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Observation on the Non-Homogeneous Binary Quadratic Diophantine Equation $7x^2 - 2y^2 = 40$

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ABSTRACT

The Quadratic Diophantine equation given by $7x^2 - 2y^2 = 40$ is studied for determining its infinitely many non-zero integral solutions. A few interesting properties among its solutions are given. Also, knowing an integral solution of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented. Also, employing the solutions of the given equation is constructed.

Key words: Binary quadratic, Hyperbola, Parabola, Pell equation, Integer solutions.

INTRODUCTION

The hyperbola represented by the Diophantine equations of the form $ax^2 + by^2 = N$, (a,b≠ 0) are rich in variety and have been analyzed by many mathematicians for their respective integer solutions for particular values of a, b and N. For an extensive review, one may refer[1-14].

This communication concerns with the problem of obtaining non-zero distinct integer solutions to the binary quadratic equation $7x^2 - 2y^2 = 40$given by representing hyperbola. A few interesting relations among its solutions are presented. Knowing an integral solutions of the given hyperbola, integer solutions for other choices of hyperbolas and parabolas are presented.

METHOD OF ANALYSIS

The Diophantine equation under consideration is

$$7x^2 - 2y^2 = 40 \tag{1}$$

It is to be noted that (1) represent a hyperbola

Taking,

$$x = X + 2T \;,\quad y = X + 7T \tag{2}$$

In (1), it reduces to the equation,

$$X^2 = 14T^2 + 8 \tag{3}$$

The smallest positive integer solution (T_0, X_0) of (3) is

$$T_0 = 2, X_0 = 8$$

To obtain, the other solutions of (3), consider the Pellian equation

$$X^2 = 14T^2 + 1 \tag{4}$$

Whose smallest positive integer solution is

$$\tilde{T}_0 = 4, \tilde{X}_0 = 15$$

The gerenal solution $\left(\tilde{T}_n, \tilde{X}_n\right)$ of (4) is given by,

$$\tilde{X}_n + \sqrt{14}\tilde{T}_n = (15 + 4\sqrt{14})^{n+1} \quad \text{, where n=0,1,2,…} \qquad (5)$$

Since, irrational roots occur in pairs, we have,

$$\tilde{X}_n - \sqrt{14}\tilde{T}_n = (15 - 4\sqrt{14})^{n+1} \quad \text{, where n=0,1,2,…} \qquad (6)$$

From (5) and (6) solving for $\left(\tilde{T}_n, \tilde{X}_n\right)$ we have,

$$\tilde{T}_n = \frac{1}{2\sqrt{14}} g_n ; \tilde{X}_n = \frac{1}{2} f_n$$

Where

$$f_n = \left(15 + 4\sqrt{14}\right)^{n+1} + \left(15 - 4\sqrt{14}\right)^{n+1},$$

$$g_n = \left(15 + 4\sqrt{14}\right)^{n+1} - \left(15 - 4\sqrt{14}\right)^{n+1}.$$

Applying Brahmagupta lemma between $\left(T_0, X_0\right)$ and $\left(\tilde{T}_n, \tilde{X}_n\right)$,the general integer solution $\left(T_{n+1}, X_{n+1}\right)$ of (3) are found to be

$$T_{n+1} = X_0 \tilde{T}_n + T_0 \tilde{X}_n$$

$$T_{n+1} = f_n + \frac{4}{\sqrt{14}} g_n \qquad (7)$$

$$X_{n+1} = X_0 \tilde{X}_n + D T_0 \tilde{T}_n$$

$$X_{n+1} = 4 f_n + \frac{14}{\sqrt{14}} g_n \qquad (8)$$

Using (7) & (8) in (2), we have

$$x_{n+1} = X_{n+1} + 2T_{n+1}$$

$$x_{n+1} = 6 f_n + \frac{22}{\sqrt{14}} g_n \qquad (9)$$

$$y_{n+1} = X_{n+1} + 7T_{n+1}$$

$$y_{n+1} = 11 f_n + \frac{42}{\sqrt{14}} g_n \qquad (10)$$

Thus, (9) and (10) represent the integer solutions of the hyperbola (1)

A few numerical examples are given in the following Table : 1

Table : 1 Numerical values

n	x_n	y_n
0	12	22
1	356	666
2	10,668	19,958
3	319,684	598,074
4	9,579,852	17,922,262

From the above table, we observe some interesting properties among the solutions which are presenting below:

i. x_n & y_n values are always even.

