International Journal of Research Publication and Reviews, Vol 5, no 3, pp 1881-1886 March 2024

International Journal of Research Publication and Reviews

Journal homepage: www.ijrpr.com ISSN 2582-7421

Visualization: Qubit, θ Degree and Certainty Amplitude with Geometrical Approach.

Santosh Ramesh Jadhav.

HOD, Prototyping, Advandes Design and Engineering Services, Pune, Maharashtra State, India.

A B S T R A C T

The fundamental building block of classical computational devices is a two-state system representing zeros and ones. In quantum computing, the quantum bit (qubit) can exist not only as zero or one but can also simultaneously occupy multiple positions, leading to a theoretical superposition condition due to the wave-like characteristics inherent in subatomic particles. However, upon measurement, it inevitably collapses into either the 0 or 1 state, disrupting the superposition. (1) Here, we propose a hypothesis suggesting the potential determination of 'certainty amplitude', representing two Latitude Lines or two angular momentum states present on a unit radius sphere, denoted as cα and cβ, consistently influence the likelihood of a qubit inevitably collapsing into either the 0 or 1 state.

Within the framework of a unit-radius sphere, a point's position on the sphere's surface is conventionally expressed as $\sqrt{(x^2 + y^2 + z^2)} = 1$. By distributing multiple points (specifically six points) on a unit-radius sphere, this paper also aims to illustrate a scientific concept: that qubits occupy multiple states during a wave function. In both the upper and lower hemispheres, a total of two latitude lines are defined by three points for each latitude lines in the three-dimensional coordinate system. Any given point from these points will always be lie on one of the two latitude lines only, thereby illustrating the scientific principle that, after measurement, qubits collapse into either the 0 or 1 state exclusively. This results in the equality of the sum of squared values for each corresponding coordinate: $(x_1^2 + x_2^2 + x_3^2) = (y_1^2 + y_2^2 + y_3^2) = (z_1^2 + z_2^2 + z_3^2) = 1$. This equation remains valid even when adjusting the relative values of x and y coordinates, particularly in relation to the z-axis. Furthermore, we have projected angular momentum vectors out of those two angular momentum states onto the complex plane in rectangular form, which can serve as a representation of Certainty Amplitude, where $c\alpha^2 + c\beta^2 = 1$. (1) (2) (3) (4)

Keywords: Certainty Amplitude, Qubit superposition, Probability Amplitude, Quantum Computing, Qubit Representation, Qubit Visualization

1. Current approach to represent qubit state.

Generally, qubit states are expressed as $\alpha + i\beta = 1$, with α and β as probability amplitudes satisfying $|\alpha|^2 + |\beta|^2 = 1$. In the complex plane with a unit radius, the real part (α) aligns with the x-axis, and the imaginary part (β) aligns with the y-axis, perpendicular to each other.

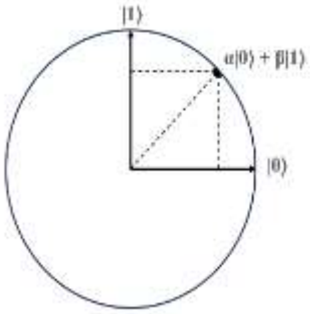

Figure 1: Qubit visualisation on Complex plane

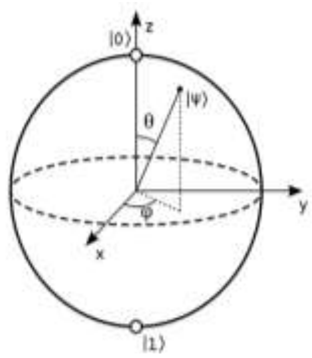

Figure 2: Bloch Sphere

In the Bloch sphere representation, the surface of a sphere with a unit radius corresponds to the set of all possible states of a qubit. The two angles, denoted as polar angle θ and the azimuth angle φ, are used to parameterize the qubit's state on the sphere. (1)

