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Visualization: Qubit, θ Degree and Proportional Probability Amplitude

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A B S T R A C T

This study delves into the representation of quantum bits (qubits) on the complex plane, investigating the intricate relationship between qubit states, probability, and probability amplitude. Our goal, through both geometric and mathematical analyses, is to ascertain the amplitudes required for the effective manipulation of qubits in quantum computational tasks. Furthermore, we seek to explore the entire spectrum of potential states denoted by θ (polar angle) and their corresponding probability amplitudes regardless azimuth angle φ. (1) (2)

In quantum computing, the superposition of qubits can be visualized on the Bloch Sphere, and probability can be calculated using trigonometry, depicting a unit-radius qubit on the complex plane. However, this hypothesis suggests a novel approach wherein specific probability amplitudes for θ degrees can be assigned within the spectrum, representing the rectangular form of a qubit, where $\alpha + i\beta = 10$.

Keywords: θ-degree argument, Probability Amplitude, Quantum Computing, Qubit Representation on Complex Plane, Qubit Visualization

1. Introduction

The fundamental building block of classical computational devices is a two-state system, representing zeros and ones. Qubits represent a pivotal departure from classical binary systems, and their unique characteristics open new vistas for computation and data processing. However, in the realm of quantum computing, a particle or qubit can exist in a superposition of states—simultaneously manifesting as an electromagnetic wave (EM wave) or a particle. But upon measurement, a qubit's state collapses, yielding only 0 or 1 outcomes, and its superposition gets measured based on multiple measurements. Controlled transitions between these two states are essential for quantum computation. (1)

Generally, qubit states are expressed as $\alpha + i\beta = 1$, with α and β as probability amplitudes satisfying $\alpha^2 + \beta^2 = 1$. In the complex plane with a unit radius, the real part (α) aligns with the x-axis, and the imaginary part (β) aligns with the y-axis, perpendicular to each other. In the Bloch sphere representation, the surface of a sphere with a unit radius corresponds to the set of all possible states of a qubit. The two angles, denoted as the polar angle θ and the azimuth angle φ, are used to parameterize the qubit's state on the sphere. (1) (2)

Our primary focus is on the methodology of calculating proportions, probabilities, percentages, and complex numbers in mathematics. Based on this foundation, we aim to establish a relationship to determine the probability amplitude. Utilizing a geometric approach, we project qubits onto the complex plane using established methodology. By connecting the qubit's polar representation to fundamental geometric constants, such as π, we aim to gain insights into effective representation and determine the amplitude to choose during quantum computational tasks for manipulating qubits.

2. Correlation between proportion, probability, percentage and the Pythagorean theorem:

1. **Proportion:** A proportion is an equation that defines two given ratios as equivalent to each other. In other words, the proportion asserts the equality of two fractions or ratios.
2. **Probability and Percentage:** In mathematics, probability is a measure of the likelihood that a particular event will occur. It is expressed as a number between 0 and 1, where 0 indicates that the event will not occur, and 1 indicates that the event will occur. The probability of an event A is denoted by P(A). The formula for probability is: P(A) = (number of favorable outcomes) / (total number of possible outcomes). Building upon the established method of calculating probability as a percentage—achieved by multiplying the fraction by 100.
3. **Pythagorean theorem and Percentage in qubit representation:** In the context of the complex plane, a qubit is represented with a unit radius. Qubit states are expressed as $\alpha + i\beta = 1$, where α and β are probability amplitudes satisfying $\alpha^2 + \beta^2 = 1$. The Pythagorean Theorem, expressed as $a^2 + b^2 = c^2$, is a foundational principle in geometry. It allows us to find the length of the hypotenuse (c) in a right triangle, given the lengths of its two legs (a and b). If we divide the equation $a^2 + b^2 = c^2$ by c^2, we obtain $(a/c)^2 + (b/c)^2 = 1$, representing 100%. (3)

3. Hypothesis:

1. Electromagnetic wave and Amplitude:

An electromagnetic (EM) wave is composed of electric and magnetic fields. In EM waves, the amplitude represents the maximum strength of these fields. The wave's energy is determined by its amplitude, with constructive interference occurring when two waves overlap to create a larger wave, and destructive interference occurring when they cancel each other out. (1)

2. Current methodology to assign amplitude:

$$\left|\cos\frac{\theta}{2}\right|^2 + \left|\sin\frac{\theta}{2}\right|^2 = 1$$

