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Metamorphosis of the Simple Simple Simple Fixed Plate Material into Unstable State Under 3rd Order Energy Functional.

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ABSTRACT

The impact of buckling load on at structural plate element which is support on four edges is the aim of the research work. The north south direction is considered to be on simple and simple supported edge while the east west axis rests on also simple and fixed boundaries, forming a plate of SSSF shape arrangement. Odd order energy Functional was adopted in the research work. The simple simple simple fixed plate was considered as the direct independent plate, meaning that the material properties are uniform round about the shape of the element. These includes the flexural rigidity, poison ratio and young elastic modulus of elasticity. Considering the plate arrangement, the shape functions were first formulated, after which the various integral values of the differentiated shape functions, of the various boundary conditions were all generated. Upon the derivation of the stiffness coefficients of the various boundary cases, the Third order strain energy equation emerged and further expansion of Third order strain energy equation cumulated in the Third Order Overall Potential Energy Functional. The Lead equation was later gotten by differentiating the Third Order Overall Potential Energy Functional, with respect to the amplitude. Further minimization of the derived Lead equation gave rise to the formulation of the vital buckling load equations, together with its coefficients. After which was the formulation of the non-dimensional buckling load parameters and upon substitution of the aspect ratios a, ranging from 1 to 2 at the increase rate of 0.1, the real relationship was established for the final derived outputs.

Symbols	Words
O_p	Overall Potential Energy Functional,
$§_h$	Normal Stress
$ð_h$	Normal Strain
bk	Buckling load parameters
L_e	Lead Equation,
V_{bk}	Vital Buckling load
S-S-S-F	Simple Simple Simple Fixed Plate
E_w	External Work
Є	Strain energy

1. Introduction

Several researches have been conducted with the target of maximizing their high values for wider structural applications; this is a result of their high importance and wide applications in many engineering materials. Their relevance in Structural, Mechanical and Aeronautic Engineering, cannot be played down on. The impact of the use of these materials cannot be over look due to their high importance in our everyday life. In 1776, when Euler carried out a free vibration analysis of plate problems. Euler motivated Chladni (a German physicist) into study which led to the discovery of the various modes of free vibration of plates. This plate element can be considered as a structural element which is either straight or curved, and also having three dimensions length, width and thickness also referred to as the primary, secondary and tertiary dimensions respectively. The smallest of the three dimensions is the tertiary dimension. This sometimes is referred to as the plate thickness, usually very small when compared to the rest of the dimensions. The isotropic rectangular SSSF plate have all their material properties in all directions as the same and so they classified as direction independent element. In structure, stability analysis can also be referred to as the plate buckling. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries Prior to this time, other researchers have gotten solution using even order energy functional for Buckling of plate, so the resolution of the buckling tendency of SIMPLE SIMPLE SIMPLE FIXED isotropic plate using odd order energy functional is the addition this study will bring to literature of plate analysis.

Step 1: The Buckling Load Equation.

Some parameters forms the basis for the buckling load equation. These includes the external work, strain energy, normal strain and normal stress. Firstly, overall potential energy O_p was gotten by adding up the Strain energy, ϵ and External Work, E_w . This is shown in Equation 1i

$$E_w + \epsilon = O_p \qquad 1i$$

Upon the formulation of the overall potential energy, the strain energy, was derived by multiplying normal stress with the normal strain, both on the horizontal axis as shown in Equation 1ii.

$$\S_h ð_h = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + \mu\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2\right) \qquad 1ii$$

The vertical direction (Y axis) given as shown the Equation 1iii

$$\S_v ð_v = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fu}{\partial y^2}\right]^2 + \mu\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2\right) \qquad 1iii$$

Also the product of the in-plane shear stress and in-plane shear strain is stated in Equation 2i

$$\tau_{hv}\gamma_{hv} = 2\,\frac{Ez^2(1-\mu)}{(1-\mu^2)}\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 \qquad 2i$$

Upon the summation and further factorization of Equations 1ii, 1iii and 2i together gives

$$\S_h ð_h + \S_v ð_v + \tau_{hv}\gamma_{hv} = \frac{Ez^2}{1-\mu^2}\left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fu}{\partial y^2}\right]^2\right) \qquad 2ii$$

with the strain Energy is given as $\epsilon = \frac{1}{2}\iint_{xy} \bar{\epsilon}\, dxdy$ 2iii

where $\bar{\epsilon} = \frac{Ez^2}{1-\mu^2} \int \left(\left[\frac{\partial^2 fu}{\partial x^2}\right]^2 + 2\left[\frac{\partial^2 fu}{\partial x \partial y}\right]^2 + \left[\frac{\partial^2 fu}{\partial y^2}\right]^2\right)$ 3i

