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Motion or a Conservative System Near Equilibrium

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Abstract:

In the preceding chapter the question as to whether a slightly disturbed conservative system remains near or departs from equilibrium was resolved but there was no attempt to discuss the actual form of the motion.

We shall now examine the dynamical behaviour near equilibrium more closely. Our considerations will be limited to conservative systems with one degree of freedom.[7] the equation of motion for a conservative system with one degree of freedom can be

written

$$M\ddot{q} + \frac{dM}{dq}q + \frac{dV}{dq} = 0(1)$$

where$M$ and$V$ are functions of q only. Let$q = \alpha$ be a position of equilibrium. Put

$q = \alpha + x$

so that

$\dot{q} = \dot{x}$ and$\ddot{q} = \ddot{x}$

At present we are interested only in motion near equilibrium so that it is plausible to assume that$x$,$\dot{x}$ and $\dot{x}$are all small quantities of the same order [7].

(Attempts to prove this rigorously are by no means straightforward.)

This suggests that we should expand in terms of the small quantities and reject all but the most important[7].

For example

$M(\alpha + x) = M(\alpha) + a$

quantity of the first order and therefore

$M\ddot{q} = M(\alpha)\ddot{x} + a$

quantity of the second order. Similarly,

$$\frac{dM}{dq}\dot{q}^2 = a$$

quantity of the second order. Also, by Taylor's theorem,

$$\left(\frac{dV}{dq}\right)_q = \alpha + x = \left(\frac{dV}{dq}\right)_q = \alpha + x\left(\frac{d^2V}{dq^2}\right)_q = \alpha$$

second order term. the term

$$\left(\frac{dV}{dq}\right)_q = \alpha$$

vanishes becauseq = α is a position of equilibrium.

Hence, when we substitute our expansions in (1), we obtain

$$M(\alpha)\ddot{x} + (\frac{d^2V}{d\alpha^2})_q = \alpha x + second\ order\ term = 0(2)$$

$$(\alpha) and (\frac{d^2V}{d\alpha^2})_q = \alpha^M$$

are constant.

0.1.1 Stability

A dynamical system in equilibrium will rarely be free from disturbances of one kind or another, e.g. that produced by a sudden gust of wind. The question therefore arises as to whether a system will remain near equilibrium when a slight impulse or slight displacement isapplied.

If, when the system is given a small kinetic energy and a small displacement from equilibrium, the subsequent kinetic energy and displacement are small the equilibrium position is called stable. Otherwise it is unstable.

Let the potential energy at the position of equilibrium be V_0.

Suppose that the system is started with kinetic energyδT_0 from a position where the potential energy is $V_0 + \delta V_0$ Then, in the subsequent motion,

$T + V = \delta T_0 + V_0 + \delta V_0$

or

$T + v = \delta T_0 + \delta V_0$

(13)

on putting$V = V_0 + v$

Now let$V$ always increase as the departure from equilibrium increases, i.e.$v \geq 0$. Then, since$T \geq 0$,(13) shows that $0 \leq v \leq \delta T_0 + \delta V_0$.

Thus v is always a small positive quantity and the system never moves very far from equilibrium.

Furthermore, since$v \geq$, (13) gives

$o \leq T \leq \delta T_0 + \delta V_0$

which demonstrates that the kinetic energy remains small. The position of equilibrium is therefore stable.

It may happen, of course, that$v > 0$ for$q > 0$ and $v \leq 0$ for $q < 0$. In that case

consideration of the side $q < 0$ alone indicates that the position is unstable. Hence a necessary and sufficient condition for a position of equilibrium to be stable is that the potential energy increases in displacements from equilibrium.

Since we are concerned only with the immediate neighbourhood of a position of equilib- rium we can ensure that the potential energy increases by requiring it to be a minimum at equilibrium.

thus

$$(\frac{d^2V}{dq2})_q = \alpha > 0 \qquad (0.4.27)$$

is sufficient to make a position of equilibrium stable.

e.i If

$$(\frac{d^2V}{dq2})_q = \alpha = 0 \qquad (0.4.28)$$

$$(\frac{d^4V}{dq4})_q = \alpha > 0 \qquad (0.4.29)$$

the position may still be stable, e.g. if and

d^3V

$(dq3)_q = \alpha = 0$ (0.4.30)

d^4V

$(dq4)_q = \alpha > 0$ (0.4.31)

The rule can be remembered easily by drawing the curve of V against q and imagining it to be a fine wire.

If a small bead is placed on the wire it will remain at rest in the position of equilibrium. If the bead is slightly disturbed it will not move very far away in the case of Fig. 1.4 where the potential energy has a minimum so that the position is stable.

In the cases of Fig. 1.5t howevert the bead will obviously slide down the lower portions and the positions are unstable.

It is important to emphasize that we have discussed stability only with reference to an initial small disturbance.

If impulses were arriving all the time they might be so synchronized that a large dis- placement was produced from an accumulation of small ones.

Moreover, although small displacements cannot occur near an unstable position, there is nothing to prevent a motion which is not infinitesimalt e.g. an oscillation from A to B which passes through 2 stable and 1 unstable position of equilibrium.

The system:

$X = f(x, y, t)$

$y = g(x, y, t)$

x and y are dependent variables.

t is independent variable. initial condition:

$x(t_0) = x_0$ $y(t_0) = y_0$

0.1.2 First Order Systems

$x = ax + by + r_1(t)$ $y = cx + dy + r_2(t)$

a,b,c and d are constant. this is linear system.

Result and discussion

We can solve second-order, linear, homogeneous differential equations with constant coefficients by finding the roots of the associated characteristic equation

. The form of the general solution varies, depending on whether the

characteristic equation has distinct, real roots; a single, repeated real root; or complex conjugate roots. The three cases are summarized

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Not applicable.

\section{Ethical approval }

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