This formula is utilized to determine the amplitude, where θ represents the polar angle of the qubit. (1)

3. Associated Amplitude:

In the realm of quantum mechanics, the likelihood of a system existing in a particular state 'x' is determined by the square of its amplitude, represented by probability(x) = $|\text{amplitude}(x)|^2$. This fundamental principle emphasizes that probability equals the square of the amplitude's magnitude. (1)

Imagine a scenario where qubit states '0' and '1' are evenly spread across the Bloch sphere. Their respective amplitudes, cos 45° and sin 45° (equivalent to 0.7071), when squared, yield 0.5. This value is shared by both qubit states. Converting this fraction into a percentage (by multiplying by 100), the hypothesis suggests that the sum of squared amplitudes (α^2 and β^2) for qubit states should equate to the real number '10^2' in trigonometric functions, indicating a probability of 100%. This implies that '100' encompasses a range of numbers symbolizing finite probabilities, encompassing all possible outcomes during qubit manipulation.

In contrast to conventional quantum mechanics, which states that the total probability for mutually exclusive states ('0' and '1') equals 1, representing 100%, this hypothesis introduces a speculative framework. Within this theoretical construct, it proposes that the sum of squared amplitudes (α^2 and β^2) for '0' and '1' should amount to 100.

4. Equation of a Circle, Pythagorean Triples and the need of a complex plane:

The equation $x^2 + y^2 = r^2$ represents the equation of a circle in a two-dimensional Cartesian coordinate system. In this equation, 'x' and 'y' are coordinates on the plane, and 'r' is the radius of the circle. It defines all the points that are at a distance 'r' from the center of the circle. (3)

- **a.** By using the first quarter of a circle with a 10-unit radius on the plane, a set of many numbers could be computed that satisfies the condition that the sum of $x^2 + y^2$ (or α^2 and β^2) would be equals to $(10)^2$ or 100 units.
- **b.** A Pythagorean triple consists of three positive integers a, b, and c, such that $a^2 + b^2 = c^2$. Such a triple is commonly written (a, b, c), and a well-known example is (3, 4, 5). By using the number (6,8,10) also, α and β value can be interpreted. (3)

However, since both the α and β are complex numbers, it is appropriate to visualize the number '10' as a real number but also associated with complex numbers on the complex plane for determine the best possible values for α and β using imaginary part, geometry and trigonometric function.

4. Components for visualizing probability amplitude unit on complex plane

1. Complex Number:

10*i*										0
9*i*									0	√19
8*i*								0	√17	**6**
7*i*							0	√3·√5	4√2	√3·√17
6*i*						0	√13	2√7	3√5	**8**
5*i*					0	√11	2√2·√3	√3·√13	2√2·√7	5√3
4*i*				0	3	2√5	√3·√11	4√3	√5·√13	2√3·√7
3*i*			0	√7	4	3√3	2√2·√5	√5·√11	3√2	√7·√13
2*i*		0	√5	2√3	√3·√7	4√2	3√5	2√3·√5	√7·√11	4√6
i	0	√3	2√2	√3·√5	2√2·√3	√5·√7	4√3	3√7	4√5	3√11
0	1	2	3	4	5	6	7	8	9	**10**

Figure 1: Integers as complex numbers using the principle of Pythagorean theorem

Complex numbers are expressed in the form a + *i*b, where 'a' and 'b' are real numbers, and '*i*' represents the imaginary unit. This unit is notable for its property as the square root of -1, denoted as '$i = \sqrt{-1}$', with 'i^2' equaling -1. (1)

2. Representing integers as complex numbers using the principle of Pythagorean theorem:

A graphical representation, such as the chart shown in Figure 1, visually demonstrates the consequences of squaring both real and imaginary numbers.

In the representation, real numbers are plotted horizontally along the x-axis, while imaginary numbers (which become negated upon squaring) are represented vertically along the y-axis. The cyclic relationship between squaring and taking square roots is intriguing; for example, it involves converting the sum value back to a square root value. Using this table makes it easier to relate the sum of squared values for both positive integers and imaginary parts, which could be associated with a number. Indeed, the numbers 6, 8, and 10 form a Pythagorean triple, constituting a right triangle.

In the context of complex numbers, this relationship can be expressed as: $6 = \sqrt{10^2 + (8i)^2}$ and $8 = \sqrt{10^2 + (6i)^2}$.