Further rearrangement of Equation 3i, gives the third order strain energy equation

as $\epsilon = \frac{G}{2}\int_0^n\int_0^m \left(\frac{\partial^3 fu}{\partial x^3}\cdot\frac{\partial fu}{\partial x} + 2\frac{\partial^3 fu}{\partial x \partial y^2}\cdot\frac{\partial fu}{\partial x} + \frac{\partial^3 fu}{\partial y^3}\cdot\frac{\partial fu}{\partial y}\right) dxdy$ 4

with the external load as $v = -\frac{bkl_x}{2}\int_0^n\int_0^m \left(\frac{\partial fu}{\partial x}\right)^2 dxdy$ 5

The third order total potential energy functional is expressed mathematically as

$$O_p = \frac{G}{2}\int\int \left(\frac{\partial^3 fu}{\partial x^3}\cdot\frac{\partial fu}{\partial x} + 2\frac{\partial^3 fu}{\partial x^2 \partial y}\cdot\frac{\partial fu}{\partial y} + \frac{\partial^3 fu}{\partial y^3}\cdot\frac{\partial fu}{\partial y}\right) dxdy - \frac{bkl_x}{2}\int\int \frac{\partial^2 fu}{\partial x^2} dxdy \qquad 6$$

Rearranging the total potential energy equation in terms of non-dimensional parameters I, J the buckling load equation is gotten as

$$bkl_{up} = \frac{G}{a^2}\int_0^1\int_0^1 \cdot\left(\left[\frac{\partial^3 fu}{\partial J^3}\right]\cdot\frac{\partial fu}{\partial J} + 2\frac{1}{p^2}\left[\frac{\partial^3 fu}{\partial J \partial I^2}\right]\cdot\frac{\partial fu}{\partial J} + \frac{1}{p^4}\left[\frac{\partial^3 fu}{\partial I^3}\right]\cdot\frac{\partial fu}{\partial I}\right) dJdI \qquad 7$$

$$bkl_{down} = \int_0^1\int_0^1 \left(\frac{\partial fu}{\partial J}\right)^2 dJdI \qquad 8$$

$$bkl_x = \frac{bkl_{up}}{bkl_{down}} \qquad 9$$

Step 2: Formulation of the Shape Function

For the derivation of the shape functions, the major support styles considered were Simple support and Fixed support system, Three out of the four supports were all simple supports with the remaining one as the fixed. For Simple support condition, the deflection equation *fw* and the 2nd order derivative of the deflection equation fw^2, were both equated to zero and these gave rise to the different simultaneous equations by considering I = 0 at the left hand support in the case of the x-axis and I = 1 at the right side of the same component. Also considering the top as J = 0 and J = 1 at the bottom support for the case of the vertical components. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions (n_1, m_1, n_2, m_2 n_3, m_3, n_4 andm_4) for the SSSF plate element. Where I and J are non-dimensional parameters parallel to the horizontal and vertical axis respectively as earlier explained.

1.3 Formulation of the Deflection Equation

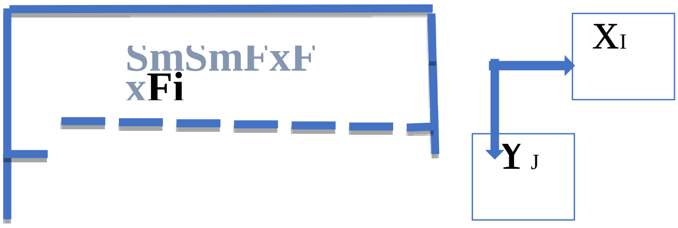


Figure i Isotropic Rectangular SmSmFxFx Plate

The case of horizontal Direction (X- X axis)