ii. $x_n \equiv 0(mod\ 2)$

iii. $x_n + y_n \equiv 0(mod\ 2)$

1. Relations between solutions

- $x_{n+1} - 30x_{n+2} + x_{n+3} = 0$
- $y_{n+1} - 30y_{n+2} + y_{n+3} = 0$
- $8y_{n+1} + 15x_{n+1} - x_{n+2} = 0$
- $8y_{n+2} + x_{n+1} - 15x_{n+2} = 0$
- $8y_{n+3} - 449x_{n+2} + 15x_{n+1} = 0$
- $240y_{n+1} - x_{n+3} + 449x_{n+1} = 0$
- $16y_{n+2} - x_{n+3} + x_{n+1} = 0$
- $240y_{n+3} - 449x_{n+3} + x_{n+1} = 0$
- $28x_{n+1} - y_{n+2} + 15y_{n+1} = 0$
- $840x_{n+1} - y_{n+3} + 449y_{n+1} = 0$
- $28x_{n+1} - 15y_{n+3} + 449y_{n+2} = 0$
- $8y_{n+1} - 15x_{n+3} + 449x_{n+2} = 0$
- $8y_{n+2} - x_{n+3} + 15x_{n+2} = 0$
- $8y_{n+3} - 15x_{n+3} + x_{n+2} = 0$
- $28x_{n+2} - 15y_{n+2} + y_{n+1} = 0$
- $56x_{n+2} - y_{n+3} + y_{n+1} = 0$
- $28x_{n+2} - y_{n+3} + 15y_{n+2} = 0$
- $y_{n+2} - 15y_{n+3} + 28x_{n+3} = 0$
- $y_{n+1} - 449y_{n+3} + 840x_{n+3} = 0$
- $28x_{n+3} - 449y_{n+2} + 15y_{n+1} = 0$

2. Each of the following expressions represents a Nasty Number

- $\frac{1}{80}\left(666x_{2n+2}-22x_{2n+3}+160\right)$
- $\frac{1}{2400}\left(19958x_{2n+2}-22x_{2n+4}+4800\right)$
- $\frac{1}{10}\left(42x_{2n+2}-22y_{2n+2}+20\right)$
- $\frac{1}{150}\left(1246x_{2n+2}-22y_{2n+3}+300\right)$
- $\frac{1}{4490}\left(37338x_{2n+2}-22y_{2n+4}+8980\right)$
- $\frac{1}{80}\left(19958x_{2n+3}-666x_{2n+4}+160\right)$
- $\frac{1}{150}\left(42x_{2n+3}-666y_{2n+2}+300\right)$
- $\frac{1}{10}\left(1246x_{2n+3}-666y_{2n+3}+20\right)$
- $\frac{1}{150}\left(37338x_{2n+3}-666y_{2n+4}+300\right)$
- $\frac{1}{4490}\left(42x_{2n+4}-19958y_{2n+2}+8980\right)$
- $\frac{1}{150}\left(1246x_{2n+4}-19958x_{2n+3}+300\right)$
- $\frac{1}{10}\left(37338x_{2n+4}-19958y_{2n+4}+20\right)$
- $\frac{1}{280}\left(42y_{2n+3}-1246y_{2n+2}+560\right)$
- $\frac{1}{8400}\left(42y_{2n+4}-37338y_{2n+2}+16800\right)$
- $\frac{1}{280}\left(1246y_{2n+4}-37338y_{2n+3}+560\right)$

