# Change Examination of Structural Clamped Clamped Fixed Fixed Edged Plate Under Stability Analysis. 

${ }^{1}$ Uzoukwи C. S., ${ }^{2}$ Ibearugbulem O. M., ${ }^{3}$ Obumseli P., ${ }^{4}$ Iroegbu R. U, ${ }^{5}$ Nwachuku C. I., ${ }^{6}$ Ubagha O. D., ${ }^{7}$ Ogbonna S. N., ${ }^{8}$ Senator P. C, ${ }^{9}$ Ezenkwa C. S<br>${ }^{1,2,3,4,5,6,7,8}$ School of Engineering and Engineering Technology, Federal University of Technology Owerri, Imo State, Nigeria<br>${ }^{9}$ Covenant University Otta Ogun State, Nigeria<br>Email: cskambassadors @ gmail.com, Phone: +2348130131114<br>DOI: https://doi.org/10.55248/gengpi.5.0324.0659

## ABSTRACT

The effect of buckling load on at structural plate element which is support on four edges. The vertical axis rest on clamped and fixed supported edge while the horizontal axis rests on also clamped and fixed boundaries, forming a plate of CCFF shape orientation. 3rd order energy Functional was adopted in the research work. The clamped clamped fixed fixed plate was considered as the direct independent plate, meaning that the material properties are uniform round about the shape of the element. These includes the flexural rigidity, poison ratio and young elastic modulus of elasticity. Considering the plate arrangement, the shape functions were first formulated, after which the various integral values of the differentiated shape functions, of the various boundary conditions were all generated. Next to this was the formulation of the stiffness coefficients for the various boundary cases. Further minimization yielded the controlling functional known as the overall potential energy functional. The differential value of the Third Order Overall Potential Energy Functional, with respect to the amplitude was further integrated. The integration gave rise to the result known as the Lead equation. From the lead equation comes the derivation of the non-dimensional buckling load parameters. The rest of the analysis which is detailed below was conducted using the buckling equation

## Key words:

- Third order Functional
* Buckling load Equation
* C-C-F-F Plate
* Overall Potential Energy Functional,
* Lead Equation,
* Vital Buckling load


## Introduction

Plate elements are mostly used as engineering materials. Due to their importance and wide application, several researches have been conducted with the target of maximizing their high values for wider structural applications. They can be either straight or curved. They known for having three dimensions called the primary, secondary and tertiary dimensions. The plate thickness is usually referred to as the tertiary dimension. They are smaller than the rest of the plate dimensions. The isotropic rectangular CCFF plate have all their material properties in all directions as the same and so they are classified as direction independent element. The subject has been a subject of study in solid structural mechanics for a long time now. Although the buckling analysis of rectangular plates has received the attention of many researchers for several centuries prior to this time, other researchers have gotten solution using even order energy functional for Buckling of plate and so the analysis of the buckling activities of CLAMPED CLAMPED FIXED FIXED isotropic plate using odd order energy functional is the gap the work tends to fill.

## Section A

The overall potential energy, $\mathrm{O}_{\mathrm{v}}$ was first formulated by the summation of External Work $\mathrm{E}_{\mathrm{w}}$ and Strain energy, $€$. This is shown in Equation i
$E_{x}+\epsilon=O_{v}$

The strain energy, is gotten from the product of normal stress and normal strain,
both in the x components as
$\S_{\mathrm{i}} \partial_{\mathrm{i}}=\frac{E z^{2}}{1-\mu^{2}}\left(\left[\frac{\partial^{2} f u}{\partial x^{2}}\right]^{2}+\mu\left[\frac{\partial^{2} f u}{\partial x \partial y}\right]^{2}\right)$
while also in vertical axis as shown in the Equation iii
$\S_{\mathrm{j}} \partial_{\mathrm{j}}=\frac{E z^{2}}{1-\mu^{2}}\left(\left[\frac{\partial^{2} f u}{\partial y^{2}}\right]^{2}+\mu\left[\frac{\partial^{2} f u}{\partial x \partial y}\right]^{2}\right)$
Considering the parallel effect of the stress and strain on the plate surface, gives the product of the in-plane shear stress and in-plane shear strain as
$\tau_{i j} \gamma_{\mathrm{ij}}=2 \frac{E z^{2}(1-\mu)}{\left(1-\mu^{2}\right)}\left[\frac{\partial^{2} f u}{\partial x \partial y}\right]^{2}$
Bringing Equations ii, iii and iv together gives
$\S_{\mathrm{i}} \mathrm{\partial}_{\mathrm{i}}+\S_{\mathrm{j}} \mathrm{\partial}_{\mathrm{j}}+\tau_{i j} \gamma_{\mathrm{ij}}=\frac{E z^{2}}{1-\mu^{2}}\left(\left[\frac{\partial^{2} f u}{\partial x^{2}}\right]^{2}+2\left[\frac{\partial^{2} f u}{\partial x \partial y}\right]^{2}+\left[\frac{\partial^{2} f u}{\partial y^{2}}\right]^{2}\right)$
v

