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Teaching Competencies of Senior High School Teachers Relative to the Content Standards on Logic in General Mathematics

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ABSTRACT

This study explored the teaching competencies of the senior high school mathematics teachers of the Division of Quirino relative to the content standard of logic in general mathematics in terms of their content knowledge, confidence in teaching logic, difficulties encountered in teaching logic, factors that limit their competence and strategies in teaching the different learning competencies. It also determined the relationship between years in teaching logic and teacher's content knowledge and teacher's confidence in teaching. The teacher-respondents answered a researcher-made survey questionnaire and an assessment test which has been tested and validated. The result of the study was the basis in proposing a learning module in logic which may help in enhancing the competency of the teachers as well as the performance of the students. The findings revealed that the Senior High School mathematics teachers are moderately confident in teaching the content standards in logic but they are poor in content knowledge. There are varied factors affecting the teaching competence of the teachers such as too much paper works, heavy teaching load, inadequate time to perform lesson plans and teaching aids, poor ability in managing time, shortage of other instructional materials for students' use, inadequate library facilities, and high student/teacher ratio. The main difficulty encountered by the teachers in teaching logic is attributed to lack of time. For those who had the opportunity to handle the topic, lecture was the most common strategy used. Respondents' content knowledge and confidence in teaching logic were found to have a positive non-significant correlation with number of years in teaching logic however, it is not significant.

Keywords: *confidence in teaching, content knowledge, limit on competence, number of years in teaching, content standards on logic, general mathematics*

INTRODUCTION

Quality education is very much dependent on the quality of the teacher. For Roxas (2005), the teacher determines to a large extent what is taught, how it is taught and how learning is measured. But, no teacher can impart what he/she does not know.

The word competence as used in education circles today refers to one's ability and measure of one's performance. It can be defined in terms of one's knowledge, skills and behaviors. Teaching competence is about how a teacher or a professor carries his/her job effectively by possessing professional skills and dispositions (Spilkova, 2001).

In all educational systems, the performance of teachers is one of a handful of factors which determine school effectiveness and learning outcomes. Teachers are the most critical component of any system of education. How well they teach depends on motivation, qualification, experience, training, aptitude and a mass of other factors, not the least of these being the environment and management structures within which they perform their role. Teachers must be seen as part of the solution, not part of the problem (Nair, 2017).

Teachers are encouraged to make their mathematics lessons more learner-centered by encouraging learners to contribute to the lesson. To achieve this kind of approach to teaching, schools need quality teachers who have the appropriate knowledge about the art of teaching. Without doubt, teachers are one of the most powerful influences on students' engagement with mathematics (Attard, 2011). Such teachers, according to Tanner, Bottoms, Bottoms, Feagin, and Bearman (2003), "create experiences that help students make sense of the knowledge and skills being studied". According to Turnuklu and Yesildere (2007), "… although a number of factors may influence the effective teaching of a particular subject, teachers play an important role in that success". Good teachers, Attard (2011) claims, can achieve high and consistent levels of engagement and effective learning. Contrary to common belief in society that a teacher who knows a particular subject very well is best suited to teach such a subject, research has shown that this belief is not necessarily true. Various researchers such as Hill, Rowan and Ball (2005) and Etkina (2010) emphasize that "teachers of a specific subject should possess special understandings and abilities that integrate their knowledge of the content of the subject that they are teaching as well as having knowledge of the learners who are learning the content". Also, number of years in teaching a certain subject should be taken into consideration, teachers with a longer length of teaching a particular subject could make them more competent in terms of content knowledge and more confident in teaching the subject (Mateo, 2000).

Aside from the teaching competency of the teachers, another factor to consider to develop the education system of the Philippines is the curriculum. Hence, in the school year 2012-2013, the education system of the Philippines was enhanced from the ten years of basic education to a 12-year program through an initiative called the K-12 Education Plan sponsored by the Department of Education. The K-12 program offers a decongested 12-year program that gives students sufficient time to master skills and absorb basic competencies. The K-12 program accelerates mutual recognition of Filipino graduates and professionals in other countries (Official Gazette, 2015).

Article XIV, Section 2 of the 1987 Constitution of the Philippines states that the State shall establish, maintain and support a complete, adequate and integrated system relevant to the needs of the people and the society. This stipulation in the country's highest law is one of the legal bases of the Republic Act 10533, also known as the Enhanced Basic Education Act of 2013 which paved the way for upgrading the 10 – year basic education to a 12 – year program. It introduced Senior High School which is from Grade 11 to Grade 12 in the country (Herrera & Dio, 2016).

Senior High School students have to take two compulsory Mathematics subjects in Grade 11. These subjects are General Mathematics which is offered in the first semester, and Statistics and Probability, offered in the second semester. The Department of Education Curriculum Guide lists the following as the covered topics under General Mathematics, namely functions, rational functions, exponential, inverse and logarithmic functions, basic business mathematics, and logic (Herrera & Dio, 2016).

Herrera and Dio (2016) studied the readiness of the Grade 10 students in the general mathematics of senior high school where competencies of the said respondents were determined in the different areas of general mathematics in senior high school. Their study reveals that among the topics in General Mathematics, it was in logic where students were not yet ready.

Competencies of students depend much on competencies of teachers. This is the reason why a study on the competencies of teachers on logic was necessary.

Logic is often used, but not always in its technical sense. According to Bayot and Damayon (2010), logic was introduced by Zeno, the founder of the Stoic School and is derived from the Greek word *logike* which means "systematized and intelligible".

Logic in Senior High School exposes learners to symbolic forms of propositions (or statements) and arguments. Through the use of symbolic logic, learners should be able to recognize equivalent propositions, identify fallacies, and judge the validity of arguments. The culminating lesson is an application of the rules of symbolic logic, as learners are taught to write their own justifications to mathematical and real-life statements (Department of Education, 2013).

This study attempted to analyze the competency level of the senior high school teachers relative to the content standards of logic in general mathematics. Specifically, it aimed to:

1. Describe the Senior High School mathematics teachers in terms of the following variables in teaching logic:
   (a) content knowledge
   (b) confidence
   (c) difficulties
   (d) factors that limit the competence
   (e) strategies
2. Determine the significant relationship between years of teaching and
   (a) teachers' content knowledge on logic
   (b) teachers' confidence in teaching logic
3. Propose supplementary activities in teaching logic

Null Hypotheses

From the objectives stated above, the null hypotheses below are tested:

1. There is no significant correlation between the teachers' number of years in teaching logic and their content knowledge.
2. There is no significant correlation between the teachers' number of years in teaching logic and their confidence in teaching logic.

METHODOLOGY

This study used the descriptive-correlational method of study. Descriptive because it described the teaching competency of the senior high school mathematics teachers through different variables particularly content knowledge, confidence, difficulties, factors limiting their competence, and strategies used in teaching. The study was also correlational as it determined the relationship of the number of years of teaching logic with teachers' content knowledge on logic, and teachers' confidence in teaching logic.

Table 1. Frequency and Percent Distribution of Respondents by School

Name of School	Number of SHS Math Teachers	Percent
Aglipay High School	1	5.88
Cabarroguis National School of Arts and Trades	2	11.76
Diffun National High School	2	11.76
Maddela Comprehensive High School	6	35.29
Magsaysay National High School	1	5.88
Nagtipunan National High School	1	5.88
Pinaripad National High School	1	5.88
Quirino General High School	2	11.76
Saguday National High School	1	5.88
Total	**17**	**100.00**

As shown in Table 1, Maddela Comprehensive High School has six (35.29%) SHS math teachers, the greatest in the province of Quirino, followed by Cabarroguis National School of Arts and Trades, Diffun National High School, and Quirino General High School having two teachers each (11.76%) and the rest of the schools have one math teacher each (5.88%).

Table 2 presents the profile of the teacher respondents in terms of sex, age, and the length of service as a teacher in logic.