Figure ii Horizontal Support

Considering the X- X axis

But $fw_x = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4 + m_5I^5$ 1

The first derivation of Equation 1 gives

$fw_x^1 = m_1 + 2m_2I + 3m_3I^2 + 4m_4I^3 + 5m_5I^4$ 2

also the second derivative of the Equation 1 gives

$fw_x^2 = 2m_2 + 6m_3I + 12m_4I^2 + 20m_5I^3$ 3

and finally the third derivative the same Equation gives

$fw_x^3 = 6m_3 + 24m_4I + 60m_5I^2$ 4

1.3.1 Analysis of the Horizontal component

Introducing the boundary conditions on the horizontal component

At the left support, I = 0

When $fw_x = 0$

$fw_x = 0 = m_o + 0 + 0 + 0 + 0$ 5

$m_o = 0$

Also when $fw_x^2 = 0$ 6

$fw_x^2 = 0 = 2m_2 + 0 + 0 + 0$ 7

$2m_2 = 0$ 8

$m_2 = 0$ 9

at the right support, I =1

$fw_x^1 = m_1 + 0 + 3m_3 + 4m_4 + 5m_5 = -\frac{2m_5}{3}$ 10

Further simplifying Equation 10 gives

$m_1 = -3m_3 - 4m_4 - 5m_5 - \frac{2m_5}{3}$ 11

Also for the second derivative of the deflection on the X axis,

$fw_x^2 = 0 = 0 + 6m_3 + 12m_4 + 20m_5$ 12

rearranging the equation and making n_3 the subject gives

$m_3 = \frac{-12m_4 - 20m_5}{6}$ 13

in simpler form as

$m_3 = \frac{-10m_5}{3} - 2m_4$ 14

Solving for the third derivative of the deflection on the horizontal component gives

$fw_x^3 = 0 = 6m_3 + 24m_4 + 60m_5$ 15

That is

$fw_x^3 = 0 = m_3 + 4m_4 + 10m_5$ 16

$n_3 = \frac{-10m_5 - 4m_4}{1}$ 17

Resolving Equation 14 and 17 together gives

$\frac{-10m_5}{3} - 2m_4 = \frac{-60m_5 - 24m_4}{6}$ 18

and further simplifying gives

$m_4 = \frac{-10m_5}{3}$ 19

But substituting Equation 19 into Equation 17 gives

$m_3 = \frac{-10m_5 - 4(\frac{-10m_5}{3})}{1}$ 20

Further simplification gives

$m_3 = \frac{10m_5}{3}$ 21

Putting Equations 19 and 21 into Equation 11 gives

$m_1 = -\frac{2m_5}{3} - 3(\frac{10m_5}{3}) - 4(\frac{-10m_5}{3}) - 5m_5$ 22

and finally

$m_1 = -\frac{7m_5}{3}$ 23

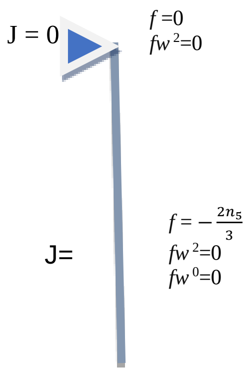
Recall that $fw_x^2 = m_o + m_1I + m_2 I^2 + m_3I^3 + m_4I^4 + m_5I^5$ 24

Putting the derived values into Equation 1 gives

$fw_x = n_5 (-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)$ 25

1.3.2 Analysis of the Vertical Component

The case of horizontal Direction (Y- Y axis)



Also introducing the boundary conditions on the vertical component

At the top support, I = 0

When $fw_y = 0$

$fw_y = 0 = n_o + 0 + 0 + 0 + 0$ 26

$m_o = 0$

Also when $fw_y^2 = 0$ 27

$fw_y^2 = 0 = 2n_2 + 0 + 0 + 0$ 28

$2n_2 = 0$ 29

$n_2 = 0$ 30

at the right support, J =1

$fw_y^1 = n_1 + 0 + 3n_3 + 4n_4 + 5n_5 = -\frac{2n_5}{3}$ 31

Further simplifying Equation 31 gives

$n_1 = -3n_3 - 4n_4 - 5n_5 - \frac{2n_5}{3}$ 32

Also for the second derivative of the deflection on the Y axis,

$fw_y^2 = 0 = 0 + 6n_3 + 12n_4 + 20n_5$ 33

Rearranging the equation and making n_3 the subject gives

$n_3 = \frac{-12n_4 - 20n_5}{6}$ 34

in simpler form as

$n_3 = \frac{-10n_5}{3} - 2n_4$ 35

Solving for the third derivative of the deflection on the vertical component gives