The strain Energy is given mathematically as $\epsilon=\frac{1}{2} \iint_{\mathrm{xy}} \bar{\epsilon}$ dxdy vi
where $\bar{\epsilon}=\frac{E z^{2}}{1-\mu^{2}} \int\left(\left[\frac{\partial^{2} f u}{\partial x^{2}}\right]^{2}+2\left[\frac{\partial^{2} f u}{\partial x \partial y}\right]^{2}+\left[\frac{\partial^{2} f u}{\partial y^{2}}\right]^{2}\right)$
vii
Further rearrangement of Equation vii, gives the third order strain energy equation
as $€=\frac{G}{2} \int_{0}^{\mathrm{n}} \int_{0}^{\mathrm{m}}\left(\frac{\partial^{3} f u}{\partial x^{3}} \cdot \frac{\partial f u}{\partial \mathrm{x}}+2 \frac{\partial^{3} f u}{\partial \mathrm{x} \partial \mathrm{y}^{2}} \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{x}}+\frac{\partial^{3} f u}{\partial y^{3}} \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{y}}\right) \mathrm{dxdy}$
with the external load as $\mathrm{v}=-\frac{b k l_{x}}{2} \int_{0}^{\mathrm{n}} \int_{0}^{\mathrm{m}}\left(\frac{\partial \mathrm{fu}}{\partial \mathrm{x}}\right)^{2}$ dxdy
The third order total potential energy functional is expressed mathematically as
$\mathrm{O}_{\mathrm{v}}=\frac{G}{2} \iint\left(\frac{\partial^{3} f u}{\partial x^{3}} \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{x}}+2 \frac{\partial^{3} f u}{\partial \mathrm{x}^{2} \partial \mathrm{y}} \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{y}}+\frac{\partial^{3} f u}{\partial y^{3}} \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{y}}\right) \mathrm{dxdy}-\frac{b k l_{x}}{2} \iint \frac{\partial^{2} f u}{\partial x^{2}} \mathrm{dxdy}$
x

Rearranging the total potential energy equation in terms of non-dimensional
parameters I , J the buckling load equation is gotten as
$b k l_{\mathrm{t}}=\frac{\mathrm{G}}{\mathrm{a}^{2}} \int_{0}^{1} \int_{0}^{1} \cdot\left(\left[\frac{\partial^{3} \mathrm{fu}}{\partial J^{3}}\right] \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{J}}+2 \frac{1}{p^{2}}\left[\frac{\partial^{3} \mathrm{fu}}{\partial \mathrm{J} \partial I^{2}}\right] \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{J}}+\frac{1}{p^{4}}\left[\frac{\partial^{3} \mathrm{fu}}{\partial I^{3}}\right] \cdot \frac{\partial \mathrm{fu}}{\partial \mathrm{I}}\right) \mathrm{dJdI}$
$b k l_{\mathrm{b}}=\int_{0}^{1} \int_{0}^{1}\left(\frac{\partial \mathrm{fu}}{\partial \mathrm{J}}\right)^{2} \mathrm{dJdI}$
xii
$b k l_{\mathrm{x}}=\frac{b k l_{\mathrm{t}}}{b k l_{\mathrm{b}}}$

## Section B

Two boundary cases were treated, in the derivation of the shape functions and they namely Clamped edge which was denoted as C and Fixed edge which is denoted as F . For the Clamped edge situation, the equation of deflection $f w$ and the $2^{\text {nd }}$ order derivative of the same equation $f w^{2}$, were equated to zero and simultaneous equations were formed by considering $\mathrm{I}=0$ at the left hand edge in the case of the horizontal axis and $\mathrm{I}=1$ at the right side of the same component. Also considering the top as $\mathrm{J}=0$ and $\mathrm{J}=1$ at the bottom support for the vertical direction. These equations were solved simultaneously to obtain the various values of the primary and secondary dimensions ( $n_{1}, m_{1}, n_{2}, m_{2} n_{3,} m_{3,}, n_{4}$ and $m_{4}$ ) for the CCFF plate element. Where I and J are nondimensional parameters parallel to horizontal and vertical axis respectively as earlier explained.