3. Each of the following expressions represents a Cubical Integer

- $\frac{1}{80}[666x_{3n+3}-22x_{3n+4}+1998x_{n+1}-66x_{n+2}]$

- $\frac{1}{2400}[19958x_{3n+3}-22x_{3n+5}+59874x_{n+1}-66x_{n+3}]$
- $\frac{1}{10}[42x_{3n+3}-22y_{3n+3}+126x_{n+1}-66y_{n+1}]$
- $\frac{1}{150}[1246x_{3n+3}-22y_{3n+4}+3738x_{n+1}-66y_{n+2}]$
- $\frac{1}{4490}[37338x_{3n+3}-22y_{3n+5}+112014x_{n+1}-66y_{n+3}]$
- $\frac{1}{80}[19958x_{3n+4}-666x_{3n+5}+59874x_{n+2}-1998x_{n+3}]$
- $\frac{1}{150}[42x_{3n+4}-666y_{3n+3}+126x_{n+2}-1998y_{n+1}]$
- $\frac{1}{10}[1246x_{3n+4}-666y_{3n+4}+3738x_{n+2}-1998y_{n+2}]$
- $\frac{1}{150}[37338x_{3n+4}-666y_{3n+5}+112014x_{n+2}-1998y_{n+3}]$
- $\frac{1}{4490}[42x_{3n+5}-19958y_{3n+3}+126x_{n+3}-59874y_{n+1}]$
- $\frac{1}{150}[1246x_{3n+5}-19958y_{3n+4}+3738x_{n+3}-59874y_{n+2}]$
- $\frac{1}{10}[37338x_{3n+5}-19958y_{3n+5}+112014x_{n+3}-59874y_{n+3}]$
- $\frac{1}{280}[42y_{3n+4}-1246y_{3n+3}+126y_{n+2}-3738y_{n+1}]$
- $\frac{1}{8400}[42y_{3n+5}-37338y_{3n+3}+126y_{n+3}-112014y_{n+1}]$
- $\frac{1}{280}[1246y_{3n+5}-37338y_{3n+4}+3738y_{n+3}-112014y_{n+2}]$

4. Each of the following expression represents a Bi-quadratic Integer

- $\frac{1}{80}[666x_{4n+4}-22x_{4n+5}+2664x_{2n+2}-88x_{2n+3}+480]$
- $\frac{1}{2400}[19958x_{4n+4}-22x_{4n+6}+79832x_{2n+2}-88x_{2n+4}+14400]$
- $\frac{1}{10}[42x_{4n+4}-22y_{4n+4}+168x_{2n+2}-88y_{2n+2}+60]$

- $\frac{1}{150}[1246x_{4n+4} - 22y_{4n+5} + 4984x_{2n+2} - 88y_{2n+3} + 600]$
- $\frac{1}{4490}[37338x_{4n+4} - 22y_{4n+6} + 149352x_{2n+2} - 88y_{2n+4} + 26940]$
- $\frac{1}{80}[19958x_{4n+5} - 666x_{4n+6} + 79832x_{2n+3} - 2664x_{2n+4} + 480]$
- $\frac{1}{150}[42x_{4n+5} - 666y_{4n+4} + 168x_{2n+3} - 2664y_{2n+2} + 900]$
- $\frac{1}{10}[1246x_{4n+5} - 666y_{4n+5} + 4984x_{2n+3} - 2664y_{2n+3} + 60]$
- $\frac{1}{150}[37338x_{4n+5} - 666y_{4n+6} + 149352x_{2n+3} - 2664y_{2n+4} + 900]$
- $\frac{1}{4490}[42x_{4n+6} - 19958y_{4n+4} + 168x_{2n+4} - 79832y_{2n+2} + 26940]$
- $\frac{1}{150}[1246x_{4n+6} - 19958y_{4n+5} + 4984x_{2n+4} - 79832y_{2n+3} + 900]$
- $\frac{1}{10}[37338x_{4n+6} - 19958y_{4n+6} + 149352x_{2n+4} - 79832y_{2n+4} + 60]$
- $\frac{1}{280}[42y_{4n+5} - 1246y_{4n+4} + 168y_{2n+3} - 4984y_{2n+2} + 1680]$
- $\frac{1}{8400}[42y_{4n+6} - 37338y_{4n+4} + 168y_{2n+4} - 149352y_{2n+2} + 50400]$
- $\frac{1}{280}[1246y_{4n+6} - 37338y_{4n+5} + 4984y_{2n+4} - 149352y_{2n+3} + 1680]$

5. Each of the following expressions represents a Quintic Integer

1. $\frac{1}{80}[666x_{5n+5} - 22x_{5n+6} + 3330x_{3n+3} - 110x_{3n+4} + 6660x_{n+1} - 220x_{n+2}]$
2. $\frac{1}{2400}[19958x_{5n+5} - 22x_{4n+7} + 99790x_{3n+3} - 110x_{3n+5} + 199580x_{n+1} - 220x_{n+3}]$
3. $\frac{1}{10}[42x_{5n+5} - 22y_{5n+5} + 210x_{3n+3} - 110y_{3n+3} - 220y_{n+1} + 420x_{n+1}]$
4. $\frac{1}{150}[1246x_{5n+5} - 22y_{5n+6} + 6230x_{3n+3} - 110y_{3n+4} + 12460x_{n+1} - 220y_{n+2}]$

