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Observation on the Non-Homogeneous Binary Quadratic Diophantine Equation $6x^2 - 5y^2 = 4$.

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ABSTRACT

This paper deal with the problem of obtaining non-zero distinct integer solutions to the Non-homogeneous binary quadratic equation represented by the Pell–like equation $6x^2 - 5y^2 = 4$. different sets of integer solutions are presented. Employing the solutions of the above equation, integer solution for other choices of hyperbolas and parabolas are obtained. A special Pythagorean triangle is also determined.

Key words: Non-homogeneous, Binary quadratic, Pell-like equation, Hyperbola, Parabola, Integral solutions.

2010 hematics subject classification:11D09.

INTRODUCTION

The binary quadratic Diophantine equations (both homogeneous and non-homogeneous) are rich in variety $[1-6]$.In $[7-14]$ the binary quadratic non-homogeneous equations representing hyperbolas respectively are studied for their non-zero integral solution of still another interesting binary quadratic equation given by $6x^2-5y^2=4$. The recurrence relations satisfied by the solution x and y are given, also a few interesting properties among the solution are exhibited.

METHOD OF ANALYSIS

The Diophantine equation under consideration is

$$6x^2 - 5y^2 = 4 \qquad (1)$$

It is to be noted that (1) represent a hyperbola

Taking,

$$x=X+5T, \quad y=X+6T \qquad (2)$$

In (1), it reduces to the equation,

$$X^2 = 30T^2 + 4 \qquad (3)$$

The smallest positive integer solution (T_0, X_0) of (3) is

$$T_0=4,\ X_0=22$$

To obtain the other solution of (1), consider the pell equation

$$X^2 = 30T^2 + 1 \qquad (4)$$

Whose smallest positive integer solution is

$$\tilde{T}_0 = 2, \tilde{X}_0 = 11$$

The gerenal solution $(\tilde{T}_0, \tilde{X}_0)$ of (4) is given by

$$\tilde{X}_n + \sqrt{30}\tilde{T}_n = \left(11 + 2\sqrt{30}\right)^{n+1}$$, where n=0,1,2,… (5)

Since, irrational roots occur in pairs, we have,

$$\tilde{X}_n - \sqrt{30}\tilde{T}_n = \left(11 - 2\sqrt{30}\right)^{n+1}$$, where n=0,1,2,… (6)

From (5) and (6) solving for $(\tilde{T}_n, \tilde{X}_n)$ we have,

$$\tilde{T}_n = \frac{1}{2\sqrt{12}} g_n ; \tilde{X}_n = \frac{1}{2} f_n$$

Where

$$f_n = \left(11 + 2\sqrt{30}\right)^{n+1} + \left(11 - 2\sqrt{30}\right)^{n+1},$$

$$g_n = \left(11 + 2\sqrt{30}\right)^{n+1} - \left(11 - 2\sqrt{30}\right)^{n+1}$$

Applying Brahmagupta lemma between (T_0, X_0) and $(\tilde{T}_n, \tilde{X}_n)$, the general integer solution (T_{n+1}, X_{n+1}) of (3) are found to be

$$T_{n+1} = X_0 \tilde{T}_n + T_0 \tilde{X}_n$$

$$T_{n+1} = 2 f_n + \frac{11}{\sqrt{30}} g_n \qquad (7)$$

$$X_{n+1} = X_0 \tilde{X}_n + D T_0 \tilde{T}_n$$

$$X_{n+1} = 11 f_n + \frac{60}{\sqrt{30}} g_n \qquad (8)$$

Using (7) & (8) in (2), we have

$$x_{n+1} = X_{n+1} + 5T_{n+1}$$

$$x_{n+1} = 11 f_n + \frac{60}{\sqrt{30}} g_n \qquad (9)$$

$$y_{n+1} = X_{n+1} + 6T_{n+1}$$

$$y_{n+1} = 23 f_n + \frac{126}{\sqrt{30}} g_n \qquad (10)$$

Thus, (9) and (10) represent the integer solutions of the hyperbola (1)