## Section C

## The Deflection Equation



Fig. i Isotropic Rectangular CCFF Plate

Considering the horizontal axis


Fig. ii Horizontal Supports
Since the left side is clamped and the right side fixed, that means that
$f w_{\mathrm{x}}=\mathrm{m}_{\mathrm{o}}+\mathrm{m}_{1} \mathrm{I}+\mathrm{m}_{2} \mathrm{I}^{2}+\mathrm{m}_{3} \mathrm{I}^{3}+\mathrm{m}_{4} \mathrm{I}^{4}+\mathrm{m}_{5} \mathrm{I}^{5} \quad 1$
The first derivation of Equation 1 gives
$f w_{x}^{I}=m_{1}+2 \mathrm{~m}_{2} \mathrm{I}+3 \mathrm{~m}_{3} \mathrm{I}^{2}+4 \mathrm{~m}_{4} \mathrm{I}^{3}+5 \mathrm{~m}_{5} \mathrm{I}^{4}$ 2
also the second derivative of the Equation 1 gives
$f w_{x}{ }^{I I}=2 \mathrm{~m}_{2}+6 \mathrm{~m}_{3} \mathrm{I}+12 \mathrm{~m}_{4} \mathrm{I}^{2}+20 \mathrm{~m}_{5} \mathrm{I}^{3} \quad 3$
and finally the third derivative the same Equation gives
$f w_{x}^{I I I}=6 \mathrm{~m}_{3}+24 \mathrm{~m}_{4} \mathrm{I}+60 \mathrm{~m}_{5} \mathrm{I}^{2}$ 4

On introducing the boundary conditions to the horizontal axis
At the left support, $I=0$
When $f w_{\mathrm{x}}=0$
$f w_{\mathrm{x}}=0=\mathrm{m}_{\mathrm{o}}+0+0+0+0$
$\mathrm{m}_{\mathrm{o}}=0$
Also when $f w_{\mathrm{x}}{ }^{11}=0$
$f w_{\mathrm{x}}{ }^{1}=0=\mathrm{m}_{1}+0+0+0+0$
$\mathrm{m}_{1}=0 \quad 8$
$\mathrm{m}_{1}=0$
at the right support, $I=1$
$f w_{\mathrm{x}}{ }^{1}=2 \mathrm{~m}_{2}+3 \mathrm{~m}_{3}+4 \mathrm{~m}_{4}+5 \mathrm{~m}_{5}=-\frac{1 \mathrm{~m}_{5}}{5}$ 10

Further simplifying Equation 10 gives
$m_{2}=\frac{-3 m_{3}-4 m_{4}-5 m_{5}-\frac{1 m_{5}}{5}}{2}$
Also for the second derivative of the deflection on the X axis,
$f w_{\mathrm{x}}{ }^{11}=0=0+2 \mathrm{~m}_{2}+6 \mathrm{~m}_{3}+12 \mathrm{~m}_{4}+20 \mathrm{~m}_{5}$
rearranging the equation and making $\mathrm{n}_{3}$ the subject gives

$$
\begin{equation*}
2 m_{2}=-6 m_{3}-12 m_{4}-20 m_{5} \tag{13}
\end{equation*}
$$

in simpler form as
$m_{2}=-3 m_{3}-6 m_{4}-10 m_{5}$
Solving for the third derivative of the deflection on the horizontal component gives
$f w_{x}{ }^{111}=0=6 m_{3}+24 m_{4}+60 m_{5}$
That is
$f w_{\mathrm{x}}^{111}=0=\mathrm{m}_{3}+4 \mathrm{~m}_{4}+10 \mathrm{~m}_{5}$ 16
$m_{3}=-10 m_{5}-4 m_{4} \quad 17$