5. Visualizing probability amplitude unit on complex plane

1. Polar form of complex number:

The form z = a + bi is the rectangular form of a complex number, where (a, b) are the rectangular coordinates. Where: z is the complex number. a is the real part and b is the imaginary part. To represent a complex number in polar form, we use the formula: z = r (cos θ + $i$sin θ). In this form: r is the magnitude of the complex number, which is the distance from the origin (0,0) to the complex number, z in the complex plane. It is given by: $r=|z|= a^2 + b^2$ and θ is the angle (or argument) of the complex number z with respect to the positive real axis, measured in radians. It's calculated as: θ=arctan(a/b).

2. Vector Selection:

In quantum mechanics, the likelihood of a system existing in a particular state 'x' is determined by the square of its amplitude. So, when visualizing probability amplitudes (α and β) on the complex plane, as detailed in sections '3.3', '3.4', and '4.2', we explore Pythagorean Triples such as 6, 8, and 10. This exploration involves representing 6 as $\sqrt{10^2 + (8i)^2}$ and 8 as $\sqrt{10^2 + (6i)^2}$. These expressions illustrate the possibility to express α and β as probability amplitudes on the complex plane, offering insights into the intriguing relationship between quantum states and Pythagorean Triples associated with the real number 10, which is the square root of the 100.

3. Representation through Geometric Transformations:

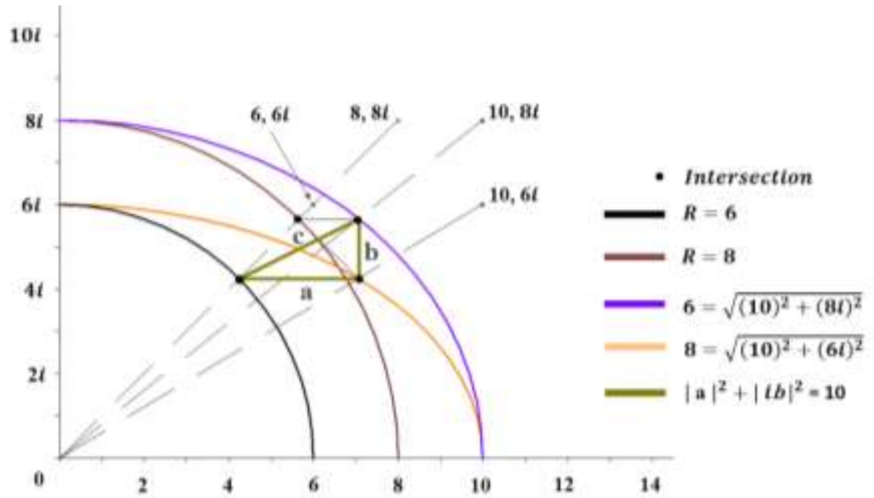

Figure 2: Representation of ‘10’ as a complex number

With the insights gained from the aforementioned topics, amplitude radius representation can be achieved through a shear transformation in a two-dimensional space. By connecting the Pythagorean theorem within the framework of circles to the polar representation of complex numbers and assigning the real and complex components, the numerical values 6, 8, and 10 find their place in their rectangular form on the complex plane.

4. Incorporating π and Complex Numbers in Geometric Calculations:

In the representation shown in Figure 1, we have incorporated the mathematical constant π, which signifies the ratio of a circle's circumference to its diameter (approximately equal to 3.14159). This constant is widely employed in calculating the circumference and area of a circle, as well as the area and volume of a sphere. In our depiction, we draw the first quarter of an arc and an ellipse, aligning them on the real and imaginary axes with the rectangular form of individual complex numbers taken into account.

a. Arcs:

In the diagram on the complex plane, the black-colored arc represents an arc with a 6-unit radius, while the brown-colored arc represents an arc with an 8-unit radius. Both arcs originate from point zero on both axes.

b. Elliptical Splines:

In the representation, we equate 6 to $6 = \sqrt{10^2 + (8i)^2}$. The magenta-colored elliptical spline illustrates the first quarter of an ellipse with a 10-unit major radius along the x-axis, representing the real part of the complex number, and an 8-unit minor radius along the y-axis, representing the imaginary part. Similarly, for the representation where we equate 8 to $8 = \sqrt{10^2 + (6i)^2}$, we depict it with a pink-colored elliptical spline. Its major radius is 10 units along the real axis, and the radius is 6 units on the imaginary axis. Both ellipses are centered at the point (0,0).