A few numerical examples are given in the following Table: 1

Table: 1 Numerical values

n	x_n	y_n
0	42	46
1	922	1010
2	20242	22174

3	444402	486818
4	9756602	10687822

From the above table, we observe some interesting properties among the solutions which are presenting below:

i. x_n & y_n values are always even

ii. $x_n \equiv 0(\mathrm{mod}\,2)$

iii. $x_n + y_n \equiv 0(\mathrm{mod}\,2)$

Relations between solutions

- $10x_{n+3} - 220x_{n+2} + 10x_{n+1} = 0$
- $10y_{n+1} - x_{n+2} + 11x_{n+1} = 0$
- $10y_{n+2} - 11x_{n+2} + x_{n+1} = 0$
- $10y_{n+3} - 241x_{n+2} + 11x_{n+1} = 0$
- $220y_{n+1} - x_{n+3} + 241x_{n+1} = 0$
- $220y_{n+2} - 11x_{n+3} + 11x_{n+1} = 0$
- $220y_{n+3} - 241x_{n+3} + x_{n+1} = 0$
- $y_{n+2} - 12x_{n+1} + 11y_{n+1} = 0$
- $y_{n+3} - 264x_{n+1} - 241y_{n+1} = 0$
- $11y_{n+3} - 12x_{n+1} - 241y_{n+2} = 0$
- $10y_{n+1} - 11x_{n+3} - 241x_{n+2} = 0$
- $10y_{n+2} - x_{n+3} + 11x_{n+2} = 0$
- $10y_{n+3} - 11x_{n+3} + x_{n+2} = 0$
- $11y_{n+2} - 12x_{n+2} + y_{n+1} = 0$
- $y_{n+3} - 12x_{n+2} - 11y_{n+2} = 0$
- $11y_{n+1} - 11y_{n+3} - 264x_{n+2} = 0$
- $241y_{n+2} - 12x_{n+3} - 11y_{n+1} = 0$
- $241y_{n+3} - 264x_{n+3} - y_{n+1} = 0$
- $12y_{n+3} - 264y_{n+2} - 12y_{n+1} = 0$
- $y_{n+2} - 11y_{n+3} - 12x_{n+3} = 0$

Each of the following expressions represents a Nasty Number

- $\frac{1}{10}\left(2525x_{2n+2}-115x_{2n+3}+20\right)$
- $\frac{1}{220}\left(55435x_{2n+2}-115x_{2n+4}+440\right)$
- $\left(126x_{2n+2}-115y_{2n+2}+2\right)$
- $\frac{1}{11}\left(2766x_{2n+2}-115y_{2n+3}+22\right)$
- $\frac{1}{241}\left(60726x_{2n+2}-115y_{2n+4}+482\right)$
- $\frac{1}{11}\left(126x_{2n+4}-2525y_{2n+2}+22\right)$
- $\frac{1}{10}\left(55435x_{2n+3}-2525x_{2n+4}+20\right)$
- $\left(2766x_{2n+3}-2525y_{2n+3}+2\right)$
- $\frac{1}{11}\left(60726x_{2n+3}-2525y_{2n+4}+22\right)$
- $\frac{1}{241}\left(126x_{2n+4}-55435y_{2n+2}+482\right)$
- $\frac{1}{11}\left(2766x_{2n+4}-55435y_{2n+3}+22\right)$
- $(60726x_{2n+4}-55435y_{2n+4}+2)$
- $\frac{1}{12}\left(126y_{2n+3}-2766y_{2n+2}+24\right)$
- $\frac{1}{12}\left(2766y_{2n+4}-60726y_{2n+3}+24\right)$
- $\frac{1}{264}\left(126y_{2n+4}-60726y_{2n+2}+528\right)$

3. **Each of the following expressions represents a cubical integer**

- $\frac{1}{10}\left[2525x_{3n+3}-115x_{3n+4}+7575x_{n+1}-345x_{n+2}\right]$
- $\frac{1}{220}[55435x_{3n+3}-115x_{3n+5}+166305x_{n+1}-345x_{n+3}]$
- $[126x_{3n+3}-115y_{3n+3}+378x_{n+1}-345y_{n+1}]$
- $\frac{1}{11}[2766x_{3n+3}-115y_{3n+4}+8298x_{n+1}-345y_{n+2}]$