Table 2. Distribution of the Respondents in terms of the Profile Variables

<table>
<tr><th>Profile</th><th>Categories</th><th>Frequency</th><th>Percent</th></tr>
<tr><td rowspan="3">Sex</td><td>Female</td><td>5</td><td>29.4</td></tr>
<tr><td>Male</td><td>12</td><td>70.6</td></tr>
<tr><td>Total</td><td>17</td><td>100</td></tr>
<tr><td rowspan="6">Age</td><td>23 – 29</td><td>5</td><td>29.4</td></tr>
<tr><td>30 – 34</td><td>5</td><td>29.4</td></tr>
<tr><td>35 – 39</td><td>3</td><td>17.6</td></tr>
<tr><td>40 – 48</td><td>4</td><td>23.5</td></tr>
<tr><td colspan="3">Min = 23, Max = 48, Mean = 34.35, SD = 7.15</td></tr>
<tr><td>Total</td><td>17</td><td>100</td></tr>
<tr><td rowspan="7">No. of Years in Teaching Logic</td><td>1</td><td>2</td><td>11.8</td></tr>
<tr><td>2</td><td>8</td><td>47.1</td></tr>
<tr><td>3</td><td>5</td><td>29.4</td></tr>
<tr><td>5</td><td>1</td><td>5.9</td></tr>
<tr><td>7</td><td>1</td><td>5.9</td></tr>
<tr><td>Total</td><td>17</td><td>100</td></tr>
<tr><td colspan="3">Min = 1, Max = 7, Mean = 2.65, SD = 1.46</td></tr>
</table>

Among the 17 respondents, 70.6% were males and 29.4% were females. This discloses that the females were outnumbered by male respondents.

Five respondents constituting 29.4% were under the age bracket of 23 – 29 and another 29.4% are 30 – 34 years old, 23.5% belonged to age bracket 40 – 48 years, and 17.6% belonged to age bracket 35 – 39 years. The youngest respondent was 23 years old and the oldest was 48 years old, while the mean age was 34.35 (SD = 7.15).

In terms of number of years in teaching, 8 or almost half (47.1%) of the teachers have been teaching logic for 2 years, 3 teachers (29.4%) have taught for 3 years, 2 (11.8%) for 1 year only and 1 (5.9%) have been teaching logic for 5 and 7 years, respectively. The teachers have been teaching for an average period of 2.65 or almost 3 years (SD=1.46). It can be noted that respondents have taught mathematics since the implementation of the K to 12 Basic Education Program, while others got their experience in teaching logic in their past teaching experience in college.

Research Instruments

To gather pertinent data and information for this study, the researcher used a survey questionnaire and an assessment test.

Part I of the instrument included the level of knowledgeability, confidence level in teaching, factors affecting the competence level of the respondents in teaching the different content standards with the different learning competencies in logic in the general mathematics of senior high school, and appropriate teaching strategy for each learning competency they have incorporated in their Daily Lesson Log.

Part II is a researcher-made test covering all learning competencies included in the content standard in logic which is cited in the senior high school core curriculum guide. A total of 50 items were used in the original form. A pilot test was conducted to senior high school math teachers at Saint Mary's University Senior High School and three schools in Santiago City namely, Patria Sable Corpus College, Sagana Integrated High School, and Santiago City National High School.

After pilot testing of the instrument, an item analysis was conducted where items which were found to be moderately difficult or difficult and with satisfactory or highly satisfactory discrimination were retained, items found to be poorly discriminating were discarded and items which were moderately difficult and poorly discriminating were revised. The test was also subjected to reliability testing. The obtained Cronbach alpha coefficient is 0.857 which implies good internal consistency. The final assessment test includes 35 items that composed of two types, multiple choice and supply test.

Treatment of Data

In descriptively analyzing the data in this study, mean, standard deviation and Pearson r were computed. The level of knowledgeability on the different content standards in logic was measured from the ratings made by the respondents on the items and was described as *outstanding*, *very satisfactory*, *satisfactory*, *fair* and *poor*. The result of the assessment test was interpreted using the table below:

Table 3. ***Level of Content Knowledge in Logic***

Mean Percent Score	**Level**
0.00 – 64.99	Poor
65.00 – 74.99	Fair
75.00 – 84.99	Satisfactory
85.00 – 94.99	Very Satisfactory
95.00 – 100.00	Outstanding

Source: Modified from Mangawil (2007)

The respondents' level of confidence in teaching the content standards in logic was based on the ratings on the items as *very confident*, *confident*, *moderately confident*, *slightly confident*, *not confident and extremely not confident* as shown in Table 4.

Table 4. ***Level of Confidence in Teaching Logic***

Rating	**Level**
1.00 – 1.49	Extremely Not Confident
1.50 – 2.49	Not Confident
2.50 – 3.49	Slightly Confident
3.50 – 4.49	Moderately confident
4.50 – 5.59	Confident
5.50 – 6.00	Very Confident

The extent by which the different content standards put limit on their competence as logic teachers may be described as *to almost none*, *small extent*, *some extent*, *moderate extent*, *large extent* and *very large extent*. The level and the corresponding rating are reflected in Table 5.

Table 5. Extent of Limit on the Competence in Teaching Logic

Rating	**Level**
1.00 – 1.49	Almost None
1.50 – 2.49	Small Extent
2.50 – 3.49	Some Extent
3.50 – 4.49	Moderate Extent
4.50 – 5.49	Large Extent
5.50 – 6.00	Very Large Extent

The Pearson moment coefficient of correlation r was used to relate the teachers' number of years of teaching with content knowledge and confidence in teaching logic.

The topics covered in the proposed supplementary materials in teaching logic were based on the competencies in which the respondents performed poorly in the assessment test.

RESULT AND DISCUSSION

Part 1. Profile of the Senior High School Mathematics Teachers

Content Knowledge of the Teacher Respondents

Table 6 shows the level of content knowledge of the respondents as revealed by the result of the assessment test in logic.

Table 6. Level of Content Knowledge in the Different Learning Competencies

Content Standards	Learning Competencies	Mean	SD	Level of Content Knowledge
Key concepts of propositional logic; syllogisms and fallacies	1. Illustrates a proposition	74.51	18.74	Fair
	2. Symbolizes propositions	60.79	21.20	Poor
	3. Distinguishes between simple and compound propositions	94.12	16.61	Very Satisfactory
	4. Performs the different types of operations on propositions	94.12	10.93	Very Satisfactory
	5. Determines the truth values of propositions	77.65	17.15	Satisfactory
	6. Illustrates the different forms of conditional propositions	77.13	14.42	Satisfactory
	7. Illustrates different types of tautologies and fallacies	54.90	15.33	Poor
	8. Determines the validity of categorical syllogisms	52.94	26.51	Poor
	9. Establishes the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies.	16.18	9.65	Poor
Appropriately apply a method of proof and disproof in real- life situations	10. Illustrates the different methods of proof (direct and indirect) and disproof (indirect and counterexample).	37.65	20.94	Poor

Content Standards	Learning Competencies	Mean	SD	Level of Content Knowledge
	11. Justifies mathematical and real-life statements using the different methods of proof and disproof	44.12	22.65	Poor
Overall		54.07	11.68	Poor

Legend: Poor: 1.00 – 64.99, Fair: 65.00 – 74.99, Satisfactory: 75.00 – 84.99, Very Satisfactory: 85.00 – 94.99, Outstanding: 95.00 – 100

Table 6 reveals that the teacher-respondents are *very satisfactory* on distinguishing simple and compound propositions and on performing different types of operations on propositions; *satisfactory* on determining the truth value and on illustrating different forms of conditional propositions; *fair* on illustrating propositions; and *poor* in symbolizing propositions, illustrating different types of tautologies and fallacies, determining the validity of categorical syllogisms, establishing the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies, illustrating the different methods of proof (direct and indirect) and disproof (indirect and counterexample) and in justifying mathematical and real-life statements using the different methods of proof and disproof.

It is for the reason that three modules were prepared in Section 3 to address the following content difficulties shown in Table 7.