Resolving Equation 11 and 14 together gives
$\frac{-3 m_{3}-4 m_{4}-5 m_{5}-\frac{1 m_{5}}{5}}{2}=\frac{-10 m_{5}}{3}-2 m_{4}$
and further simplifying gives
$-1.5 m_{3}-2 m_{4}-2.6 m_{5}=-3 m_{3}-6 m_{4}-10 m_{5}$
Bringing the like terms together
$1.5 \mathrm{~m}_{3}+4 \mathrm{~m}_{4}+7.4 \mathrm{~m}_{5}=0$
$m_{3}=\frac{-4 m_{4}-7.4 m_{5}}{1.5}$
Further simplification gives
$m_{3}=-2.66667 m_{4}-4.9333 m_{5}$
Solving Equations 17 and Equation 21 together gives
$-10 m_{5}-4 m_{4}=-2.66667 m_{4}-4.9333 m_{5}$
Collecting the like terms together gives
$-4 \mathrm{~m}_{4}+2.66667 \mathrm{~m}_{4}=10 \mathrm{~m}_{5}-4.9333 \mathrm{~m}_{5} 23 \mathrm{i}$
$-1.3333 m_{4}=5.06667 m_{5}$
That means $\mathrm{m}_{4}=\frac{5.06667}{-1.3333} \mathrm{~m}_{5}=-3.73337 \mathrm{~m}_{5}$ 23ii

In order to obtain the values of the $m_{2}$ and $m_{3}$ interms of $m_{5}$, substitute
Equation 23ii into Equation 11 and 21 and gives
$m_{3}=-2.66667\left(-3.73337 m_{5}\right)-4.9333 m_{5}$
$\mathrm{m}_{3}=9.955666 \mathrm{~m}_{5}-4.9333 \mathrm{~m}_{5}$
$m_{3}=9.955666 m_{5}-4.9333 m_{5}=5.022366 m_{5}$ 23iii
$\mathrm{m}_{2}=\frac{-3\left(5.022366 \mathrm{~m}_{5}\right)-4\left(-3.73337 \mathrm{~m}_{5}\right)-5 \mathrm{~m}_{5}-\frac{1 \mathrm{~m}_{5}}{5}}{2}$
$m_{2}=\frac{-15.0671 m_{5}+14.933 m_{5}-85 m_{5}-0.2 m_{5}}{2}$
$m_{2}=-2.667 m_{5}$ or $-2.7 m_{5}$ 23iv

When Equations 23ii , 23iii and 23iv substituted back into Equation 1 gives
$f w_{\mathrm{x}}=\left(-2.7 \mathrm{~m}_{5} \mathrm{I}^{2}+5.02 \mathrm{~m}_{5} \mathrm{I}^{3}-3.7 \mathrm{~m}_{5} \mathrm{I}^{4}+\mathrm{m}_{5} \mathrm{I}^{5}\right)$
or $\quad=\mathrm{m}_{5}\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)$
when multiplied by negative one.

### 1.3.2 Analysis of the Vertical Component

The case of horizontal Direction (Y-Y axis)


Similarly the Top edge is clamped and the bottom edge fixed,
that means that
$f w_{\mathrm{y}}=\mathrm{n}_{\mathrm{o}}+\mathrm{n}_{1} \mathrm{~J}+\mathrm{n}_{2} \mathrm{~J}^{2}+\mathrm{n}_{3} \mathrm{~J}^{3}+\mathrm{n}_{4} \mathrm{~J}^{4}+\mathrm{n}_{5} \mathrm{~J}^{5} \quad 25$
The first derivation of Equation 25 gives
$f w_{y}{ }^{I}=\mathrm{n}_{1}+2 \mathrm{n}_{2} \mathrm{~J}+3 \mathrm{n}_{3} \mathrm{~J}^{2}+4 \mathrm{n}_{4} \mathrm{~J}^{3}+5 \mathrm{n}_{5} \mathrm{~J}^{4} \quad 26$
also the second derivative of the Equation 25 gives
$f w_{y}{ }^{I I}=2 n_{2}+6 n_{3} J+12 n_{4} J^{2}+20 n_{5} J^{3}$
and finally the third derivative the same Equation gives
$f w_{x}{ }^{111}=6 n_{3}+24 n_{4} \mathrm{~J}+60 \mathrm{n}_{5} \mathrm{~J}^{2}$ 28

