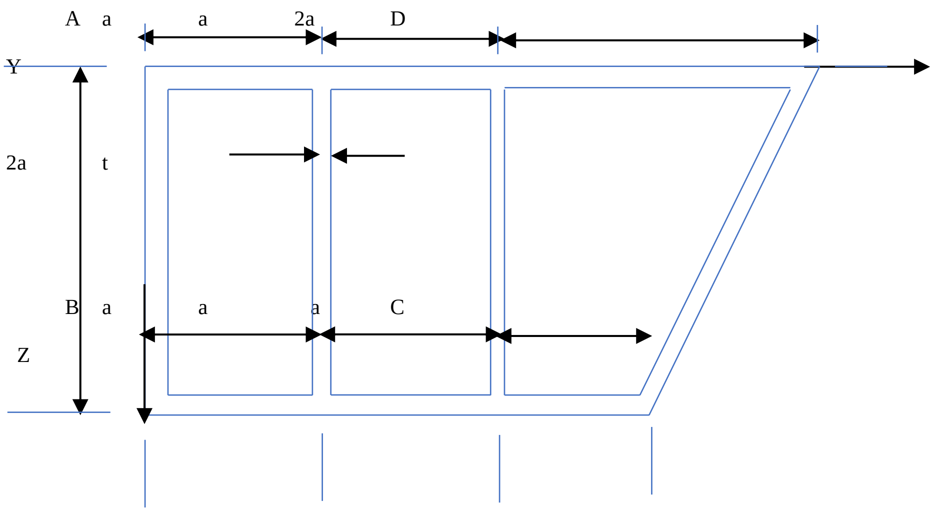

Fig.1: An ASM thin-walled box column cross section

(a).CROSS- SECTIONAL AREA, A^{ASM} AND CENTROID

Applying thin- walled assumptions, we have in Fig.2 as follows:

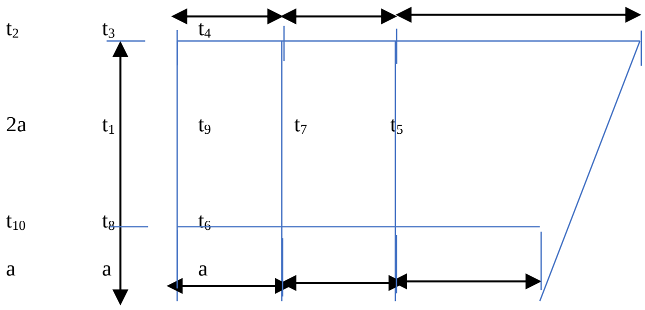

Fig.2 : Thin – Walled Assumption for ASM

From Figs.1 and 2,

$$CD = \sqrt{a^2 + (2a)^2} = \sqrt{5a^2} = a\sqrt{5} \qquad \textbf{(6)}$$

Then, the centroid, cross- sectional area, and moment of inertia are evaluated in Table 1.

Table 1: Centroid, Cross- Sectional Area and moment of inertia for ASM

Section	$\bar{y}_i$	$\bar{z}_i$	A_i	I_{Yci}	I_{Zci}
1	0	$-\frac{2a}{2}$	2at	$\frac{(2a)^3t}{12}$	$\frac{(t)^3 2a}{12}$
2	$\frac{a}{2}$	0	at	$\frac{(a)^3t}{12}$	$\frac{(t)^3 a}{12}$
3	$\frac{a}{2}$	0	at	$\frac{(a)^3t}{12}$	$\frac{(t)^3 a}{12}$
4	$\frac{2a}{2}$	0	2at	$\frac{(2a)^3t}{12}$	$\frac{(t)^3 2a}{12}$
5	0	$\frac{-a\sqrt{5}}{2}$	$at\sqrt{5}$	$\frac{(a\sqrt{5})^3t}{12}$	$\frac{(t)^3 a\sqrt{5}}{12}$
6	$\frac{a}{2}$	0	at	$\frac{(a)^3t}{12}$	$\frac{(t)^3 a}{12}$
7	0	$-\frac{2a}{2}$	2at	$\frac{(2a)^3t}{12}$	$\frac{(t)^3 2a}{12}$
8	$\frac{a}{2}$	0	at	$\frac{(a)^3t}{12}$	$\frac{(t)^3 a}{12}$
9	0	$-\frac{2a}{2}$	2at	$\frac{(2a)^3t}{12}$	$\frac{(t)^3 2a}{12}$
10	$\frac{a}{2}$	0	at	$\frac{(a)^3t}{12}$	$\frac{(t)^3 a}{12}$
SUM	$\sum \bar{y}_i = \frac{7a}{2}$	$\sum \bar{z}_i = -3a - \frac{a\sqrt{5}}{2}$	$\sum A_i = 13at + at\sqrt{5}$		