- $\frac{1}{241}[60726x_{3n+3} - 115y_{3n+5} + 182178x_{n+1} - 345y_{n+3}]$
- $\frac{1}{10}[55435x_{3n+4} - 2525x_{3n+5} + 166305x_{n+2} - 7575x_{n+3}]$
- $\frac{1}{11}[126x_{3n+4} - 2525y_{3n+3} + 378x_{n+2} - 7575y_{n+1}]$
- $[2766x_{3n+4} - 2525y_{3n+4} + 8298x_{n+2} - 7575y_{n+2}]$
- $\frac{1}{11}[60726x_{3n+4} - 2525y_{3n+5} + 182178x_{n+2} - 7575y_{n+3}]$
- $\frac{1}{241}[126x_{3n+5} - 55435y_{3n+3} + 378x_{n+3} - 166305y_{n+1}]$
- $\frac{1}{11}[2766x_{3n+5} - 55435y_{3n+4} + 8298x_{n+3} - 166305y_{n+2}]$
- $[60726x_{3n+5} - 55435y_{3n+5} + 182178x_{n+3} - 166305y_{n+3}]$
- $\frac{1}{12}[126y_{3n+4} - 2766y_{3n+3} + 378y_{n+2} - 8298y_{n+1}]$
- $\frac{1}{12}[2766y_{3n+5} - 60726y_{3n+4} + 8298y_{n+3} - 182178y_{n+2}]$
- $\frac{1}{126}[126y_{3n+5} - 60726y_{3n+3} + 378y_{n+3} - 182178y_{n+1}]$

4. Each of the following expression represents a Bi-quadratic Integer

- $\frac{1}{10}[2525x_{4n+4} - 115x_{4n+5} + 10100x_{2n+2} - 460x_{2n+3} + 60]$
- $\frac{1}{220}[55435x_{4n+4} - 115x_{4n+6} + 221740x_{2n+2} - 460x_{2n+4}1320]$
- $[126x_{4n+4} - 115y_{4n+4} + 504x_{2n+2} - 460y_{2n+2} + 6]$
- $\frac{1}{11}[2766x_{4n+4} - 115y_{4n+5} + 11064x_{2n+2} - 460y_{2n+3} + 66]$
- $\frac{1}{241}[60726x_{4n+5} - 115y_{4n+6} + 242904x_{2n+2} - 460y_{2n+4} + 1446]$
- $\frac{1}{10}[55435x_{4n+5} - 2525x_{4n+6} + 221740x_{2n+3} - 10100x_{2n+4} + 60]$
- $\frac{1}{11}[126x_{4n+5} - 2525y_{4n+4} + 504x_{2n+3} - 10100y_{2n+2} + 66]$
- $[2766x_{4n+5} - 2525x_{4n+5} + 11064x_{2n+3} - 10100y_{2n+3} + 6]$
- $\frac{1}{11}[60726x_{4n+5} - 2525y_{4n+6} + 242904x_{2n+3} - 10100y_{2n+4} + 66]$

- $\frac{1}{241}[126x_{4n+6} - 55435y_{4n+4} + 504x_{2n+4} - 221740y_{2n+2} + 1446]$
- $\frac{1}{11}[2766x_{4n+6} - 55435y_{4n+5} + 11064x_{2n+4} - 221740y_{2n+3} + 66]$
- $[60726x_{4n+6} - 55435y_{4n+4} + 242904x_{2n+4} - 221740y_{2n+4} + 6]$
- $\frac{1}{12}[126y_{4n+5} - 2766y_{4n+4} + 504y_{2n+3} - 11064y_{2n+2} + 72]$
- $\frac{1}{12}[2766y_{4n+6} - 60726y_{4n+5} + 11064y_{2n+4} - 242904y_{2n+3} + 72]$
- $\frac{1}{246}[126y_{4n+6} - 60726y_{4n+4} + 504y_{2n+4} - 242904y_{2n+2} + 1584]$