On introducing the boundary conditions to the horizontal axis
At the left support, $\mathrm{J}=0$
When $f w_{y}=0$
$f w_{\mathrm{y}}=0=\mathrm{m}_{\mathrm{o}}+0+0+0+0$
$\mathrm{m}_{0}=0$
Also when $f w_{y}{ }^{11}=0$ 30
$F w_{\mathrm{y}}{ }^{1}=0=\mathrm{m}_{1}+0+0+0+0$ 31
$\mathrm{m}_{1}=0$
$\mathrm{m}_{1}=0$
at the right support, $\mathrm{J}=1$
$f w_{\mathrm{y}}{ }^{1}=2 \mathrm{n}_{2}+3 \mathrm{n}_{3}+4 \mathrm{n}_{4}+5 \mathrm{n}_{5}=-\frac{1 \mathrm{n}_{5}}{5}$
Further simplifying Equation 34 gives
$\mathrm{n}_{2}=\frac{-3 \mathrm{n}_{3}-4 \mathrm{n}_{4}-5 \mathrm{n}_{5}-\frac{1 \mathrm{n}_{5}}{5}}{2}$
Also for the second derivative of the deflection on the X axis,
$f w_{\mathrm{x}}{ }^{11}=0=0+2 \mathrm{n}_{2}+6 \mathrm{n}_{3}+12 \mathrm{n}_{4}+20 \mathrm{n}_{5}$
rearranging the equation and making $n_{3}$ the subject gives
$2 \mathrm{n}_{2}=-6 \mathrm{n}_{3}-12 \mathrm{n}_{4}-20 \mathrm{n}_{5}$
in simpler form as
$n_{2}=-3 n_{3}-6 n_{4}-10 n_{5}$
Solving for the third derivative of the deflection on the horizontal component gives
$f w_{\mathrm{x}}{ }^{111}=0=6 \mathrm{n}_{3}+24 \mathrm{n}_{4}+60 \mathrm{n}_{5} \quad 39$
That is
$f w_{\mathrm{x}}{ }^{111}=0=\mathrm{n}_{3}+4 \mathrm{n}_{4}+10 \mathrm{n}_{5} \quad 40$
$n_{3}=-10 n_{5}-4 n_{4} \quad 41$
Resolving Equation 35 and 38 together gives
$\frac{-3 n_{3}-4 n_{4}-5 n_{5}-\frac{1 n_{5}}{5}}{2}=-3 n_{3}-6 n_{4}-10 n_{5}$ 42
and further simplifying gives
$-1.5 n_{3}-2 n_{4}-2.6 n_{5}=-3 n_{3}-6 n_{4}-10 n_{5}$
Bringing the like terms together
$1.5 \mathrm{n}_{3}+4 \mathrm{n}_{4}+7.4 \mathrm{n}_{5}=0$
$\mathrm{n}_{3}=\frac{-4 \mathrm{n}_{4}-7.4 \mathrm{n}_{5}}{1.5}$
Further simplification gives
$n_{3}=-2.66667 n_{4}-4.9333 n_{5}$
Solving Equations 41 and Equation 45 together gives
$-10 n_{5}-4 n_{4}=-2.66667 n_{4}-4.9333 n_{5} 46$
Collecting the like terms together gives
$-4 n_{4}+2.66667 n_{4}=10 n_{5}-4.9333 n_{5} \quad 47$
$-1.3333 \mathrm{n}_{4}=5.06667 \mathrm{n}_{5}$
That means $\mathrm{n}_{4}=\frac{5.06667}{-1.3333} \mathrm{n}_{5}=-3.73337 \mathrm{n}_{5} \quad 48$
In order to obtain the values of the $n_{2}$ and $n_{3}$ interms of $n_{5}$, substitute
Equation 48 into Equation 45 and 35 and gives
$n_{3}=-2.66667\left(-3.73337 n_{5}\right)-4.9333 n_{5}$
$n_{3}=9.955666 n_{5}-4.9333 n_{5}$
$n_{3}=9.955666 n_{5}-4.9333 n_{5}=5.022366 n_{5}$
$n_{2}=\frac{-3\left(5.022366 n_{5}\right)-4\left(-3.73337 n_{5}\right)-5 n_{5}-\frac{1 n_{5}}{5}}{2}$
$\mathrm{n}_{2}=\frac{-15.0671 \mathrm{n}_{5}+14.933 \mathrm{n}_{5}-85 \mathrm{n}_{5}-0.2 \mathrm{n}_{5}}{2}$
$\mathrm{n}_{2}=-2.667 \mathrm{n}_{5}$ or $-2.7 \mathrm{n}_{5}$
Similarly when Equations 50,49 and 48 substituted back into Equation 25, that gives