$fw_y^3 = 0 = 6n_3 + 24n_4 + 60n_5$ 36

That is

$fw_y^3 = 0 = n_3 + 4n_4 + 10n_5$ 37

$n_3 = \frac{-10n_5 - 4n_4}{1}$ 38

Resolving Equation 35 and 38 together gives

$\frac{-10n_5}{3} - 2n_4 = \frac{-60n_5 - 24n_4}{6}$ 39

and further simplifying gives

$n_4 = \frac{-10n_5}{3}$ 40

But substituting Equation 19 into Equation 17 gives

$n_3 = \frac{-10n_5 - 4(\frac{-10n_5}{3})}{1}$ 41

Further simplification gives

$n_3 = \frac{10n_5}{3}$ 42

Putting Equations 19 and 21 into Equation 11 gives

$n_1 = -\frac{2n_5}{3} - 3(\frac{10n_5}{3}) - 4(\frac{-10n_5}{3}) - 5n_5$ 43

and finally

$n_1 = -\frac{7n_5}{3}$ 44

Recall that $fw_y^2 = n_o + n_1J + n_2J^2 + n_3J^3 + n_4J^4 + n_5J^5$ 45

Putting the derived values into Equation 1 gives

$fw_y = n_5\,(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 46

That means

$fw = fw_x * fw_y = m_4\,(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5) * n_5(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 47

Factorizing further gives the Amplitude and the shape function

$= m_4n_5\,(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 48

The shape function *f* is give as $(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5)(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5)$ 49

1.4 Derivation of The Stiffness Coefficients

Equation 49 is further differentiated at different stages, from where the stiffness coefficients were derived. These includes

$$\frac{\partial f}{\partial I} = \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) \quad 50$$

$$\frac{\partial^2 f}{\partial I\,\partial J} = \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right) \quad 52$$

$$\frac{\partial f}{\partial I\,\partial J^2} = \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3\right) \quad 53$$

$$\frac{\partial^2 k}{\partial I^2} = \left(\frac{60I^1}{3} - \frac{120I^2}{3} + 20I^3\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) \quad 54$$

$$\frac{\partial^3 f}{\partial I^3} = \left(\frac{60}{3} - \frac{240I}{3} + 60I^2\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) \quad 55$$

also

$$\frac{\partial f}{\partial J} = \left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right) \quad 56$$

$$\frac{\partial^2 f}{\partial J^2} = \left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3\right) \quad 57$$

$$\frac{\partial^3 f}{\partial J^3} = \left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(\frac{60}{3} - \frac{240J}{3} + 60J^2\right) \quad 58$$

Integrating the product of the Equation 55 by 50 gives the first stiffness coefficient.

That is

$$k_{ssff1} = \int_0^1\int_0^1 \frac{\partial^3 f}{\partial I^3} * \frac{\partial f}{\partial I}\, dIdJ \quad 59$$

$$k_{ssff1} = \int_0^1\int_0^1 \left[\left(\frac{60}{3} - \frac{240I}{3} + 60I^2\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) * \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ \quad 60$$

bringing the like terms together gives

$$= \int_0^1\int_0^1 \left[\left(\frac{60}{3} - \frac{240I}{3} + 60I^2\right)\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) * \left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ \quad 60a$$

multiplying them gives

$$= \int_0^1\int_0^1 \left[\left(\frac{60}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) - \frac{240I}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + 60I^2\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\right) * \left(-\frac{7J}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) + \frac{10J^3}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) - \frac{10J^4}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) + J^5\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right)\right] dIdJ \quad 60b$$

further minimization yields

$k_{ssff1} = 0.65455$

also integrating the product Equation 53 by 51 give the second stiffness coefficient.

That is

$$k_{ssff2} = \int_0^1\int_0^1 \frac{\partial^3 f}{\partial I\,\partial J^2} * \frac{\partial f}{\partial I}\, dIdJ \quad 61$$

$$k_{ssff2} = \int_0^1\int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3\right) * \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ \quad 62$$

Bring the like terms together gives

$$\int_0^1\int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) * \left(\frac{60J^1}{3} - \frac{120J^2}{3} + 20J^3\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ \quad 62a$$

Multiplying the like terms gives

$$= \int_0^1\int_0^1 \left[\left(-\frac{7}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + \frac{30I^2}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) - \frac{40I^3}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + 5I^4\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\right) * \left(\frac{60J^1}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) - \frac{120J^2}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) + 20J^3\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right)\right] dIdJ \quad 62b$$

$k_{ssff2} = 0.04043$

Furthermore integrating the product Equation 58 by 56 give the third stiffness coefficient. That is