5. Each of the following expressions represents a Quintic integer

- $\frac{1}{10}[2525x_{5n+5} - 115x_{5n+6} + 12625x_{3n+3} - 575x_{3n+4} + 25250x_{n+1} - 1150x_{n+2}]$
- $\frac{1}{220}[55435x_{5n+5} - 115x_{5n+7} + 277175x_{3n+3} - 575x_{3n+5} + 554350x_{n+1} - 1150x_{n+3}]$
- $[126x_{5n+5} - 115y_{5n+5} + 630x_{3n+3} - 575y_{3n=3} + 1260x_{n+1} - 1150y_{n+1}]$
- $\frac{1}{241}[60726x_{5n+5} - 115y_{5n+7} + 303630x_{3n+3} - 575y_{3n+5} + 607260x_{n+1} - 1150y_{n+3}]$
- $\frac{1}{10}[55435x_{5n+6} - 2525x_{5n+7} + 277175x_{3n+4} - 12625x_{3n+5} + 554350x_{n=2} - 25250x_{n+3}]$
- $\frac{1}{11}[126x_{5n+6} - 2525y_{5n+5} + 630x_{3n+4} - 12625y_{3n+3} + 1260x_{n+2} - 25250y_{n+1}]$
- $[2766x_{5n+6} - 2525y_{5n+6} + 13830x_{3n+4} - 12625y_{3n+4} + 27660x_{n+2} - 25250y_{n+2}]$
- $\frac{1}{11}[60726x_{5n+6} - 2525y_{5n+7} + 303630x_{3n+4} - 12625y_{3n+5} + 607260x_{n+2} - 2525y_{n+3}]$
- $\frac{1}{241}[126x_{5n+7} - 55435y_{5n+5} + 630x_{3n+5} - 277175y_{3n+3} + 1260x_{n+3} - 554350y_{n+1}]$
- $\frac{1}{11}[2766x_{5n+7} - 55435y_{5n+6} + 13830x_{3n+5} - 277175y_{3n+4} + 27660x_{n+3} - 554350y_{n+2}]$
- $[60726x_{5n+7} - 55435y_{5n+7} + 303630x_{3n+5} - 277175y_{3n+5} + 607260x_{n+3} - 554350y_{n+3}]$
- $\frac{1}{12}[126y_{5n+6} - 2766y_{5n+5} + 630y_{3n+4} - 13830y_{3n+3} + 1260y_{n+2} - 276600y_{n+1}]$
- $\frac{1}{264}[126y_{5n+7} - 60726y_{5n+5} + 630y_{3n+5} - 303630y_{3n+3} + 1260y_{n+3} - 607260y_{n+1}]$
- $\frac{1}{12}[2766y_{5n+7} - 60726y_{5n+6} + 13830y_{3n+5} - 303630y_{3n+4} + 27660y_{n+3} - 607260y_{n+2}]$

REMARKABLE OBSERVATIONS:

1. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below

Table: 2 Hyperbola

S. No	Hyperbola	(P,Q)
1.	$P^2 - Q^2 = 400$	$P = 2525x_{n+1} - 115x_{n+2}$ $Q = \sqrt{30}[21x_{n+2} - 461x_{n+1}]$
2.	$P^2-Q^2=193600$	$P = 55435x_{n+1} - 115x_{n+3}$ $Q = \sqrt{30}[10121x_{n+1} - 21x_{n+3}]$
3.	$P^2-Q^2=4$	$P = 126x_{n+1} - 115y_{n+1}$ $Q = \sqrt{30}[21y_{n+1} - 23x_{n+1}]$
4.	$P^2 - Q^2 = 484$	$P = 2766x_{n+1} - 115y_{n+2}$ $Q = \sqrt{30}[21y_{n+2} - 505x_{n+1}]$
5.	$P^2-Q^2=232324$	$P = 60726x_{n+1} - 115y_{n+3}$ $Q = \sqrt{30}[21y_{n+3} - 11087x_{n+1}]$
6.	$P^2-Q^2= 400$	$P = 55435x_{n+2} - 2525x_{n+3}$ $Q = \sqrt{30}[461x_{n+3} - 10121x_{n+2}]$
7.	$P^2 - Q^2 = 484$	$P = 126x_{n+2} - 2525y_{n+1}$ $Q = \sqrt{30}[461y_{n+1} - 23x_{n+2}]$
8.	$P^2-Q^2=4$	$P=2766x_{n+2}-2525y_{n+2}$ $Q = \sqrt{30}[-505x_{n+2} + 461y_{n=2}]$
9.	$P^2-Q^2=484$	$P = 60726x_{n+2} - 2525y_{n+3}$ $Q = \sqrt{30}[461y_{n+3} - 11087x_{n+2}]$
10.	$P^2 - Q^2 = 232324$	$P = 126x_{n+3} - 55435y_{n+1}$ $Q = \sqrt{30}[10124y_{n+1} - 23x_{n=3}]$
11.	$P^2 - Q^2 = 484$	$P = 2766x_{n+3} - 55435y_{n+2}$ $Q = \sqrt{30}[10121y_{n+2} - 505x_{n+3}]$