Table 7. Content of Proposed Supplementary Materials in Logic

Content	Module
Symbolizing Propositions	1
Logical Fallacies, Tautologies, Contradiction and Contingency Validity of categorical syllogisms	2
Different Methods of Proof and Disproof	3

Table 8. Over-all Level of Content Knowledge

Percent Score	Level of Content Knowledge	Frequency	Percent
0.00 – 64.99	Poor	13	76.5
65.00 – 74.99	Fair	2	11.8
75.00 – 84.99	Satisfactory	2	11.8
Total		17	100
Mean=54.07, SD=11.68, Level: Poor			

Legend:

Poor: 0.00 – 64.99, Fair: 65.00 – 74.99, Satisfactory: 75.00 – 84.99, Very Satisfactory: 85.00 – 94.99, Outstanding: 95.00 – 100

Table 11 reveals that 76.5% of the respondents were *poor* and an equal percent of 11.8% of the respondents have *fair* and *satisfactory* ratings. Many of the respondents did not have answers on the proving part of the assessment. The overall rating of 54.07% indicates that the respondents are generally *poor* in terms of their content knowledge in logic.

Teachers who have inadequate meaningful mathematical content knowledge and/or poor attitudes toward the subject often exacerbate the problems that students experience in learning (Dullas, 2003).

Confidence in Teaching Logic

Table 9 below shows the level of confidence of the respondents in teaching the different learning competencies in logic.

Table 9. Confidence in Teaching Logic in the Different Learning Competencies

Content Standards	Learning Competencies	Mean	SD	Level of Confidence
	Illustrates a proposition	5.47	.800	Confident

Content Standards	Learning Competencies	Mean	SD	Level of Confidence
Key concepts of propositional logic; syllogisms and fallacies	Symbolizes propositions	5.00	.707	Confident
	Distinguishes between simple and compound propositions	4.76	.664	Confident
	Performs the different types of operations on propositions	4.65	.493	Confident
	Determines the truth values of propositions	4.53	.800	Confident
	Illustrates the different forms of conditional propositions	4.35	.996	Moderately Confident
	Illustrates different types of tautologies and fallacies	4.00	4.00	Moderately Confident
	Determines the validity of categorical syllogisms	3.88	1.453	Moderately Confident
	Establishes the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies.	3.35	1.618	Slightly Confident
Appropriately apply a method of proof and disproof in real- life situations	Illustrates the different methods of proof (direct and indirect) and disproof (indirect and counterexample).	2.41	1.543	Not Confident
	Justifies mathematical and real-life statements using the different methods of proof and disproof	2.41	1.543	Not Confident
Overall		4.07	1.33	Moderately Confident

Legend:

Not Very Confident: 1.00 – 1.49, Not Confident: 1.50 – 2.49, Slightly Confident: 2.50 – 3.49, Moderately Confident: 3.50 – 4.49, Confident: 4.50 – 5.59, Very Confident: 5.50 – 6.00

Table 12 reveals that the overall level of confidence of the senior high school Mathematics teachers in teaching the learning competencies included in the content standard of logic in the general mathematics subject is *moderate*. In terms of the specific learning competencies, they are *confident* in teaching propositions, symbolizing propositions, distinguishing simple and compound propositions, performing different types of operations in propositions and determining truth values of a given proposition; *moderately confident* in teaching how to illustrate different forms of conditional propositions, illustrating different forms of conditional propositions, different types of tautologies and fallacies and determining the validity of categorical syllogisms; *slightly confident* in teaching how to establish the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies; lastly, they are *not confident* in teaching how to illustrate the different methods of proof and disproof and on how to justify mathematical and real-life statements using the different methods of proof and disproof.

The frequency distribution of the respondents according to the level of confidence in teaching logic is reflected in Table 10.

Table 10. Overall Level of Confidence in Teaching Logic

Level of Confidence	Frequency	Percent
Slightly Confident	3	17.6
Moderately Confident	9	52.9
Confident	5	29.4
Total	17	100
Mean = 4.07; SD = .75, Level: Moderately confident		

Legend:

Extremely Not Confident: 1.00 – 1.49, Not Confident: 1.50 – 2.49, Slightly Confident: 2.50 – 3.49, Moderately confident: 3.50 – 4.49, Confident: 4.50 – 5.59, Very Confident: 5.50 – 6.00

Table 13 reveals that that more than half (9 or 52.9%) of the respondents are *moderately confident* in teaching logic and followed respectively by 5 (29.4%) of them who are *confident*, and 3 (17.6%) who are *slightly confident*. It has a mean of 4.07 (SD=.75) corresponding to an overall *moderately confident*.

The over-all confidence level of the respondents in teaching logic is *moderate*. This may be attributed to the relatively low number of years of teaching experience in logic. There is a need for the teachers to strengthen their confidence by gaining additional knowledge and skills to confidently teach the topic.

Self-confidence towards teaching different topics in mathematics specifically logic is an important part to successes in mathematics achievement. Zan and Di Martino (2014) mentioned that low self-confidence toward mathematics defeats the purpose of learning in mathematics.

According to Kenny (2010) and Kind (2009), three major variables that contribute to enhancing and strengthening confidence for teachers are increasing subject matter knowledge, enhancing the readiness and preparation, and engaging teachers in various professional development opportunities to build their confidence.

Difficulties Encountered in Teaching Logic

Table 11 shows the frequency distribution of the respondents in terms of the different difficulties they have encountered in teaching logic.

Table 11. Frequency Distribution in terms of the Difficulties Encountered in Teaching Logic

Difficulties	Frequency	Percent
Did not have time to discuss the topic logic at all.	9	52.9
There is not much time to further discuss the different learning competencies.	5	29.4
Lack of time due to many seminars/professional development activities attended by the teachers.	2	11.8
Find difficulty in teaching the last competency which is to justify mathematical and real-life statements using different methods of proof and disproof.	1	5.9
Total	17	100

Most of the respondents (9 or 52.9%) listed that their difficulty is that, they did not have time to discuss the topic logic at all while five of them indicated that there is no plenty of time to further discuss the different learning competencies. It can be inferred that the 11 learning competencies on logic may be too many to be accomplished for a period of 16 hours or 4 weeks.

Two of the respondents stated that their difficulty arises from the lack of time due to many seminars they have attended and lastly, one of them expressed that she found it difficult to teach the last competency which is to justify mathematical and real life statements using different methods of proof and disproof.

As can be seen from the answers of the respondents, insufficient time to discuss topics in logic is a main difficulty arising from disruptions of classes due to their attendance to seminars and other professional development activities. The follow-up interview revealed that the disruptions or suspensions of classes are also caused by typhoons, town and provincial fiestas, and sports activities like provincial meet.

Factors that Limit the Competence in Teaching Logic

Table 12 shows the extent to which the cited factors put limits on the competence of the respondents.