2. Navigation and Terminology

In this exploration, we delve into the concept that a qubit can manifest in multiple states during the wave function. Throughout the text, we employ interchangeable terms such as 'point,' 'position,' or ‘vertex corners’ to signify the various states occupy by the qubit. While these terms are contextually appropriate, it's important to note that when these specifically mentioned, it indicates that the qubit’s Z-axis is aligned to that particular point, position or vertex corner respectively. This alignment originates from the center (0,0,0), in the context of Bloch Sphere. (1)

3. Logic.

1) Distribution:

In a configuration where a qubit can manifest in multiple states during the wave function, each state from a set of determined states should be evenly spread across the surface of a unit-radius sphere. To ensure equal distribution, the relative angular distance (radian) between any two adjacent states should be equal. (4)

2) Total latitude line in accounting:

In the Bloch sphere representation, states 0 and 1 are evenly distributed on opposite sides of the sphere's surface along the z-axis, precisely represented by θ angles of 0° and 180. (1) Although a qubit may occupy multiple states, it ultimately collapses into either 0 or 1 after measurement. To mathematically represent this as angular momentum states, a numerical value for the representative latitude line for states 0 and 1 should be two.

3) Total vortex corners in accounting

In our hypothesis, we define two latitude lines (θ degrees) on a unit-radius sphere. These lines determine the angular momentum state, influencing whether the qubit is measured as state 0 or 1. Following the constraints of three-dimensional coordinate systems, a minimum of three points is needed in both hemispheres to define a section on the sphere that represents the states 0 and 1 as angular momentum states. This is because at least three points are necessary to accurately define a circle. As a result, six points are assigned, geometrically aligned on the surface of a unit-radius sphere.

4) Geometrical representation of occupying multiple state by qubit

In our methodology, we have chosen the Octahedron geometry and aligned it with a unit-radius sphere. This alignment positions the octahedron's six vertex corners on the sphere, allowing us to represent the two latitude lines or Angular Momentum States as representative for qubit states 0 and 1. This representation occurs when the sum of points presents on those two latitudes.

4. Visualizing geometry: representing two latitude lines as state |0⟩ and |1⟩ on sphere.

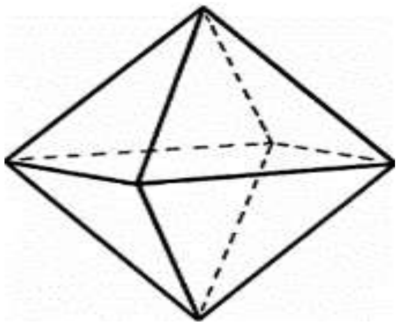

Figure 3: Octahedron

1) Uniqueness in geometry

In geometry, the term 'octahedron' typically refers to the regular octahedron, consisting of eight triangular faces, twelve straight edges, and six vertex corners. This geometric shape holds significance in the context discussed in the topic '3. Logic’, particularly when the octahedron has equal edge lengths. The following key geometric properties are discussed in this context:

a. Equal Distance from Vortex Corner to Origin:

The distance from any cortex corner to its origin is equal

b. Equal Distance between Adjacent Vertex Corners:

The distance between any two adjacent vertex corners of the octahedron is found to be equal. This allows for the equal distribution of all six vertex corners on the surface of a unit-radius sphere.

c. Distribution on Latitude Lines:

Although there are six vertex corners, geometrically they can be distributed on two latitude lines that are equally distanced from a unit-radius sphere's origin, and those two latitude lines are parallel to the x-y plane.

d. Opposing Vortex Corners and comparison with Tetrahedron Geometry:

With specific alignment in a three-dimensional coordinate system, the geometry of a tetrahedron also presents the same two latitude lines as the octahedron geometry. However, the focus often leans towards octahedron geometry, especially when all six vertex corners are equally distributed on the surface of a unit-radius sphere. In this distribution, a vertex corner consistently appears exactly opposite the center of the sphere along those two latitude lines. This alignment can be verified using a three-dimensional computer-aided model. Such a configuration bears resemblance to the pure states $|0\rangle$ and $|1\rangle$, positioned precisely opposite each other, within the context of the Bloch sphere.

e. Summary:

The octahedron's geometric properties, when applied to visualizing two angular moment states representative of the states $|0\rangle$ and $|1\rangle$, offer insights into equal distribution and opposing vortex corners. The specific angular intervals in octahedron geometry contribute to the understanding of occupying multiple states at the same time by aligning with the z-axis of the Bloch Sphere.