$k_{ssff3} = \int_0^1 \int_0^1 \frac{\partial^3 f}{\partial J^3} * \frac{\partial f}{\partial J} dIdJ$ 63

$k_{ssff3} = \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(\frac{60}{3} - \frac{240J}{3} + 60J^2\right) * \left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right)\right] dIdJ$ 64

$k_{ssff3} = \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) * \left(\frac{60}{3} - \frac{240J}{3} + 60J^2\right)\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right)\right] dIdJ$

$$= \int_0^1 \int_0^1 \left[\left(-\frac{7I}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) + \frac{10I^3}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) - \frac{10I^4}{3}\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right) + I^5\left(-\frac{7I}{3} + \frac{10I^3}{3} - \frac{10I^4}{3} + I^5\right)\right) * \left(\frac{60}{3}\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right) - \frac{240J}{3}\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right) + 60J^2\left(-\frac{7}{3} + \frac{30J^2}{3} - \frac{40J^3}{3} + 5J^4\right)\right)\right] dIdJ$$

$k_{ssff3} = 0.006047$

And finally integrating the product Equation 51 by 51 give the sixth stiffness coefficient.

That is

$k_{ssff\,6} = \int_0^1 \int_0^1 \left(\frac{\partial f}{\partial I} * \frac{\partial f}{\partial I}\right) dIdJ$ 63

$k_{ssff6} = \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) * \left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ$ 64

Collecting the like terms together gives

$= \int_0^1 \int_0^1 \left[\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) * \left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right] dIdJ$ 65

Opening the brackets gives

$= [[\left(-\frac{7}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + \frac{30I^2}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) - \frac{40I^3}{3}\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right) + 5I^4\left(-\frac{7}{3} + \frac{30I^2}{3} - \frac{40I^3}{3} + 5I^4\right)\right) * \left(-\frac{7J}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) + \frac{10J^3}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) - \frac{10J^4}{3}\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right) + J^5\left(-\frac{7J}{3} + \frac{10J^3}{3} - \frac{10J^4}{3} + J^5\right)\right)]^1]^1$ 65b

Putting the upper and lower limit values gives

$k_{ssff\,6} = 0.0159444$

Reducing Equation xiii in terms of the stiffness coefficients gives

$bkl_x = \frac{D(kssff_1 + 2\frac{1}{p^2}kssff_2 + \frac{1}{p^4}kssff_3)}{kssff_6 m^2}$ 65

Substituting the real values in to Equation 65 gives

$bkl_x = \frac{D(0.67096 + 2\frac{1}{p^2}(0.04043) + \frac{1}{p^4}(0.006047))}{(0.0159444)m^2}$ 66

RESULTS AND DISCUSSION.

The results for the stiffness coefficients and the critical buckling load coefficients were derived. The critical buckling load coefficients were considered at different aspect ratios. The first table represents the values of the stiffness coefficients while the other contains the critical buckling coefficients for the aspect ratio of m/n, both for the previous and present study. The values of the aspect Ratios ranges from 2.0 to 1.0 with arithmetic increase of 0.1. From the values generated in the tables, it was observed that as the aspect ratio increases from 1.0 to 2.0, the critical buckling load decreases. This occurred both in the present and previous results.

Table 1.1 Stiffness Coefficients from Previous researchers

Stiffness coefficients, sc	Derived values
k_{ssff1}	0.67096
k_{ssff2}	0.04043
k_{ssff3}	0.006047
k_{ssff6}	0.0159444

Table 1.2 Stiffness Coefficients from Present Work

Stiffness coefficients, k	Derived values
k_{ssff1}	0.65455
k_{ssff2}	0.0400137
k_{ssff3}	0.0059651
k_{ssff6}	0.0153545

Table 1.3 Critical buckling load values for CSCF Plate from Previous/Present.

m/n		2	1.9	1.8	1.7	1.6
B		43.9346	44.0759	44.2415	44.4373	44.6711
B_x	Previous	43.3728	43.5151	43.6826	43.8814	44.1201
	Present	43.9346	44.0759	44.2415	44.4373	44.6711

Table 1.3 cont'd.

m/n		1.5	1.4	1.3	1.2	1.1	1
B		44.9533	45.2985	45.7268	46.2674	46.9632	47.88
$B_{x}\frac{G}{n^2}$	Previous	44.4101	44.7674	45.2148	45.7859	46.5315	47.5319
	Present	44.9533	45.2985	45.7268	46.2674	46.9632	47.88

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