c. Dotted lines and Points:

Among the three dotted lines, one covers the magnitude of a 6-unit and 8-unit radius arc, while the other two represent the magnitudes of their respective complex numbers: $6 = \sqrt{10^2 + (8i)^2}$ and $8 = \sqrt{10^2 + (6i)^2}$. We have identified four points, marked by four black dots, where two arcs and two elliptical splines intersect the line that defines their polar form.

d. Right Tringles:

After connecting these four points, three right triangles are formed with side lengths √2:√2:2, 2:2:2√2, and √2:2√2:√10. However, we have highlighted a particular triangle in dark gray, where the sum of the squares of its two legs equals 10, aligning with the topic discussed in '3. 3. Hypothesis.' This specific triangle represents the interpreted amplitude for manipulating qubit states, with its sides designated as a, b, and c based on the lengths representing the ground state and excited state of the qubit. By utilizing geometric transformations, this application allows us to illustrate how these values interact and evolve within the given mathematical framework. This method aligns with the broader visualization approach employed for representing qubits on the complex plane.

5. Readings and Factor:

As per section '6.3. Representation through Geometric Transformations' regarding measurements on the graph, the values are located as follows:

a ≈ -2√2, b ≈ √2, c ≈ √10. To align these with the condition Probability (100%) = |amplitude (10/10) $|^2$, by multiplying the factor √10 to both components (a and b), the values of α and β can be calculated.

- **Real part: α** ≈ a · √10 ≈ -2√2 · √10 ≈ -2√20 ≈ **-4√5** ≈ -8.94427191
- **Imaginary part: β** ≈ b · √10 ≈ √2 · √10 ≈ √20 ≈ **2√5** ≈ 4.472135955

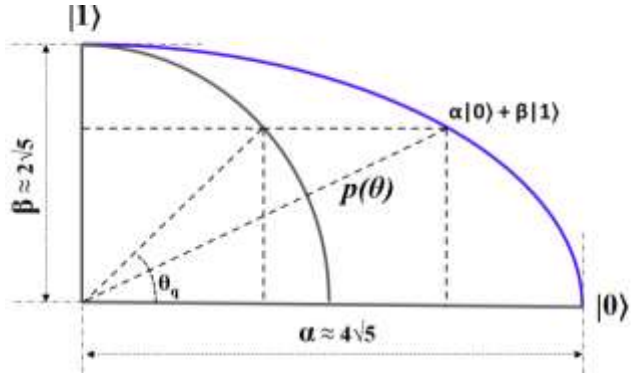

Figure 3: Polar form of Qubit

6. Polar form of qubit

Drawing from references in '5.5. Readings and Factor' and employing the polar form, when considering the absolute value of a complex number to determine the probability amplitude, it appears feasible to establish a geometrical framework on the plane. This framework, as illustrated in Figure 3, not only encompasses the fundamental 0 and 1 states but also spans the entire spectrum of potential θ states that can exist between 0 and 1 with the help of shear transformation. (1) (2) (4)

1. **Arc radius:** In the complex plane diagram, the black arc signifies an arc with a 2√5-unit radius centered at the origin (point '0') on both axes.
2. **θ_q:** In the context of the Bloch sphere, θq represents the angle that denotes the qubit's θ (polar angle) on the plane, specifically related to the qubit's z-axis. In this representation, the angle θq = θ/2.

3. **Elliptical Spline:** The blue-colored elliptical spline illustrates the first quarter of an ellipse with a 4√5-unit major radius along the x-axis and a 2√5-unit minor radius along the y-axis. This elliptical shape is centered at the origin (0,0).

4. **Probability:** Probability can be calculated using trigonometry by representing 2√5 unit-radius qubit on the plane.

5. **Probability Amplitude:** When 2·θq equals θ, then based on the current methodology to assign amplitude, the probability amplitude for θ (polar angle) can be mathematically expressed as: $\mathbf{p(\theta)} = \sqrt{(|\mathbf{sin}\frac{\theta}{2}| \cdot \mathbf{4\sqrt{5}})^2 + (|\mathbf{sin}\frac{\theta}{2}| \cdot \mathbf{2\sqrt{5}})^2}$

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