$$
\begin{aligned}
f w_{y} & =\left(-2.7 \mathrm{n}_{5} \mathrm{~J}^{2}+5.02 \mathrm{n}_{5} \mathrm{~J}^{3}-3.7 \mathrm{n}_{5} \mathrm{~J}^{4}+\mathrm{n}_{5} \mathrm{~J}^{5}\right) \\
& =\mathrm{n}_{5}\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)
\end{aligned}
$$

when multiplied by negative one.
Then the Shape function is expressed as
$f w=\mathrm{m}_{5} \mathrm{n}_{5}\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)^{*}\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) \quad 52 \mathrm{i}$

## Section D

## The Stiffness Coefficients

The shape functions were further differentiated at different stages, and the
integration of the differential values gave the stiffness coefficients. These includes
$\mathrm{f}=\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)^{*}\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)$
$\frac{\partial \mathrm{f}}{\partial \mathrm{I}}=\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) \quad 52 \mathrm{ii}$
$\frac{\partial^{2} \mathrm{f}}{\partial \mathrm{I} \partial \mathrm{J}}=\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(5.4 \mathrm{~J}+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4}\right) \quad 52 \mathrm{iii}$
$\frac{\partial^{3} \mathrm{f}}{\partial \mathrm{I} \partial \mathrm{J}^{2}}=\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(5.4+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4}\right) \quad 53$
$\frac{\partial^{2} \mathrm{f}}{\partial \mathrm{I}^{2}}=\left(5.4+30.12 \mathrm{I}-45.6 \mathrm{I}^{2}+20 \mathrm{I}^{3}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) \quad 54$
$\frac{\partial^{3} \mathrm{f}}{\partial \mathrm{I}^{3}}=\left(30.12-91.2 \mathrm{I}+60 \mathrm{I}^{2}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) \quad 55$
also
$\frac{\partial \mathrm{f}}{\partial \mathrm{J}}=\quad\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)^{*}\left(5.4 \mathrm{~J}+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4} \quad 56\right.$
$\frac{\partial^{2} \mathrm{f}}{\partial \mathrm{J}^{2}}=\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)\left(5.4+30.12 \mathrm{~J}-45.6 \mathrm{~J}^{2}+20 \mathrm{~J}^{3}\right)$ 57
$\frac{\partial^{3} \mathrm{f}}{\partial \mathrm{J}^{3}}=\quad\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)\left(30.12-91.2 \mathrm{~J}+60 \mathrm{~J}^{2}\right) \quad 58$
Integrating the product of the Equation 55 and 50 gives the first stiffness coefficient.
That is
$\mathrm{k}_{\text {ccffl }}=\int_{0}^{1} \int_{0}^{1} \frac{\partial^{3} \mathrm{f}}{\partial \mathrm{I}^{3}} * \frac{\partial \mathrm{f}}{\partial \mathrm{I}} \mathrm{dIdJ}$
$\mathrm{k}_{\text {ccffI }}=\int_{0}^{1} \int_{0}^{1}\left[\left(30.12-91.2 \mathrm{I}+60 \mathrm{I}^{2}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) *\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\right]$ dIdJ 60
bringing the like terms together gives

$$
\left.=\int_{0}^{1} \int_{0}^{1}\left[\left(30.12-91.2 \mathrm{I}+60 \mathrm{I}^{2}\right) 5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right) *\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\right] \mathrm{dIdJ}
$$

$$
61
$$

further minimization and substitution of the upper and lower limits yields
$\mathrm{k}_{\text {ccffl }}=0.15792$
also integrating the product Equation 53 and 51 give the second stiffness coefficient.
That is
$\mathrm{k}_{\text {ccff2 }}=\int_{0}^{1} \int_{0}^{1} \frac{\partial^{3} \mathrm{f}}{\partial \mathrm{I} \partial \mathrm{J}^{2}} * \frac{\partial \mathrm{f}}{\partial \mathrm{I}} \mathrm{dId} \mathrm{J}$
Fixing the real values gives
$\mathrm{k}_{\text {ccff2 }}=\int_{0}^{1} \int_{0}^{1}\left[\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(5.4+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4}\right) *\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\right] d \mathrm{IId}$ 63