Table 12. ***Extent of the Factors Affecting Competence in Teaching Logic***

Factor	Mean	SD	Extent of Limit
1. Deficiency in knowledge of logic	3.18	1.629	Some Extent
2. Inability to motivate students	3.18	1.590	Some Extent
3. Difficulty in disciplining students	4.06	1.298	Moderate Extent
4. Difficulty in explaining and communicating to students	3.76	1.200	Moderate Extent
5. Inability to engage students in class activities	3.82	1.286	Moderate Extent
6. Poor ability in managing time	4.94	0.748	Large Extent
7. Limited teaching strategies	2.76	1.300	Some Extent
8. Shortage of other instructional materials for students' use	4.88	0.600	Large Extent

Factor	Mean	SD	Extent of Limit
9. Inadequate library facilities	4.82	0.636	Large Extent
10. Inadequate time to perform lesson plans and teaching aids	4.94	0.772	Large Extent
11. High student/teacher ratio	4.88	0.600	Large Extent
12. Assessing students' needs, interest and difficulties	3.82	1.015	Moderate Extent
13. No knowledgeable co-teacher to seek help	3.47	1.419	Some Extent
14. Too much paper work	5.24	0.562	Large Extent
15. Unreasonable demands from school administrators/ coordinators	4.12	0.697	Moderate Extent
16. Dealing with parents of students	3.94	1.029	Moderate Extent
17. Students who come from a wide range of backgrounds	4.35	0.702	Moderate Extent
18. Heavy teaching load	4.88	0.697	Large Extent
19. Inability to relate with other co-teachers in school	3.00	1.00	Some Extent
Overall	4.11	0.988	Moderate Extent

Legend:

Almost None: 1.00 – 1.49, Small Extent: 1.50 – 2.49, Some Extent: 2.50 – 3.49,

Moderate Extent: 3.50 – 4.49, Large Extent: 4.50 – 5.49, Very Large Extent: 5.50 – 6.00

There are a number of factors that influence the performance of the teachers. As shown in Table 15, too much paper work, poor ability in managing time, shortage of other instructional materials for students' use, inadequate library facilities, inadequate time to perform lesson plans and teaching aids, high student/teacher ratio, heavy teaching load put a limit on their competence in handling logic to a *large extent*; while difficulty in disciplining students, difficulty in explaining and communicating to students, inability to engage students in class activities, assessing students' needs, interest and difficulties, unreasonable demands from school administrators/ coordinators, dealing with parents of students, students who come from a wide range of backgrounds limit the respondents to a *moderate extent*; and lastly, deficiency in knowledge of logic, inability to motivate students, limited teaching strategies, no knowledgeable co-teacher to seek help, inability to relate with other co-teachers in school put a limit on their competency to some extent.

Factors cited above involve students (inability to motivate students, difficulty in disciplining, in explaining and communicating to students, inability to engage students in class activities, assessing students' needs, interest and difficulties, and students who come from a wide range of backgrounds; teacher (limited teaching strategies, no knowledgeable co-teacher to seek help, and inability to relate with other co-teacher in school), time, facilities/physical resources (shortage of other instructional materials for students' use and inadequate library facilities), school system/administrators (high student/teacher ratio, too much paper work, heavy teaching load and unreasonable demands from school administrators/coordinators), and lastly, the parents of students.

Teacher competence is one aspect that affects the educational quality in many countries all over the world and it can be affected by some factors. Competent and professional teachers are teachers who are skilled in performing their duties, but they are influenced by many factors. Some are the expectations of the community, the particular school system in which the teacher is employed, the school itself the grade policies (paper works), the parents and the students, the individual teacher's beliefs about how children learn most effectively, how to teach in particular discipline or key learning area. The match between in individual teacher's beliefs about best teaching practice and whether they can personally meet these demands in the classroom is crucial. A teacher's own preferred ways of thinking, acting and seeing the world, learners and learning will also be affected by the availability of resources both human and physical (Hasan, 2004).

The evaluation study conducted by Bibi (2005) reveals that one of the reasons for weak competencies is the excess of periods being taught, ineffective teaching methods and habit of imposing their own views and ideas to students which are similar to this study.

Teaching Strategies in Logic

Table 13 shows the frequency distribution of the respondents in terms of the different strategies they have used in the different learning competencies included in the content standards of logic in the general mathematics.

Table 13. ***Strategies Used in Teaching the Different Learning Competencies***

Learning Competency	Strategies	Frequency	Percent
Illustrates a proposition	Watch and learn!	1	6.3
	Lecture	14	87.5
	Think-pair-share	1	6.3
Symbolizes propositions	Show me	1	6.3
	Lecture	8	50.0
	Games/Drills	4	25.0
	Flash cards	1	6.3
	Active learning	1	6.3
	Number heads	1	6.3
Distinguishes between simple and compound propositions	Pick, Read and Tell	1	6.3
	Lecture	13	81.3
	Active learning	1	6.3
	Number heads	1	6.3
Performs the different types of operations on propositions	Task cards	1	6.3
	Lecture	10	62.5
	Games/Drills	2	12.5
	Active learning	2	12.5
	Lecture with drills	1	6.3
Determines the truth values of propositions	Lecture	14	87.5
	Active learning	2	12.5
Illustrates the different forms of conditional propositions	Lecture	14	87.5
	Think and write the form	1	6.3
	Active learning	1	6.3
Illustrates different types of tautologies and fallacies	Lecture	14	87.5
	Think-pair-share	1	6.3
	Active learning	1	6.3
Determines the validity of categorical syllogisms	Lecture	13	81.3
	Show and Tell	1	6.3
	Problem solving	1	6.3
	Active learning	1	6.3
Establishes the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies	Lecture	11	68.8
	Case method	2	12.5
	Problem solving	3	18.8
	Lecture	12	75.0
	Think-pair-share	1	6.3

Learning Competency	Strategies	Frequency	Percent
Illustrates the different methods of proof (direct and indirect) and disproof (indirect and counterexample)	Games/Drills	1	6.3
	Student Board work	1	6.3
	Lecture with drills	1	6.3
Justifies mathematical and real-life statements using the different methods of proof and disproof	Lecture	8	50.0
	Collaborative/ Cooperative Learning	3	18.8
	Games/Drills	1	6.3
	Student Board work	3	18.8
	Problem solving	1	6.3
Total		16	100

It is evident that the lecture method is employed by majority of the teachers. Other strategies being used are active learning, think-pair-share, number heads, collaborative/cooperative learning, board work, problem solving, games/drills, lecture with drills, show and tell, case method, watch and learn, the use of flash cards, pick, read and tell, and think and write the form.

The study of Mangawil (2007) about the pedagogical content knowledge and praxis in subject matter, curriculum and teaching of pre-service biology teachers, lecture was the predominant strategy employed by the respondents.

Teaching strategies vary from one age group to another and on the learning style of the learners. No one method or strategy is the best. Teacher must understand that the amount of students' learning in a class also depends on their native ability of cognition and as well as their prior preparation (Jalbani, 2014).

2. Correlation between Number of Years in Teaching Logic and Content Knowledge and Confidence in Teaching Logic

Number of Years in Teaching Logic and Content Knowledge

Table 14 shows the correlations on number of years in teaching logic with content knowledge on the concepts and with methods included in the relative standards in logic of general mathematics of the Senior High School Grade 11.

Table 14. ***Correlations between Number of Years in Teaching Logic and Content Knowledge on Logic***

		Content Knowledge (Concepts)	Content Knowledge (Methods)
Number of Years in Teaching Logic	Pearson r	.467	.219
	Sig. (2-tailed)	.059	.399
	N	17	17

As shown in Table 17, the Pearson r for the respondents' content knowledge in logic and number of years in teaching is 0.467 (p-value = 0.059) for concepts and 0.219 (p-value = 0.399) for methods which both indicate positive correlation. These suggest that if a teacher has a greater number of years in teaching logic, the higher content knowledge he/she will have in logic concepts and methods. However, the p-values show that the correlations are not significant.

These findings are contrary to those of Lopez (1995), Mateo (2000), and Sadler (2013) who found that teachers with a longer period of teaching a particular subject are them more competent in terms of content knowledge.

Number of Years in Teaching Logic and Confidence in Teaching Logic

Table 15 shows the correlations on number of years in teaching logic and teacher's confidence in teaching the concepts and methods included in the relative standards in logic of general mathematics of the Senior High School Grade 11.

Table 15. ***Correlations between Number of Years in Teaching Logic and Teacher's Confidence in Teaching Logic***

		Teacher's Confidence in Teaching (Concepts)	Teacher's Confidence in Teaching (Methods)
Number of Years in Teaching Logic	Pearson r	.424	.291
	Sig. (2-tailed)	.090	.257
	N	17	17

As shown in Table 18, the Pearson r for the respondents' confidence in teaching logic and number of years in teaching is 0.424 (p-value = 0.090) for concepts and .291 (p-value = 0.257) which both indicate positive correlation. These values indicate that if a teacher has a greater number of years in teaching logic, the more confidence he/she will have in teaching logic concepts and methods. The more practice, the more mastery the teacher will have. If a teacher already knows exactly what he is about to teach then he has the confidence to teach the certain topic either it could be a concept or a method. However, the correlations are not significant.