Thus, $\boldsymbol{A^{ASM}} = \sum A_i = 13at + at\sqrt{5} = 15.236at$ **(7)**

$\bar{y}_G = 3.5a$ and $\bar{z}_G = 4.12a$ **8 (a-b)**

(b). MOMENT OF INERTIA, I^{ASM}

Since there is no axis of symmetry, the moment of inertia will be based on I_{YZ}. Using thin – walled assumption,

$$I_{YZ} = 3I_{YZ1} + 5I_{YZ2} + I_{YZ4} + I_{YZ5} \quad \textbf{(9)}$$

Where $I_{YZ1} = I_{Yc1} * I_{Zc1} + A_1[-(\bar{Z}_G - \bar{z}_1)][-\bar{Y}_G]]$

$$= \frac{(2a)^3t}{12} * \frac{(t)^3 2a}{12} + 2at\ [\ -(4.12a - a)][\ -3.5a] = \frac{(2a)^3t}{12} * \frac{(t)^3 2a}{12} + 21.84a^3\ t \quad \textbf{(10)}$$

But by thin – walled assumption,

$\frac{(2a)^3t}{12} * \frac{(t)^3 2a}{12} \approx 0$ that is $I_{Yc1} * I_{Zc1} \approx 0$

Thus $3I_{YZ1} = 3 * 21.84a^3\ t = 65.52a^3$ **(11)**

Similarly,

$$I_{YZ2} = I_{Yc1} * I_{Zc1} + A_2[(\bar{Y}_2 - \bar{Y}_G)[\bar{Z}_G]] = at[4.12a][3.5a - 0.5a] = 12.36a^3\ t \quad \textbf{(12)}$$

Thus $5I_{YZ2} = 61.8a^3\ t$ **(13)**

Again , $I_{YZ4} = A_4[(\bar{Y}_G - \bar{Y}_4)][\bar{Z}_G] = 2at[4.12a][3.5a - a] = 20.6a^3\ t$ **(14)**

And finally, $I_{YZ5} = A_5[-(\bar{Z}_G - \bar{Z}_5)][\overline{-Y}_G] = a\,t\sqrt{5}\ [-(4.12a - \frac{a\sqrt{5}}{2})][-3.5a] = 23.49a^3\ t$ **(15)**

Thus, $\boldsymbol{I^{ASM}} = I_{YZ} = 171.41a^3\ t$ **(16)**

3.2. FORMULATION OF PECULIAR TPEF FOR ASM DIFFERENT BOUNDARY CONDITION CASES.

(a).CASE 1: PINNED-PINNED(S-S)- ASM- TWBC.

Following the works of Nwachukwu and others (2017), Nwachukwu and others (2021a) , Nwachukwu and others (2021b) and Nwachukwu and others (2022 a&b),the peculiar total potential energy functional (TPEF) for ASM-[S-S] thin-walled box column (TWBC), π_{ASM}^{S-S} can be formulated as follows:

$$\pi_{ASM}^{S-S} = {k_1}^{ASM}{\varphi_1}^{S-S} + {k_2}^{ASM}{\varphi_2}^{S-S} + {k_3}^{ASM}{\varphi_3}^{S-S} - {k_4}^{ASM}{\varphi_4}^{S-S} \quad \textbf{(17)}$$