5. $\frac{1}{4490}[37338x_{5n+5}-22y_{5n+7}+186690x_{3n+3}-110y_{3n+5}+37338x_{n+1}-220y_{n+3}]$

6. $\frac{1}{80}[19958x_{5n+6}-666x_{5n+7}+99790x_{3n+4}-3330x_{3n+5}+199580x_{n+2}-6660x_{n+3}]$

7. $\frac{1}{150}[42x_{5n+6}-666y_{5n+5}+210x_{3n+4}-3330y_{3n+3}+420x_{n+2}-6660y_{n+1}]$

8. $\frac{1}{10}[1246x_{5n+6}-666y_{5n+6}+6230x_{3n+4}-3330y_{3n+4}+12460x_{n+2}-6660y_{n+2}]$

9. $\frac{1}{150}[37338x_{5n+6}-666y_{5n+7}+186690x_{3n+4}-3330x_{3n+5}+373380x_{n+2}-6660y_{n+3}]$

10. $\frac{1}{4490}[42x_{5n+7}-19958y_{5n+5}+210x_{3n+5}-99790y_{3n+3}+420x_{n+3}-199580y_{n+1}]$

11. $\frac{1}{150}[1246x_{5n+7}-19958y_{5n+6}+6230x_{3n+5}-99790y_{3n+4}+12460x_{n+3}-199580y_{n+2}]$

12. $\frac{1}{10}[37338x_{5n+7}-19958y_{5n+7}+186690x_{3n+5}-99790y_{3n+5}+373380x_{n+3}-199580y_{n+3}]$

13. $\frac{1}{280}[42y_{5n+6}-1246y_{5n+5}+210y_{3n+4}-6230y_{3n+3}+420y_{n+2}-12460y_{n+1}]$

14. $\frac{1}{8400}[42y_{5n+7}-37338y_{5n+5}+210y_{3n+5}-186690y_{3n+3}+420y_{n+3}-373380y_{n+1}]$

15. $\frac{1}{280}[1246y_{5n+7}-37338y_{5n+6}+6230y_{3n+5}-186690y_{3n+4}+12460y_{n+3}-373380y_{n+2}]$

REMARKABLE OBSERVATIONS:

1.Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of hyperbola which are presented in the Table: 2 below

Table: 2 Hyperbola

S. No	**Hyperbola**	**(P,Q)**
1.	$P^2-Q^2=25600$	$P=666x_{n+1}-22x_{n+2}$ $Q=\sqrt{14}\left[6x_{n+2}-178x_{n+1}\right]$
2.	$P^2-Q^2=23040000$	$P=19958x_{n+1}-22x_{n+3}$ $Q=\sqrt{14}\left[6x_{n+3}-5334x_{n+1}\right]$
3.	$P^2-Q^2=400$	$P=42x_{n+1}-22y_{n+1}$ $Q=\sqrt{14}\left[6y_{n+1}-11x_{n+1}\right]$

4.	$P^2 - Q^2 = 90000$	$P = 1246x_{n+1} - 22y_{n+2}$ $Q = \sqrt{14}[6y_{n+2} - 333x_{n+1}]$
5.	$P^2 - Q^2 = 80640400$	$P = 37338x_{n+1} - 22y_{n+3}$ $Q = \sqrt{14}[6y_{n+3} - 9979x_{n+1}]$
6.	$P^2 - Q^2 = 25600$	$P = 19958x_{n+2} - 666x_{n+3}$ $Q = \sqrt{14}[178x_{n+3} - 5334x_{n+2}]$
7.	$P^2 - Q^2 = 90000$	$P = 42x_{n+2} - 666y_{n+1}$ $Q = \sqrt{14}[178y_{n+1} - 11x_{n+2}]$
8.	$P^2 - Q^2 = 400$	$P = 1246x_{n+2} - 666y_{n+2}$ $Q = \sqrt{14}[178y_{n+2} - 333x_{n+2}]$
9.	$P^2 - Q^2 = 90000$	$P = 37338x_{n+2} - 666y_{n+3}$ $Q = \sqrt{14}[178y_{n+3} - 9979x_{n+2}]$
10.	$P^2 - Q^2 = 80640400$	$P = 42x_{n+3} - 19958y_{n+1}$ $Q = \sqrt{14}[5334y_{n+1} - 11x_{n+3}]$
11.	$P^2 - Q^2 = 90000$	$P = 1246x_{n+3} - 19958y_{n+2}$ $Q = \sqrt{14}[5334y_{n+2} - 333x_{n+3}]$
12.	$P^2 - Q^2 = 400$	$P = 37338x_{n+3} - 19958y_{n+3}$ $Q = \sqrt{14}[5334y_{n+3} - 9979x_{n+3}]$
13.	$P^2 - Q^2 = 313600$	$P = 42y_{n+2} - 1246y_{n+1}$ $Q = \sqrt{14}[333y_{n+1} - 11y_{n+2}]$
14.	$P^2 - Q^2 = 282240000$	$P = 42y_{n+3} - 37338y_{n+1}$ $Q = \sqrt{14}[9979y_{n+1} - 11y_{n+3}]$
15.	$P^2 - Q^2 = 313600$	$P = 1246y_{n+3} - 37338y_{n+2}$ $Q = \sqrt{14}[9979y_{n+2} - 333y_{n+3}]$