From the study of Sadler (2013), experience in teaching by having years of teaching a specific course or a topic makes one more confident in teaching that certain topic. For exercising to teach a certain topic makes you develop teaching style which will boost self-confidence.

3.3. Supplementary Activities in Teaching Logic

Three modules were prepared to address the following content difficulties of the teachers namely, symbolize propositions; logical fallacies, tautologies, contradiction and contingency, validity of categorical syllogisms; and the different methods of proof and disproof.

Introduction

Logic is a language for reasoning. It is a collection of rules we use in doing logical reasoning. Human reasoning has been observed over centuries from at least the time of the Greeks, and patterns appearing in reasoning have been extracted, abstracted, and streamlined.

In logic, we are interested in true or false statements, and how the truth/falsehood of a statement can be determined from other statements.

Logic in the Senior High School Curriculum exposes learners to symbolic forms of propositions (or statements) and arguments. Through the use of symbolic logic, learners should be able to recognize equivalent propositions, identify fallacies, and judge the validity of arguments. The following lesson is an application of the rules of symbolic logic, as learners are taught to write their own justifications to mathematical and real-life statements (SHS Teaching Guide in General Mathematics, 2013).

Learning Outcome: At the end of the lesson, the learner is able to:

- illustrate and symbolize propositions - Code: M11GM-Iig-1

Lesson Outline:

Symbolizing proposition

If a proposition is compound, then it can be any of the following: negation, conjunction, disjunction, conditional, and biconditional.

The following examples are given:

p: Baguio is the summer capital of the Philippines. (T)

q: Five is a prime number. (T)

r: Rhombuses are squares. (F)

s: 10 + 4 = 14 (T)

- Negation of a proposition p is denoted by ~p, where ~ symbolizes "not".

Examples: ~p: Baguio is not the summer capital of the Philippines.

~q: Five is not a prime number.

- Conjunction of propositions p and q, called conjuncts is denoted by p ∧ q (read as *p and q*), where ∧ symbolizes "and". Some of the words used to denote conjunctions, aside from "and" are, "but", "yet", "while", and "even though".

Examples: p ∧ s: Baguio is the summer capital of the Philippines and 10 + 4 = 14.

q ∧ r: Five is a prime number and rhombuses are squares.

- Disjunction of propositions p or q is the compound proposition "p ∨ q" (read as *p or q*), where ∨ symbolize "or". In ordinary language, the word "or" has several meanings. In logic, there is inclusive disjunction (inclusive or) and exclusive disjunction (exclusive or). Unless stated, a disjunction is considered inclusive by default.

Examples: p ∨ q: Baguio is the summer capital of the Philippines or five is a prime number.

r ∨ s: Rhombuses are squares or 10 + 4 = 14.

- Conditional (implication) of propositions p and q is compound proposition "if p then q." Symbolically, p → q, where → means "if then", p is called hypothesis (or antecedent or premise) and q is called conclusion (or consequent or consequence).

Examples: p → q: If Baguio is the summer capital of the Philippines, then five is a prime number.

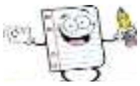
s → r: If 10 + 4 = 14 then rhombuses are squares.

- Biconditional proposition is a compound which is derived from two conditional propositions. It denoted by p ↔ q which is read as "p if and only if q".

Examples: p ↔ q: Baguio is the summer capital of the Philippines if and only if five is a prime number.

p ↔ s: Baguio is the summer capital of the Philippines if and only if 10 + 4 = 14.

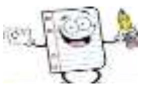
Let's Warm Up



Vocabulary and Concepts

Directions: Match each proposition in Column A to its respective symbol in Column B.

A	B
1. Biconditional	a. ∧
2. Negation	b. →
3. Conjunction (and)	c. ∨
4. Disjunction (or)	d. ↔
5. Conditional	e. +
	f. ~



Let's Do This!

A. Directions: Given the propositions below, write each of the following statements in symbolic form.

p: Mathematics is a challenging subject.
q: Manila is the capital of the Philippines.
r: 11 is a prime number.

1. If Manila is the capital of the Philippines, then Mathematics is a challenging subject.
2. It is not the case that Mathematics is a challenging subject.
3. Mathematics is not a challenging subject if and only if 11 is a prime number.
4. Manila is not the capital of the Philippines or Mathematics is not a challenging subject.
5. Mathematics is a challenging subject and Manila is the capital of the Philippines.
6. If 11 is a prime number then Mathematics is a challenging subject.
7. 11 is not a prime number.
8. Manila is the capital of the Philippines or Mathematics is a challenging subject.
9. If 11 is prime number and Mathematics is a challenging subject then Manila is the capital of the Philippines.
10. It is not the case that 11 is a prime number and Mathematics is a challenging subject.

B. Directions: Given the following propositions below, write each of the following statements in words.

p: Aristotle is a Philosopher.
q: Square is a rhombus.
r: 0 is an even number.

1. $p \wedge r$
2. $p \leftrightarrow (\sim r \vee q)$
3. $r \rightarrow q$
4. $\sim q \wedge r$
5. $p \vee r$
6. $\sim p \vee (q \rightarrow r)$
7. $\sim (p \wedge r)$
8. $(q \leftrightarrow r) \vee p$
9. $p \rightarrow q$
10. $p \leftrightarrow r$

Module 2: Logical Fallacies, Tautologies, Contradiction and Contingency Validity of Categorical Syllogisms

Logic is the study of reasoning in general, and in the first place, the concern is the difference between good reasoning and bad reasoning.

This second module discusses logical fallacies, tautologies, contradiction and contingency, also includes validity of categorical syllogisms. The focus of this is on reasoning about certain arguments. We understand an argument to be something that provides reasons to believe some claim. An argument is a list of statements, one of which is the conclusion and the others are the premises or assumptions of the argument. To give an argument is to provide a set of premises as reasons for accepting the conclusion.

Learning Outcome(s): At the end of the lesson, the learner is able to:

- illustrate different types of tautologies and fallacies;
- determine the validity of categorical syllogisms; and
- establish the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies.

Code: M11GM-Iii-1 - M11GM-Iih-3

Lesson Outline:

1. Logical Fallacies
2. Tautologies, Contradiction and Contingency
3. Validity of categorical syllogisms.

Logical Fallacies

A logical fallacy is an error in reasoning in logic of an argument. It is advantageous to know logical fallacies in order to avoid them in an argument. There are different types of fallacies that we might use to present our position. The following are the common types of fallacies with their corresponding examples.

1. Ad hominem

A personal attack which is a Latin for "against the man." Instead of advancing good sound reasoning, *ad hominems* replace logical argumentation with attack-language unrelated to the truth of the matter. An *ad hominem* is more than just an insult. It is an insult used as if it were an argument or evidence in support of a conclusion. Verbally attacking people proves nothing about the truth or falsity of their claims.

Example: You can't believe that a presidential candidate is going to lower taxes. He's a liar.

2. Strawman Fallacy

It is aptly named after a harmless, lifeless, scarecrow. It is an argument based on misrepresentation of an opponent's position.

Example: A mandatory helmet law for motorcycle drivers could never be enforced. You can't issue tickets to dead people.

3. **Appeal to Ignorance (*argumentum ad ignorantiam or argumentum ex silentio*)**

It is used as a major premise in support of an argument, it is liable to be a fallacious appeal to ignorance. It is an argument supporting a claim merely because there is no proof that it is wrong.

Example: If you can't say that there aren't Martians living in Mars, so it's safe for me to accept there are.

4. **False Dilemma/False Dichotomy**

This fallacy has a few other names: "black-and-white fallacy," "either-or fallacy," "false dichotomy," and "bifurcation fallacy." This line of reasoning fails by limiting the options to two when there are in fact more options to choose from.