$$= {k_1}^{ASM}\Big[24{c_1}^2L^{10} - 60{c_1}^2L^{11} + \frac{390{c_1}^2L^{12}}{7} - 10{c_1}^2L^{13} + \frac{10{c_1}^2L^{14}}{3} - \frac{8c_1c_2\sqrt{6300}\,L^{10}}{5} + \frac{20c_1c_2\sqrt{6300}\,L^{11}}{3} - \frac{74c_1c_2\sqrt{6300}\,L^{12}}{7} + 8c_1c_2\sqrt{6300}\,L^{13} - \frac{26c_1c_2\sqrt{6300}\,L^{14}}{9} + \frac{2c_1c_2\sqrt{6300}\,L^{15}}{5} + 168{c_2}^2L^{10} - 980{c_2}^2L^{11} + 2310{c_2}^2L^{12} - \frac{5565{c_2}^2L^{13}}{2} + \frac{5390{c_2}^2L^{14}}{3} - 588{c_2}^2L^{15} + \frac{840{c_2}^2L^{16}}{11}\Big]$$

$$+ {k_2}^{ASM}\big[30{c_1}^2 - 60{c_1}^2L + 40{c_1}^2L^2 - 2c_1c_2\sqrt{6300} + 8c_1c_2\sqrt{6300}\,L - 12c_1c_2\sqrt{6300}\,L^2 + 6c_1c_2\sqrt{6300}\,L^3 + 210{c_2}^2 - 1260{c_2}^2L + 3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\big]$$

$$+ {k_3}^{ASM}\Big[\frac{120{c_1}^2}{L^2} - \frac{24c_1c_2\sqrt{6300}}{L^2} + \frac{24c_1c_2\sqrt{6300}}{L} + \frac{7560{c_2}^2}{L^2} - \frac{15120{c_2}^2}{L} + 10080{c_2}^2\Big]$$

$$-{k_4}^{ASM}\big[30{c_1}^2 - 60{c_1}^2L + 40{c_1}^2L^2 - 2c_1c_2\sqrt{6300} + 8c_1c_2\sqrt{6300}\,L - 12c_1c_2\sqrt{6300}\,L^2 + 6c_1c_2\sqrt{6300}\,L^3 + 210{c_2}^2 - 1260{c_2}^2L + 3360{c_2}^2L^2 - 3780{c_2}^2L^3 + 1512{c_2}^2L^4\big] \quad \textbf{(18)}$$

Where , ${k_1}^{ASM} = \frac{A^{ASM}p^2}{8EI^{2(ASM)}}$, ${k_2}^{ASM} = \frac{A^{ASM}G}{2}$, ${k_3}^{ASM} = \frac{E\,I^{ASM}}{2}$ & ${k_4}^{ASM} = \frac{P}{2}$ **19(a-d)**

A^{ASM} and I^{ASM} are defined in Eqns.(7) and (16) respectively.

(b).CASE 2: FIXED-FIXED[C-C]- ASM- TWBC.

The peculiar TPEF for ASM-[C-C] TWBC can be obtained as follows:

$$\pi_{ASM}^{C-C} = {k_1}^{ASM}{\varphi_1}^{C-C} + {k_2}^{ASM}{\varphi_2}^{C-C} + {k_3}^{ASM}{\varphi_3}^{C-C} - {k_4}^{ASM}{\varphi_4}^{C-C} \quad \textbf{(20)}$$

$$= {k_1}^{ASM}\Big[360{c_1}^2L^{12} - 1575{c_1}^2L^{13} + 2870{c_1}^2L^{14} - 2772{c_1}^2L^{15} + \frac{16380{c_1}^2L^{16}}{11} - 420{c_1}^2L^{17} + \frac{630{c_1}^2L^{18}}{13} - \frac{72c_1c_2\sqrt{53900}\,L^{12}}{7} + 63c_1c_2\sqrt{53900}\,L^{13} - 162c_1c_2\sqrt{53900}\,L^{14} + \frac{2028c_1c_2\sqrt{53900}\,L^{15}}{10} - \frac{1836c_1c_2\sqrt{53900}\,L^{16}}{11} + 87c_1c_2\sqrt{53900}\,L^{17} - 24c_1c_2\sqrt{53900}\,L^{18} + \frac{36c_1c_2\sqrt{53900}\,L^{19}}{14} + 3960{c_2}^2L^{12} - 31185{c_2}^2L^{13} + 105490{c_2}^2L^{14} - 1995840{c_2}^2L^{15} + 230580{c_2}^2L^{16} - 166320{c_2}^2L^{17} + \frac{949410{c_1}^2L^{18}}{13} - 17820{c_2}^2L^{19} + 1848{c_2}^2L^{20}$$