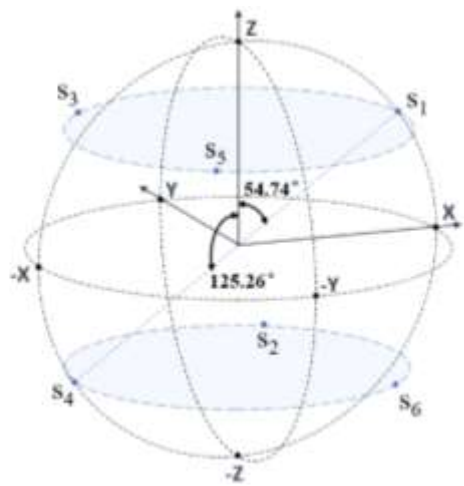

Figure 4: Projection of Octahedron's vortex corners on a unit radius sphere.

2) Geometry

With the help of a three-dimensional computer-aided model, as illustrated in Figure 4, all six vortices (s_1, s_2, s_3, s_4, s_5 and s_6) have been positioned on the sphere. To visualize the multiple positions the qubit can occupy, none of the vortices are exclusively positioned on any axis. However, two vortex corners, denoted as s_1 and s_4 (following the standard method for measuring angles from 0° to 360° in the x-y plane), are strategically positioned on the x-z plane to align with the φ measuring method from the Bloch Sphere. These are present in the first and second quarters of the x-z plane. (1)

The remaining four states, referred to as s_3, s_5 and s_2, s_6 are strategically placed so that two of them always appear in the upper hemisphere, while the other two appear in the lower hemisphere. This corresponds to the positive and negative sides of the x, y, and z-axes.

3) Geometrical Coordinates of vortex corners

a. φ and θ values (approximate):

- s_1: $\varphi = 0°$, $\theta \approx 54.74°$
- s_2: $\varphi = 60°$, $\theta \approx 125.26°$
- s_3: $\varphi = 120°$, $\theta \approx 54.74°$
- s_4: $\varphi = 180°$, $\theta \approx 125.26°$
- s_5: $\varphi = 240°$, $\theta \approx 54.74°$
- s_6: $\varphi = 300°$, $\theta \approx 125.26°$

Based on the measurements, it is evident that when one of two adjacent states are approximately 0.954 radians closer to θ 0° (from the positive z-axis), while the other state is equidistant from θ 180° (from the negative z-axis) or vice versa. (1)

b. Three-dimensional coordinates (approximate):

- $\mathbf{s_1}$: (0.8164, 0, 0.5774), $\mathbf{s_3}$: (-0.4082, 0.7071, 0.5774), $\mathbf{s_5}$: (-0.4082, -0.7071, 0.5774)
  - $\mathbf{s_2}$: (0.4082, 0.7071, -0.5774), $\mathbf{s_4}$: (-0.8164, 0, -0.5774), $\mathbf{s_6}$: (0.4082, -0.7071, -0.5774)

5. Certainty Amplitude

1. Overall magnitude distribution equals to 1

a. Column representation of the vector s_1, s_3 and s_5's three-dimensional coordinates with their Squared value:

$$\text{format: } \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix} \qquad s_1 \approx \begin{bmatrix} 0.6667 \\ 0 \\ 0.3333 \end{bmatrix}, s_3 \approx \begin{bmatrix} 0.1667 \\ 0.5 \\ 0.3333 \end{bmatrix}, s_5 \approx \begin{bmatrix} 0.1667 \\ 0.5 \\ 0.3333 \end{bmatrix}$$

b. Column representation of the vector s_2, s_4 and s_6's three-dimensional coordinates with their Squared value:

$$\text{format: } \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix} \qquad s_2 \approx \begin{bmatrix} 0.1667 \\ .5 \\ 0.3333 \end{bmatrix}, s_4 \approx \begin{bmatrix} 0.6667 \\ 0 \\ 0.3333 \end{bmatrix}, s_6 \approx \begin{bmatrix} 0.1667 \\ 0.5 \\ 0.3333 \end{bmatrix}$$

c. Sum of corresponding Coordinates

In the context of a unit-radius sphere, the magnitude of a point on surface is represented as: $\sqrt{(x^2 + y^2 + z^2)} = 1$. As per shown in figure.4, the sum of corresponding Coordinates of the points presents in the upper and lower hemisphere:

$(xs_1^2 + ys_1^2 + zs_1^2) \approx (xs_3^2 + ys_3^2 + zs_3^2) \approx (xs_5^2 + ys_5^2 + zs_5^2) \approx (xs_2^2 + ys_2^2 + zs_2^2) \approx (xs_4^2 + ys_4^2 + zs_4^2) \approx (xs_6^2 + ys_6^2 + zs_6^2) \approx 1$, and

$(xs_1^2 + xs_3^2 + xs_5^2) \approx (ys_1^2 + ys_3^2 + ys_5^2) \approx (zs_1^2 + zs_3^2 + zs_5^2) \approx (xs_2^2 + xs_4^2 + xs_6^2) \approx (ys_2^2 + ys_4^2 + ys_6^2) \approx (zs_2^2 + zs_4^2 + zs_6^2) \approx 1$ also.

These equations collectively illustrate that the sum of squared values along each coordinate for the respective points in both hemispheres results in an overall magnitude equal to 1 on the unit-radius sphere.

2. Current methodology to assign probability amplitude

$\left|\cos\frac{\theta}{2}\right|^2 + \left|\sin\frac{\theta}{2}\right|^2 = 1$. This formula is utilized to determine the probability amplitude, where θ represents the polar angle of the qubit. (1)

3. Certainty amplitude on a unit circle

As discussed in section '**4.2**' and illustrated in Figure 4, six points are equally distributed on a unit-radius sphere to demonstrate that qubits occupy multiple states within a wave function. These points are confined to two latitude lines, positioned approximately at 54.74° and 125.26° from the positive Z-axis, as discussed in section 5.1.c, representing the overall magnitude of 1 for the upper and lower hemispheres for states 0 and 1, respectively. By choosing any point (from the 6 points), we select one of the two latitudes only. This is methodology get used to represent that after measurement, qubits collapse into either the 0 or 1 state, disrupting superposition.

In current methodology, state |0⟩ and |1⟩ are called normal basis. |0⟩ and |1⟩ are orthogonal, that is, perpendicular to each other. The normalization condition for probability amplitude is generally given by $|\alpha|^2 + |\beta|^2 = 1$.

Therefore, based on this formula, utilized to determine the probability amplitude, to align the certainty amplitudes representation on the first quarter of a unit circle with current methodology could get help of the Pythagorean theorem applied to a circle and formula discussed in the topic '**5.2**'.

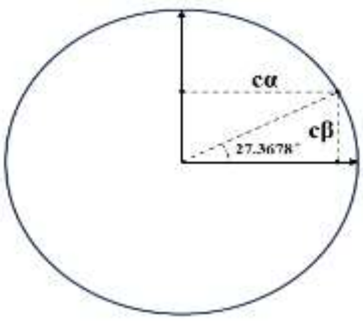

Figure 5: Certainty Amplitude on A Unit Circle

a. **cα**: Certainty amplitude for the qubit being in the State **|0⟩**

$\approx \left(\cos\left|\frac{54.73561032°}{2}\right|\right) \approx (\cos 27.36780516°) \approx$ **0.8880738340**

cα ≈ 0.8880738340

b. **cβ**: Certainty amplitude for the qubit being in the State **|1⟩**

$\approx \left(\sin\left|\frac{54.73561032°}{2}\right|\right) \approx (\sin 27.36780516°) \approx$ **0.4597008434**

cβ ≈ 0.4597008434

4. Qubits state representation: `

A qubit with state |0⟩ is commonly represents by the column vector $\begin{bmatrix}1\\0\end{bmatrix}$ and a qubit with state |1⟩ is represented by the column vector $\begin{bmatrix}0\\1\end{bmatrix}$, that is