Example: If you don't vote for this candidate you must be anti-Christ.

5. **Slippery Slope** (or **snowball/domino theory**)

An argument which claims a sort of chain reaction, usually ending in some extreme and often ludicrous outcomes will happen, but there is really not enough evidence for such assumption.

Example: If I fail Logic, I won't be able to graduate. If don't graduate, I probably won't be able to get a job, and may very well end up like a beggar.

6. **Circular Argument (*petitio principii*)**

A premise of an argument presupposes the truth of its conclusion; meaning the argument takes for granted what it's supposed to prove.

Circular arguments are also called *Petitio principii,* meaning "Assuming the initial [thing]" (commonly mistranslated as "begging the question"). You can recognize a circular argument when the conclusion also appears as one of the premises in the argument.

Example: God exists because the Holy Bible says so. The Holy Bible is true. Therefore, God exists.

7. **Hasty Generalization**

It is an argument that a general conclusion on a certain condition is always true based on insufficient or biased evidence.

Example: My friend says that Mathematics is hard, and the one I'm enrolled in is hard, too. All Mathematics classes must be hard.

8. **Red Herring**

An argument which introduces a topic related to the subject at a hand. It is diversionary tactic to avoid key issues, often a way of avoiding opposing argument rather than addressing them.

Example: I know I forget to clean the toilet yesterday. But, nothing I do pleases you.

9. **Fallacy of division**

Argument assuming that something true of a thing must also be true of all or some of its parts.

Example: The University of the Philippines is the best university in the country. Therefore, every student from the UP is better than a student of any other university in the country.

10. **Appeal to Authority (*argumentum ad verecundiam*)**

An argument that occurs when we accept or reject a claim merely because of the sources or authorities who made their positions on a certain argument.

Example: The government should not impose death penalty. Many respected people such as the former Secretary of Justice, have publicly stated her opposition to it.

11. **Equivocation**

This argument comes from the roots "equal" and "voice" and refers to two-voices; a single word can say two different things. It happens when a word, phrase, or sentence is used deliberately to confuse, deceive, or mislead by sounding like it's saying one thing but actually saying something else.

Example: Some real numbers is less than any number. Therefore, some real number is less than itself.

12. **Appeal to Pity (*argumentum ad misericordiam*)**

Argumentum ad misericordiam is Latin for "argument to compassion". Like the *ad hominem* fallacy above, it is a fallacy of relevance. Personal attacks, and emotional appeals aren't strictly relevant to whether something is true or false.

Example: The City Engineer is a vital part of this city. If he is sent to prison, the city and his family will suffer. Therefore, you must find in your heart to forgive him.

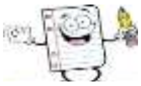
13. Bandwagon Fallacy

It assumes something is true (or right, or good) because other people agree with it. A couple different fallacies can be included under this label, since they are often indistinguishable in practice. The *ad populum* fallacy (Lat., "to the populous/popularity") is when something is accepted because it is popular. The *concensus gentium* (Lat., "consensus of the people") is when something is accepted because the relevant authorities or people all agree on it. The status appeal fallacy is when something is considered true, right, or good because it has the reputation of lending status, making you look "popular," "important," or "successful."

Example: Most Filipinos like soda. Therefore, soda is good.

14. Appeal to Force (or *Argumentum Ad Baculum*)

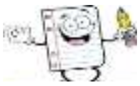
This argument attempts to establish a conclusion by threat or intimidation. Example: If you don't believe in God, you won't go to heaven.

Let's Warm Up! 

Vocabulary and Concepts

Directions: Explain the following types of fallacies based on your own understanding, through an example which is different from the above examples.

1. Ad hominem
2. Strawman Fallacy
3. Appeal to Ignorance (*argumentum ad ignorantiam or argumentum ex silentio*)
4. False Dilemma/False Dichotomy
5. Slippery Slope (or snowball/domino theory)
6. Circular Argument (*petitio principii*)
7. Hasty Generalization
8. Red Herring
9. Fallacy of division
10. Appeal to Authority (*argumentum ad verecundiam*)
11. Equivocation
12. Appeal to Pity (*argumentum ad misericordiam*)
13. Bandwagon Fallacy
14. Appeal to Force (or Argumentum Ad Baculum)

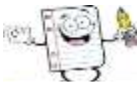
Let's Do This! 

Directions: Indicate the type of fallacy in the following arguments/passages.

1. We have no evidence that Greek gods do not exist. Therefore, they must exist.
2. If the president will allow death penalty, then eventually there would be no cases of rape.
3. You've been told by an elder not to go outside while it's raining because you will get sick.
4. Lorna is worried about her sister who is about to get married. She investigated the past of the groom-to-be and found out that he had five girlfriends in the past where he cheated on three of them. Lorna told her sister that her groom-to-be will do it again.
5. If one loves algebra, then he loves mathematics.
6. Practice makes perfect but nobody's perfect, so why practice?
7. All movie actresses are rude. I asked a movie star at Star Magic studio for her autograph, and she told me to get lost.
8. Sexual assault is immoral because it is just plain wrong.
9. Water extinguishes fire. Hydrogen is part of water. Therefore, a hydrogen bomb will extinguish fire.

10. People know that evolution is false because humans did not evolve from monkeys.
11. Of course, God answers prayers! After all, sometimes the answer may be “no” or “wait awhile” so you can’t prove he doesn’t answer them.
12. Abortion is murder, since killing of a baby is an act of murder.
13. Every Siamese cat I’ve met has been temperamental and so they all are.
14. Allowing parents to use vouchers that enable their children to attend private instead of public schools would improve the quality of education. After all, vouchers would promote competition, and competition is what makes businesses excel.
15. If I were you, I’d rethink your criticisms of this class before I need to turn in final grades. Just some friendly advice.”

Let’s Act it Out!



(Group Activity with 5 members)

1. Look for advertisements, blogs, post, tweets, news articles which you think are fallacious arguments. Make a booklet of them.
2. Make a blog or an advertisement that has a fallacious arguments/passage.

Tautology

The famous saying 'I cannot tell a lie' may come to mind when studying tautologies. A **tautology** is a formula which is "always true" --- that is, it is true for every assignment of truth value to its simple components. You can think of a tautology as a *rule of logic*.

The opposite of a tautology is a **contradiction**, a formula which is "always false". In other words, a contradiction is false for every assignment of truth values to its simple components.

Example: Show that $(p \rightarrow q) \vee (q \rightarrow p)$ is a tautology.

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \vee (q \rightarrow p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

The last column contains only T's. Therefore, the formula is a tautology.

If you construct a truth table for a statement and all of the column values for the statement are true (T), then the statement is a tautology because it is always true!

The statement 'I will either get paid or not get paid' is a tautology since it is always true. Most of the time the logic statements or arguments that we are trying to analyze are more complicated than this, or we are only given the symbolic representation of the statement and not the statement itself.

If you are given a statement and want to determine if it is a tautology, then all you need to do is construct a truth table for the statement and look at the truth values in the final column. If all of the values are T (for true), then the statement is a tautology.

Let's look at an example.

'I will either get paid or not get paid', use *p* to represent the statement 'I will get paid' and not *p* (written ~*p*) to represent 'I will not get paid.'

p: I will get paid

~*p*: I will not get paid

So, $p \vee {\sim}p$: I will either get paid or not get paid.

A truth table for the statement would look like:

Truth Table for p ∨ ~p		
p	~p	p ∨ ~p
T	F	T
F	T	T

Looking at the final column in the truth table, you can see that all the truth values are T (for true). Whenever all of the truth values in the final column are true, the statement is a tautology. So, our statement 'I will either get paid or not get paid' is always a true statement, a tautology.