$$+{k_2}^{ASM}\big[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8\big]$$

$$+ {k_3}^{ASM}\Big[\frac{2520{c_1}^2}{L^2} - \frac{8820{c_1}^2}{L} + 40320{c_1}^2 - 45360{c_1}^2L + 18144{c_1}^2L^2 - \frac{72c_1c_2\sqrt{53900}}{L^2} + \frac{648c_1c_2\sqrt{53900}}{L} - 2592c_1c_2\sqrt{53900} + 4896c_1c_2\sqrt{53900}\,\mathrm{L} - 4320c_1c_2\sqrt{53900}L^2 + 1440c_1c_2\sqrt{53900}L^3 + \frac{27720{c_2}^2}{L^2} - \frac{332640{c_2}^2}{L} + 1884960{c_2}^2 - 5266800{c_2}^2L + 7650720{c_2}^2L^2 - 5544000{c_2}^2L^3 + 1584000{c_2}^2L^4\Big]$$

$$-{k_4}^{ASM}\big[840{c_1}^2L^2 - 3780{c_1}^2L^3 + 6552{c_1}^2L^4 - 5040{c_1}^2L^5 + 1440{c_1}^2L^6 - 24c_1c_2\sqrt{53900}L^2 + 171c_1c_2\sqrt{53900}L^3 - 432c_1c_2\sqrt{53900}L^4 + 564c_1c_2\sqrt{53900}L^5 - 360c_1c_2\sqrt{53900}L^6 + 90c_1c_2\sqrt{53900}L^7 + 9240{c_2}^2L^2 - 83160{c_2}^2L^3 + 310464{c_2}^2L^4 - 600600{c_2}^2L^5 + 633600{c_2}^2L^6 - 346500{c_2}^2L^7 + 77000{c_2}^2L^8\big] \quad \textbf{(21)}$$

Where ${k_1}^{ASM}$, ${k_2}^{ASM}$, ${k_3}^{ASM}$ and ${k_4}^{ASM}$ are defined in Eqns. 19(a-d) respectively.

©.CASE 3: FIXED-PINNED[C-S]- ASM TWBC

The peculiar TPEF for ASM-[C-S] TWBC can be obtained as follows:

$$\pi_{ASM}^{C-S} = {k_1}^{ASM}{\varphi_1}^{C-S} + {k_2}^{ASM}{\varphi_2}^{C-S} + {k_3}^{ASM}{\varphi_3}^{C-S} - {k_4}^{ASM}{\varphi_4}^{C-S} \quad \textbf{(22)}$$

$$= k_1^{ASM} \left[\frac{22680c_1^2 L^{12}}{133} - \frac{98280c_1^2 L^{13}}{152} + \frac{174510c_1^2 L^{14}}{171} - \frac{162540c_1^2 L^{15}}{190} + \frac{83790c_1^2 L^{16}}{209} - \frac{22680c_1^2 L^{17}}{228} + \frac{2520c_1^2 L^{18}}{247} - \frac{216}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{12} + \frac{10728}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{13} - \frac{39078}{9}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{14} + \frac{58500}{10}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{15} - \frac{44640}{11}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{16}\right.$$

$$+ \frac{18000}{12}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{17} - \frac{3546}{13}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{18} + \frac{252}{14}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{19} + \frac{27720c_2^2 L^{12}}{1729}$$

$$\left. - \frac{2633400c_2^2 L^{13}}{1976} + \frac{66784410c_2^2 L^{14}}{2223} - \frac{203312340c_2^2 L^{15}}{2470} + \frac{250637310c_2^2 L^{16}}{2717} - \frac{151295760c_2^2 L^{17}}{2964} + \frac{45952830c_2^2 L^{18}}{3211} - \frac{6500340c_2^2 L^{19}}{3458} + \frac{339570c_2^2 L^{20}}{3705}\right]$$