Bring the like terms together gives


Multiplying and further integrating the differential values gives
$\mathrm{k}_{\text {ccff2 }}=0.035689$
Furthermore integrating the product Equation 58 and 56 give the third stiffness coefficient.
That is
$\mathrm{k}_{\text {ccff3 }}=\int_{0}^{1} \int_{0}^{1} \frac{\partial^{3} \mathrm{f}}{\partial \mathrm{J}^{3}} * \frac{\partial \mathrm{f}}{\partial \mathrm{J}} \mathrm{dIdJ}$
$\mathrm{k}_{\mathrm{ccff} 3}=\int_{0}^{1} \int_{0}^{1}\left[\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)\left(30.12-91.2 \mathrm{~J}+60 \mathrm{~J}^{2}\right) *\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right) *\left(5.4 \mathrm{~J}+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4}\right)\right] \mathrm{dIdJ}$
$\mathrm{k}_{\text {ccff3 }}=\int_{0}^{1} \int_{0}^{1}\left[\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right)\left(2.7 \mathrm{I}^{2}-5.02 \mathrm{I}^{3}+3.7 \mathrm{I}^{4}-\mathrm{I}^{5}\right) *\left(30.12-91.2 \mathrm{~J}+60 \mathrm{~J}^{2}\right)\left(5.4 \mathrm{~J}+15.06 \mathrm{~J}^{2}-15.2 \mathrm{~J}^{3}+5 \mathrm{~J}^{4}\right)\right] \mathrm{dIdJ}$
integrating Equation 65 and introducing the upper and lower limits gives
$\mathrm{k}_{\text {cfff3 }}=0.15792$
and finally integrating the product Equation 51 and 51 gives the sixth stiffness coefficient.
That is
$\mathrm{k}_{\mathrm{ccfff}}=\int_{0}^{1} \int_{0}^{1}\left(\frac{\partial \mathrm{f}}{\partial \mathrm{I}} * \frac{\partial \mathrm{f}}{\partial \mathrm{I}}\right) \mathrm{dIdJ}$
That is
$\mathrm{k}_{\text {ccff6 } 6}=\int_{0}^{1} \int_{0}^{1}\left[\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right) *\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\right] \mathrm{dIdJ}$ 70

Collecting the like terms together gives
$=\int_{0}^{1} \int_{0}^{1}\left[\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right)\left(5.4 \mathrm{I}+15.06 \mathrm{I}^{2}-15.2 \mathrm{I}^{3}+5 \mathrm{I}^{4}\right) *\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\left(2.7 \mathrm{~J}^{2}-5.02 \mathrm{~J}^{3}+3.7 \mathrm{~J}^{4}-\mathrm{J}^{5}\right)\right] \mathrm{dIdJ}$
71
Putting the upper and lower limit values gives
$\mathrm{k}_{\text {cfff } 6}=0.011423$
Reducing Equation (xiii) in terms of the stiffness coefficients gives
$\mathrm{bkl}_{\mathrm{x}}=\frac{\mathrm{D}\left(\mathrm{kccff}_{1}+2 \frac{1}{p^{2}} \mathrm{kccff}_{2}+\frac{1}{p^{4}} \mathrm{kccff}_{3}\right)}{\mathrm{kccff}_{6} \mathrm{~m}^{2}}$
65
Substituting the real values in to Equation 65 gives
$\mathrm{bkl}_{\mathrm{x}}=\frac{\mathrm{D}\left(0.15792+2 \frac{1}{p^{2}} 0.035689+\frac{1}{p^{4}} 0.15792\right)}{0.011423 \mathrm{~m}^{2}}$
66

## RESULTS AND DISCUSSION.

The actual values of the stiffness coefficients and the critical buckling load coefficients were derived. Considering the critical buckling load coefficients at different aspect ratios, the values of the critical buckling loads were calculated. The first table represents the values of the stiffness coefficients while the other contains the critical buckling coefficients for the aspect ratio of $\mathrm{m} / \mathrm{n}$, both for the previous and present study. The values of the aspect Ratios ranges from 2.0 to 1.0 with arithmetic increase of 0.1 . From the values generated in the tables, it was observed that as the aspect ratio increases from 1.0 to 2.0 , the critical buckling load decreases. This occurred both in the present and previous results.