List of Tautologies

1. p ∨ ~p	Law of the excluded middle
2. ~ (p ∧ ~ p)	Contradiction
3. [(p → q) ∧ ~q]→~p	*Modus tollens*
4. ~~p ↔ p	Double Negation
5. [(p→q) ∨ (q → r) → (p→r)	Law of syllogism
6. (p ∧ q) → p (p ∧ q) → q	Decomposing a conjunction Decomposing a conjunction
7. p → (p ∨ q) q → (p ∨ q)	Constructing a disjunction Constructing a disjunction
8. (p ↔ q) ↔ [(p→q) ∧ (q →p)]	Definition of the biconditional
9. (p ∧ q) ↔ (q ∧ p)	Commutative law for conjunction
10. (p ∨ q) ↔ (q ∨ p)	Commutative law for disjunction
11. [(p ∧ q) ∧ r] ↔ [p ∧ (q ∧ r)]	Associative law for conjunction
12. [(p ∨ q) ∨ r] ↔ [p ∨ (q ∨ r)]	Associative law for disjunction
13. ~ (p ∨ q) ↔ (~p ∧ ~q)	DeMorgan’s law
14. ~ (p ∧ q) ↔ (~p ∨ ~q)	DeMorgan’s law
15. [p ∧ (q ∨ r)] ↔ [(p ∧ q) ∨ (p ∧ r)]	Distributivity
16. [p ∨ (q ∧ r)] ↔ [(p ∨ q) ∧ (p ∨ r)]	Distributivity
17. (p → q) ↔ (~q → ~p)	Contrapositive
18. (p → q) ↔ (~p ∨ q)	Conditional disjunction
19. [(p ∨ q) ∧ ~p] → q	Disjunctive syllogism
20. (p ∨ p) ↔ p	Simplification
21. (p ∧ p) ↔ p	Simplification

Let’s Write Up!

(Group Activity with 5 members)

1. Extract twenty (20) examples of tautology from famous speeches, literary pieces, and popular music/songs.
2. Write a literary piece or a song or a speech consisting of an example of a tautology to be shared in the class.

Categorical Syllogisms

A syllogism is a deductive argument in which a conclusion is inferred from two premises. A categorical syllogism is a deductive argument consisting of three categorical propositions that contain exactly three terms, each of which occurs in exactly two of the constituent propositions. The major premise is the premise that contains the major term, while the minor premise is the premise that contains the minor term. The conclusion is the third proposition whose meaning and truth are implied in the premises.

The subject of the conclusion is the minor term while the predicate will be the major term. Middle term is the remaining term which does not (and cannot) appear in the conclusion.

Example:

All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.

Major term is “mortal” since it is the predicate of the conclusion.

Minor term is “Socrates” for it is the subject of the conclusion.

Middle term is “man” or “men” for it does not appear in the conclusion and the only remaining term.

Rules of Syllogism

In determining the validity of categorical syllogisms, the following rules must be followed. It is important to remember that all of the eight rules of syllogism must be met or satisfied for the argument or syllogism to be valid. If at least one of the eight rules of syllogism is violated, then the argument or syllogism is considered invalid.

The eight rules of syllogism are as follow:

1. **There should only be three terms in the syllogism, namely: the major term, the minor term, and the middle term. T**he meaning of the middle term in the first premise should not be changed in the second premise; otherwise, the syllogism will have four terms.
2. **The major and the minor terms should only be universal in the conclusion if they are universal in the premises.** In other words, if the major and the minor terms are universal in the conclusion, then they must also be universal in the premises for the argument to be valid. Hence, if the major and minor terms are particular in the conclusion, then rule #2 is not applicable.
3. **The middle term must be universal at least once.** Or, at least one of the middle terms must be universal.
4. **If the premises are affirmative, then the conclusion must be affirmative.**
5. **If one premise is affirmative and the other negative, then the conclusion must be negative.**
6. **The argument is invalid whenever the premises are both negative.** This is because we cannot draw a valid conclusion from two negative premises.
7. **One premise at least must be universal.**
8. **If one premise is particular, then the conclusion must be particular.**

Let’s Warm Up!

Directions: Identify the major, minor and middle term of the given syllogism. Determine its validity.

All who completely pay taxes are honest.

No Politician is honest.

Therefore, no politician pays taxes completely.

Let’s Do This!

A. Directions: Explain by giving what rule is violated by the following invalid arguments.

1. All parts of a living organism are inside the human body.

The fetus is inside the (mother’s) human body.

Therefore, the fetus is a part of the living organism (mother’s body).

2. Nothing easy is worthwhile.

Nothing good is easy.

Therefore, nothing good is worthwhile.

3. Mathematicians know what mathematics is.

No philosopher is a mathematician.

No philosopher knows what mathematics is.

4. No idiot is rational.

Kurt is not an idiot.

Therefore, Kurt is rational.

5. Some students are lazy.

But some Filipinos are students.

Therefore, some Filipinos are not lazy.

B. Directions: Indicate the validity of each argument. Justify your answer.

1. All patriots are voters.

Some citizens are not voters.

Therefore, some citizens are not patriots.

2. All human action is conditioned by circumstances.

All human action involves morality.

Therefore, all that involves morality is conditioned by circumstances.

3. All men are cheater.

Kakai is not a man.

Therefore, Kakai is not a cheater.

4. No man is perfect.

Some men are presidents.

Therefore, some presidents are not perfect.

5. Some lawyers are professionals.

But no criminals is professional.

Therefore, some criminals are lawyers.

Let's Explore!

(Group Activity with 5 members)

Directions: In a one-week time, observe people and things around you. List down all the syllogisms you have heard and seen. Make a diary about your observation starting from the first up to the last day.

Module 3: Methods of Proof and Disproof

Module 3 includes the different methods of proof and disproof. In giving or stating proofs, one must be knowledgeable of the different basic concepts of all branches of mathematics like geometry.

This module discusses another method to establish the validity of an argument by creating what mathematicians call a proof. In addition, we are also going to look at how to use proof in order to show that a mathematical proposition is true.

Learning Outcome(s): At the end of the lesson, the learner is able to:

- illustrate different methods of proof (direct and indirect) and disproof (indirect and counterexample); and
- justify mathematical and real-life statements using the different methods of proof and disproof.

Code: M11GM-Iij-1 – M11GM-Ij-2

Lesson Outline:

1. Methods of Proof (Direct and Indirect)
2. Methods of Disproof (Indirect and Counterexample)
3. Proving Mathematical and real-life statements using the different methods of proof and disproof.

A. Direct Proof

A direct proof is a way of showing that a given statement is true or false by a straightforward combination of facts, axioms, theorems, and existing lemmas. It assumes that a hypothesis is true, and then uses a series of logical inferences from previous statements to prove that its conclusion is true.

Follow these steps in performing a direct proof:

1. Assume the hypothesis to be true.
2. Use definitions, properties, axioms, theorems, and postulates to make a series of inferences that eventually prove the conclusion to be true.
3. State that by direct proof, the conclusion of the statement must be true.

Example: Prove that "If n is an odd integer, then n^2 is also an odd integer".

Proof: If n is an odd integer, then $n = 2k + 1$ for some integer k.

Showing that n^2 is an odd integer,

$n^2 = (2k + 1)^2$

$= 4k^2 + 4k + 1$

$= 2(2k^2 + 2k) + 1$

Since n^2 is one more than twice $2k^2 + 2k$, therefore it is an odd integer.

B. Indirect Proof

An indirect proof (Latin, *reductio ad absurdum*) uses a contradiction to prove a hypothesis. It first assumes that the hypothesis is not true, and then using logical inferences to arrive at a contradiction, proving that the hypothesis must be true.

There are two methods of indirect proof: *proof of the contrapositive* and *proof by contradiction* where both methods start by assuming the denial of the conclusion.

Example: If a quadrilateral has three right angles, then it is a rectangle. In quadrilateral ABCD, $m\angle A = 90°$, $m\angle B = 90°$ and $m\angle C = 85°$. Then, ABCD is not a rectangle.

Assume that ABCD is a rectangle. Then it has three right angles. But since $\angle C$ is not a right angle, then the three right angles must be $\angle A$, $\angle B$, and $\angle D$.