$$+k_2^{ASM} \left[\frac{22680c_1^2 L^{2}}{57} - \frac{113400c_1^2 L^{3}}{76)} + \frac{202230c_1^2 L^{4}}{95} - \frac{151200c_1^2 L^{5}}{114} + \frac{40320c_1^2 L^{6}}{133} - \frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{2} + \frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{3} - \frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{4} + \frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{5} - \frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{6} + \frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{7}\right.$$

$$+ \frac{27720c_2^2 L^{2}}{741} - \frac{3908520c_2^2 L^{3}}{988} + \frac{143497970c_2^2 L^{4}}{1235} - \frac{415273320c_2^2 L^{5}}{1482}$$

$$\left. + \frac{379861020c_2^2 L^{6}}{1729} - \frac{102841200c_2^2 L^{7}}{1976} + \frac{8489250c_2^2 L^{8}}{2223}\right]$$

$$+ k_3^{ASM} \left[\frac{22680c_1^2}{19L^2} - \frac{226800c_1^2}{38L} + \frac{748440c_1^2}{57} - \frac{907200c_1^2 L}{76} + \frac{362880c_1^2 L^2}{95} - \frac{216}{L^2}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}} + \frac{31536}{2L}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}} - \frac{221832}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}} + \frac{480384}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L - \frac{350352}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^2 + \frac{60480}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^3\right.$$

$$+ \frac{24640c_2^2}{247L^2} - \frac{7817040c_2^2}{494L} + \frac{568731240c_2^2}{741} - \frac{2489699520c_2^2 L}{988}$$

$$\left. + \frac{3350350080c_2^2 L^2}{1235} - \frac{123094400c_2^2 L^3}{1482} + \frac{135828000c_2^2 L^4}{1729}\right]$$

$$-k_4^{ASM} \left[\frac{22680c_1^2 L^{2}}{57} - \frac{113400c_1^2 L^{3}}{76)} + \frac{202230c_1^2 L^{4}}{95} - \frac{151200c_1^2 L^{5}}{114} + \frac{40320c_1^2 L^{6}}{133} - \frac{216}{3}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{2} + \frac{15768}{4}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{3} - \frac{61254}{5}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{4} + \frac{81324}{6}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{5} - \frac{39978}{7}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{6} + \frac{5040}{8}c_1c_2\frac{\sqrt{53900}}{\sqrt{4693}}L^{7}\right.$$

$$+ \frac{27720c_2^2 L^{2}}{741} - \frac{3908520c_2^2 L^{3}}{988} + \frac{143497970c_2^2 L^{4}}{1235} - \frac{415273320c_2^2 L^{5}}{1482}$$

$$\left. + \frac{379861020c_2^2 L^{6}}{1729} - \frac{102841200c_2^2 L^{7}}{1976} + \frac{8489250c_2^2 L^{8}}{2223}\right] \quad \textbf{(23)}$$

Where k_1^{ASM}, k_2^{ASM}, k_3^{ASM} and k_4^{ASM} are defined in Eqns.19 (a-d) respectively.

4. CONCLUSIONS

So far in this study, the peculiar TPEF for ASM- TWBC, based on Rayleigh –Ritz Method (RRM) as a classical energy method for resolving structural stability problems was formulated and presented. Firstly, the study evaluated the cross sectional properties of ASM cross sections, which are moment of Inertia and cross sectional area, given in Eqns.(7) and (16). Secondly, the study formulated peculiar TPEF for ASM cross-section at different boundary conditions. The formulated Rayleigh- Ritz based ASM- TPEF given in Eqns.(18), (21) and (23) are found handy and convenient to be used in the bulking/stability analysis of ASM TWBC cross- section. . The Rayleigh- Ritz based formulated TPEF equations are also found suitable, handy and simple to be used in the Flexural (F), Flexural- Torsional (FT) and Flexural- Torsional- Distortional (FTD) buckling/stability analysis of ASM cell TWBC cross-section in subsequent papers where data obtained (critical bulking loads) after solving series of stability matrices will provide TWC designers with ideas of the load required to select suitable cross section profiles for the stability of TWC.

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