$|\,0\,\rangle = \begin{bmatrix}1\\0\end{bmatrix} \qquad |\,1\,\rangle = \begin{bmatrix}0\\1\end{bmatrix}$

By putting the derived value from '4.3.a and 4.3.b':

cα $\approx \begin{bmatrix}1\\0\end{bmatrix} \cdot 0.8880738340 \approx \begin{bmatrix}0.8880738340\\0\end{bmatrix}$

cβ $\approx \begin{bmatrix}0\\1\end{bmatrix} \cdot 0.4597008434 \approx \begin{bmatrix}0\\0.4597008434\end{bmatrix}$

By this way we can we can represent qubits state as vector. (1)

5. Equation of Ellipse:

The equation of a standard ellipse cantered at the origin with width 2a and hight 2b is: $x^2/a^2 + y^2/b^2 = 1$.

6. Spectrum of Certainty Amplitude:

Classical bit can only be in a single state whereas a qubit cannot only be in one of the two discrete states, it can also exist simultaneously in a blend of these state. The proportions of |0⟩ and |1⟩ in the blend need not be equal, and can be arbitrary. (1)

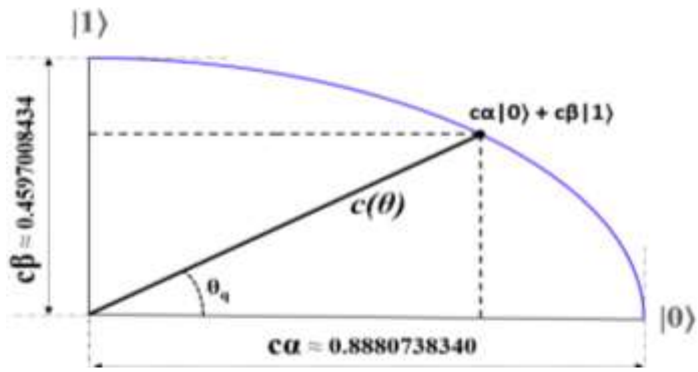

Figure 6: Spectrum of Certainty Amplitude for θ (polar angle)

Since, cα > cβ > 0, then based on equation of a standard ellipse, spectrum of certainty amplitude can be representation on the first quarter of an ellipse. In this representation, the blue-colored elliptical spline depicts the first quarter of an ellipse. It has an approximately 0.8880738340-unit major radius along the x-axis, representing the certainty amplitude alpha (cα), the real part of complex numbers, and an approximately 0.4597008434-unit minor radius along the y-axis, representing the certainty amplitude beta (cβ), the imaginary part. The ellipse is centered at the origin (0,0). Where, $|c\alpha|^2 + |c\beta|^2 = 1$, satisfying that hypotenuse equals one, sum of the square of the vertical and horizontal sides. Meanwhile, $x^2/ c\alpha^2 + y^2/ c\beta^2 = 1$.

Probability can be calculated using trigonometry by representing a unit-radius qubit on the plane. When $2{\cdot}\theta_q$ equals θ, then according to the proposed methodology, assigning the certainty amplitude for placing the qubit at θ degrees (polar angle) can be mathematically expressed as:

$$c(\theta) \approx \sqrt{(|\cos\frac{\theta}{2}| \cdot c\alpha)^2 + (|\sin\frac{\theta}{2}| \cdot c\beta)^2}$$

References

1. Lala, Parag K. Quantum Computing A Beginner's introduction. 2020. Chennai : McGraw Hill, 2019. ISBN 93-90385-26-1.

2. Griffiths, David J. and Schroeter, Darrel F. Introduction to Quantum Mechanics. third edition. New Delhi : Cambridge University Press, 2018. ISBN 978-1-108-79110-6.

3. Lay, David C., Lay, Steven R. and McDonald, Judi J. Linear Algebra and its application. fith edition. Chennai : Pearson India Education Services Pvt. Ltd., 2016. ISBN 978-93-570-5968-8.

4. Silverman, Joseph H. A friendly introduction to number theory. 2019 edition. Chennai : Pearson India Education Services Pvt. Ltd, 2018. ISBN 978-93-534-3307-9.