Table 1.1 Stiffness Coefficients from Present Work/ Previous researchers

| Stiffness coefficients, sc | Present Work | Previous Work |
| :--- | :--- | :---: |
| $\mathrm{k}_{\text {ssff1 }}$ | 0.15792 | 0.158015 |
| $\mathrm{k}_{\text {ssff2 }}$ | 0.035689 | 0.0357990 |
| $\mathrm{k}_{\text {sfff3 }}$ | 0.15792 | 0.158015 |
| $\mathrm{k}_{\text {ssff6 }}$ | 0.011423 | 0.011434 |

Table 1.2 Critical buckling load values for CCFF Plate from Previous/Present.

| $\mathrm{m} / \mathrm{n}$ |  | 2 | 1.9 | 1.8 | 1.7 | 1.6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| B |  | 43.9346 | 44.0759 | 44.2415 | 44.4373 | 44.6711 |
| $\mathrm{B}_{\mathrm{x}}$ | Previous | 43.3728 | 43.5151 | 43.6826 | 43.8814 | 44.1201 |
|  | Present | 43.9346 | 44.0759 | 44.2415 | 44.4373 | 44.6711 |

Table 1.2 cont'd.


## REFERENCES

[1] Shinde, B.M., Sayyad, A.S. \& Kowade, A.B. (2013). Thermal Analysis Of Isotropic Plates Using Hyperbolic Shear Deformation Theory. Journal of Applied and Computational Mechanics 7 (2013) 193-204
[2] Da-Guang Zhang (2014). "Nonlinear Bending Analysis of FGM Rectangular Plates with Various Supported Boundaries Resting on Two-Parameter Elastic". Archive of Applied Mechanics. Vol. 84, Issue: 1, Pp.1-20
[3] An-Chien W., Pao-Chun L. and Keh-Chynan, T. (2013)." High - Mode Buckling-restrained Brace Core Plates".Journals of the International Association for Earthquake Engineering.
[4] Ali Reza Pouladkhan (2011). "Numerical Study of Buckling of Thin Plate". International Conference on Sustainable Design and Construction Engineering. Vol. 78,Issue: 1, Pp. $152-157$.
[5] Singh, S.K \& Chakrabarti, A. (2012). Buckling Analysis of Laminated Composite Plates Using an Efficient C ${ }^{0}$ FE Model. Latin American Journal of Solids and Structures. Soni, S.R. (1975), Vibrations of Elastic Plates and Shells of Variables Thickness. Ph.D. Thesis. University of Roarkee.
[6] Srinivasa,C.V., Suresh, Y.J. and Prema, W.P. (2012). "Buckling Studies onLaminated Composite Skew Plate". International Journal of ComputerApplications. Vol. 37, Issue:1, Pp. 35-47.
[7] AydinKomur and Mustafa Sonmez (2008). "Elastic Buckling of Rectangular Plates Under Linearly Varying In-plane Normal Load with a Circular Cutout". International Journal of Mechanical Sciences. Vol. 35, Pp. 361-371.
[8] Ahmed Al-Rajihy (2008). "The Axisymmetric Dynamics of Isotropic Circular Plates with Variable Thickness Under the Effect of Large Amplitudes". Journal of Engineering, Vol. 14, Issue :1, Pp. 2302-2313.
[9] Audoly, B., Roman, B. and Pocheau, A. (2002). Secondary Buckling Patterns of a Thin Plate under In-plane Compression. The European Physical Journal BCondensed Matter and Complex Systems, Vol. 27, No. 1 (May).
[10] Azhari, M, Shahidi, A.R, Saadatpour, M.M (2004) "Post Local Buckling of Skew and Trapezoidal Plate". Journal of Advances in Structural Engineering, Vol. 7, Pp 61-70.
[11] Azhari, M. and Bradford, M.A. (2005), "The Use of Bubble Functions for the Post-Local Buckling of Plate Assemblies by the Finite Strip Method", International Journal for Numerical Methods in Engineering, Vol.38, Issue 6.
[12] Bhaskara, L.R. and Kameswara, C.T. (2013)." Buckling of Annular Plate with Elastically Restrained External and Internal Edges". Journal of Mechanics Based Design of Structures and Machines. Vol. 41, Issue 2. Pp. 222-235.