12.	$P^2 - Q^2 = 4$	$P = 60726x_{n+3} - 55435y_{n+3}$ $Q = \sqrt{30}[10121y_{n+3} - 11087x_{n+3}]$
13.	$P^2 - Q^2 = 576$	$P = 126y_{n+2} - 2766y_{n+1}$ $Q = \sqrt{30}[505y_{n=1} - 23y_{n+2}]$
14.	$P^2 - Q^2 = 278784$	$P = 126y_{n+3} - 60726y_{n+1}$ $Q = \sqrt{30}[11087y_{n+1} - 23y_{n+3}]$
15.	$P^2 - Q^2 = 576$	$P = 2766y_{n+3} - 60726y_{n+2}$ $Q = \sqrt{30}[11087y_{n+2} - 505y_{n+3}]$

2. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table :3 Parabola

S. No	**Parabola**	**(R,Q)**
1.	$10R - Q^2 = 400$	$R = 2525x_{2n+2} - 115x_{2n+3} + 20$ $Q = \sqrt{30}[21x_{n+2} - 461x_{n+1}]$
2.	$220R-Q^2=193600$	$R = 55435x_{2n+2} - 125x_{2n+4} + 440$ $Q = \sqrt{30}[10121x_{n+1} + 21x_{n+3}]$
3.	$R-Q^2=4$	$R = 126x_{2n+2} - 115y_{2n+2} + 2$ $Q = \sqrt{30}[21y_{n+1} + 23x_{n+1}]$
4.	$11R-Q^2=484$	$R = 2766x_{2n+2} - 115y_{2n+3} + 22$ $Q = \sqrt{30}[21y_{n+2} - 505x_{n+1}]$
5.	$241R-Q^2=232324$	$R = 60726x_{2n+2} - 115y_{2n+4} + 482$ $Q = \sqrt{30}[21y_{n+3} - 11087x_{n+1}]$
6.	$10R - Q^2 = 400$	$R = 55435x_{2n+3} - 2525x_{2n+4} + 20$ $Q = \sqrt{30}[461x_{n+3} - 10121x_{n+2}]$
7.	$11R - Q^2 = 484$	$R = 126x_{2n+3} - 2525y_{2n+2} + 22$ $Q = \sqrt{30}[461y_{n+1} - 23x_{n+2}]$

8.	$R-Q^2=4$	$R=2766x_{2n+3}-2525y_{2n+3}+2$ $Q=\sqrt{30}[461y_{n+2}-505x_{n+2}]$
9.	$11R-Q^2=484$	$R=60726x_{2n+3}-2525y_{2n+4}+22$ $Q=\sqrt{30}[461y_{n+3}-11087x_{n+2}]$
10.	$121R-Q^2=232324$	$R=126x_{2n+4}-55435y_{2n+2}+482$ $Q=\sqrt{30}[10121y_{n+1}-23x_{n+3}]$
11.	$11R-Q^2=484$	$R=2766x_{2n+4}-55435x_{2n+3}+22$ $Q=\sqrt{30}[10121y_{n+2}-505x_{n+3}]$
12.	$R-Q^2=4$	$R=60726y_{2n+4}-55435x_{2n+4}+2$ $Q=\sqrt{30}[10121y_{n+3}-11087x_{n+3}]$
13.	$126R-Q^2=576$	$R=126y_{2n+3}-2766y_{2n+2}+24$ $Q=\sqrt{30}[505y_{n+1}-23y_{n+2}]$
14.	$264R-Q^2=278784$	$R=126y_{2n+4}-60726y_{2n+2}+528$ $Q=\sqrt{30}[11087y_{n+1}-23y_{n+3}]$
15.	$320R-Q^2=409600$	$R=2766y_{2n+4}-60726y_{2n+3}+24$ $Q=\sqrt{30}[11087y_{n+2}-505y_{n+3}]$

CONCLUSION:

In this paper, we have presented infinitely many integer solutions for the Diophantine equations represented by hyperbola is given $6x^2-5y^2=4$. As the binary quadratic

Diophantine equations are rich in variety, one may search for the other choices of equations and determine their solutions among the suitable properties.

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