Solving for $m\angle C$ in the equation $m\angle A + m\angle B + m\angle C + m\angle D = 360°$ (given $m\angle A = 90°$, $m\angle B = 90°$ and $m\angle D = 90°$), we find that $m\angle C = 90°$. This contradicts the fact that $m\angle C = 85°$, which is the given. Therefore, ABCD is not a rectangle.

Proof by Contradiction

This is done by assuming the opposite of the hypothesis (the one that is trying to prove) and using logical inferences to show that this leads to a contradiction.

Example: The sum of two even numbers is always even.

Proof: The opposite is "The sum of two even numbers is not always even".

Assume that a and b are even numbers and c is odd, hence, $2a + 2b = c$.

$2(a + b) = c$

$a + b = c/2$

Even and odd numbers are always integers, and the sum of two integers is always an integer. Both *a* and *b* are even and *c* is odd, which is not evenly divisible by 2. Hence, c/2 is not an integer.

Therefore, the original proposition is true, the sum of two even numbers is always even.

Proof of the Contrapositive

Any statement and its contrapositive are logically equivalent, but it is usually easier to prove the contrapositive of a statement. In a proof of the contrapositive, we assume that the conclusion is false and try to prove that the hypothesis is false.

Example: If *mn* is even, then neither *m* or *n* is even.

Proof: Assume *m* and *n* are both odd.

Since the product of two odd numbers is odd, therefore *mn* is odd.

Disproof

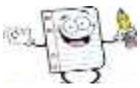
Disproof by Counterexample

This is a method of proving that a given statement is NOT true.

Method: We find one example where the statement does not hold and we have done enough to show that it is not always true.

Example: Show that the statement: $a > b \Rightarrow a^2 > b^2$ is not true.

Any pair of negative numbers with a > b will do, so our counterexample could be $a = -1$, $b = -2$ (so $a > b$), then $a^2 = 1$ and $b^2 = 4$, so $a^2 < b^2$.



Let's Warm Up!

Directions: For each of the statements below, determine the method of proof or disproof. Fully explain your answer.

Statements	**Answer**
1. 12 + 18 = 32	
2. Which day is the Chinese New Year this year?	
3. Today is a sunny day.	
4. 4x + 2y = 5	
5. Roses are red.	

Let's Do This!

Directions: Prove the following by the given method of proof.

1. Prove by contradiction: "The difference of any rational number and any irrational number is irrational."

2. Prove by counterexample: If n is an integer and n^2 is divisible by 4, then n is divisible by 4.

Let's Think and Compare!

Directions: In the table below, write your answer. Find a partner and compare your answers.

1. Show that "if x and y are sum of two integer squares, then xy is also a sum of two integer squares."
Your Answer:
2. Prove or disprove by mathematical induction.

6 divides $n^3 - n$ for $n \geq 2$.
Your Answer:
3. Prove or disprove by counterexample: “For any real numbers a and b, If $a = a^2 = b^2$ then $a = b$.”
Your Answer:

Let’s Write Up! 

Directions: Read the list of claims below. Write a journal by giving your ideas whether these claims are valid? How would you disprove them by counterexample? Reflect.

Note: The counterexample is at least one and should be a “well-known” entity as well.

1. Engineers are good in math. 2. Not intelligent people do not succeed. 3. Doing mathematics is difficult.

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RESULT AND DISCUSSION

Findings

1. *a. Content Knowledge*

The teacher respondents have *very satisfactory* level of content knowledge on distinguishing simple and compound propositions and on performing different types of operations on propositions; *satisfactory* on determining the truth value and illustrating different forms of conditional propositions; *fair* on illustrating propositions; and *poor* in symbolizing propositions, illustrating different types of tautologies and fallacies, determining the validity of categorical syllogisms, establishing the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies, illustrating the different methods of proof (direct and indirect) and disproof (indirect and counterexample) and in justifying mathematical and real-life statements using the different methods of proof and disproof. Overall, the performance of the respondents is *poor* in terms of content knowledge.

b. Level of Confidence in Teaching Logic

The overall level of confidence of the senior high school Mathematics teachers in teaching the learning competencies included in the content standard of logic in the general mathematics subject is *moderate*. In terms of the specific learning competencies, they are *confident* in teaching propositions, symbolizing propositions, distinguishing simple and compound propositions, performing different types of operations in propositions and determining truth values of a given proposition; *moderately confident* on teaching how to illustrate different forms of conditional propositions, illustrating different forms of conditional propositions, different types of tautologies and fallacies and determining the validity of categorical syllogisms; *slightly confident* in teaching how to establish the validity and falsity of real-life arguments using logical propositions, syllogisms, and fallacies; lastly, they are *not confident* in teaching how to illustrate the different methods of proof and disproof and on how to justify mathematical and real-life statements using the different methods of proof and disproof.

c. Difficulties Encountered in Teaching Logic

More than half of the respondents listed that they did not have time to discuss the topic logic at all, the others indicated that there is no plenty of time to further discuss the different learning competencies, their difficulty arises from the lack of time due to many seminars they have attended and, one teacher expressed that she found it difficult to teach the last competency which is to justify mathematical and real life statements using different methods of proof and disproof.

d. Limit in Competence

There are some factors that affect the teacher respondents' competence in teaching logic. Too much paper works, poor ability in managing time, shortage of other instructional materials for students' use, inadequate library facilities, inadequate time to perform lesson plans and teaching aids, high student/teacher ratio, heavy teaching load limit their competence to a *large extent*; while difficulty in disciplining students, difficulty in explaining and communicating to students, inability to engage students in class activities, assessing students' needs, interest and difficulties, unreasonable demands from school administrators/ coordinators, dealing with parents of students, students who come from a wide range of backgrounds limit the respondents' competence to a *moderate extent*; and lastly, deficiency in knowledge of logic, inability to motivate students, limited teaching strategies, no knowledgeable co-teacher to seek help, inability to relate with other co-teachers in school limit their competency to *some extent*.

e. Strategies in Teaching the Different Learning Competencies

Lecture method is the strategy used by three-fourths of the teachers' respondents in teaching the different learning competencies in logic. Other strategies used are active learning, think-pair-share, number heads, collaborative/cooperative learning, board work, problem solving, games/drills, lecture with drills, show and tell, case method, watch and learn, the use of flash cards, pick, read and tell, and think and write the form.

2. a. The number of years in teaching logic and the teachers' content knowledge on logic have a positive but non-significant correlation.

b. The number of years in teaching and the respondents' confidence in teaching logic have a positive but non-significant correlation.

3. The proposed supplementary activities in this study focused on symbolizing propositions, logical fallacies, tautologies, contradiction and contingency, validity of categorical syllogisms, and methods of proofs.

Conclusions

Based on the aforementioned findings, the following conclusions are derived:

1. The overall performance of the teachers indicates lack of competencies in terms of content knowledge. They know the basic concepts in logic but they find it hard to prove some mathematical and real-life statements using proof and disproof. The teachers have yet to develop full confidence in teaching logic. Lack of time was the main reason why logic is not usually discussed in class. Lecture was the primary strategy used in teaching logic. Varied factors affecting the teaching competence of the teachers include as too much paper works, heavy teaching load, inadequate time to perform lesson plans and teaching aids, poor ability in managing time, shortage of other instructional materials for students' use, inadequate library facilities, and high student/teacher ratio.
2. The teacher's content knowledge on logic and teacher's confidence in teaching logic are not correlated to the number of years of teaching logic.
3. The proposed supplementary material on symbolizing propositions, fallacies, tautology, syllogisms and methods of proof is seen as relevant to improve the teaching competency of the teachers.

Recommendations

On the basis of the key findings and established conclusions of the study, the following recommendations are advanced:

1. For curriculum planners to consider logic in the middle part of the semester and not in the last part.
2. For the developed supplementary activities on logic to be tried out.
3. For mathematics teachers to attend seminars and lectures to enhance the content knowledge in logic or to undertake further studies.
4. For future researchers to conduct the study in other schools, consider specific logic competencies.

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