2. Employing linear combinations among the solutions of (1), one may generate integer solutions for other choices of parabola which are presented in the Table: 3 below:

Table :3 Parabola

S. No	Parabola	(R,Q)
1.	$80R - Q^2 = 25600$	$R = 666x_{2n+2} - 22x_{2n+3} + 160$ $Q = \sqrt{14}[6x_{n+2} - 178x_{n+1}]$
2.	$2400R - Q^2 = 23040000$	$R = 19958x_{2n+2} - 22x_{2n+4} + 4800$ $Q = \sqrt{14}[6x_{n+3} - 5334x_{n+1}]$
3.	$10R - Q^2 = 400$	$R = 42x_{2n+2} - 22y_{2n+2} + 20$ $Q = \sqrt{14}[6y_{n+1} - 11x_{n+1}]$
4.	$150R - Q^2 = 90000$	$R = 1246x_{2n+2} - 22y_{2n+3} + 300$ $Q = \sqrt{14}[6y_{n+2} - 333x_{n+1}]$
5.	$4490R - Q^2 = 80640400$	$R = 37338x_{2n+2} - 22y_{2n+4} + 8980$ $Q = \sqrt{14}[6y_{n+3} - 9979x_{n+1}]$
6.	$80R - Q^2 = 25600$	$R = 19958x_{2n+3} - 666x_{2n+4} + 160$ $Q = \sqrt{14}[178x_{n+3} - 5334x_{n+2}]$
7.	$150R - Q^2 = 90000$	$R = 42x_{2n+3} - 666y_{2n+2} + 300$ $Q = \sqrt{14}[178y_{n+1} - 11x_{n+2}]$
8.	$10R - Q^2 = 400$	$R = 1246x_{2n+3} - 666y_{2n+3} + 20$ $Q = \sqrt{14}[178y_{n+2} - 333x_{n+2}]$
9.	$150R - Q^2 = 90000$	$R = 37338x_{2n+3} - 666y_{2n+4} + 300$ $Q = \sqrt{14}[178y_{n+3} - 9979x_{n+2}]$
10.	$4490R - Q^2 = 80640400$	$R = 42x_{2n+4} - 19958y_{2n+2} + 8980$ $Q = \sqrt{14}[5334y_{n+1} - 11x_{n+3}]$
11.	$150R - Q^2 = 90000$	$R = 1246x_{2n+4} - 19958y_{2n+3} + 300$ $Q = \sqrt{14}[5334y_{n+2} - 333x_{n+3}]$
12.	$10R - Q^2 = 400$	$R = 37338x_{2n+4} - 19958y_{2n+4} + 20$ $Q = \sqrt{14}[5334y_{n+3} - 9979x_{n+3}]$

13.	$280R - Q^2 = 313600$	$R = 42y_{2n+3} - 1246y_{2n+2} + 560$ $Q = \sqrt{14}[333y_{n+1} - 11y_{n+2}]$
14.	$8400R - Q^2 = 282240000$	$R = 42y_{2n+4} - 37338y_{2n+2} + 16800$ $Q = \sqrt{14}[9979y_{n+1} - 11y_{n+3}]$
15.	$280R - Q^2 = 313600$	$R = 1246y_{2n+4} - 37338y_{2n+3} + 560$ $Q = \sqrt{14}[9979y_{n+2} - 333y_{n+3}]$

CONCLUSION:

In this paper, we have presented infinitely many integer solutions for the Diophantine equations represented by hyperbola is given $7x^2 - 2y^2 = 40$. As the binary quadratic Diophantine equations are rich in variety, one may search for the other choices of equations and determine their solutions among the suitable properties.

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